Power Estimation for Piezoelectric Energy Harvesters in Flexure Mode with Large Displacement Amplitude

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Abstract. We propose a new method to estimate the output power from piezoelectric energy harvesters using flexure mode of the cantilever. In the energy harvester, displacement amplitude is too large for Bernoulli-Euler hypothesis to hold true. Hence, it is not easy to derive a theoretical solution of output power. In this study, applying the correction coefficient which is a function of flexure rigidity to a conventional theoretical solution, simple equation to estimate the output power was derived. This equation was applied to design a practical (K,Na)NbO_3 based energy harvester. Measured output from the harvester was in good agreement with the calculated value.

1. Introduction

Recently, cantilever type micro generators using piezoelectric material are actively researched to harvest the energy from environmental vibrations [1], and spotlighted as a power source for various applications, such as tire pressure monitoring systems [2] or implantable medical devices [3]. For these applications, it is required for the harvester to simultaneously meet many specifications, which are size, height, resonant frequency and output power. However, a theoretical solution to relate these specifications is not established well. In the micro energy harvester, since amplitude of vibration is too large to apply Bernoulli-Euler hypothesis, deriving the solution from the strain in the piezoelectric film becomes complicated. Therefore we have to spend much time for numerical calculation to design such harvester.

In this study, applying the correction coefficient which is a function of flexure rigidity to a conventional theoretical solution, simple equation to estimate the output power was derived. Then, the derived equation was applied to design a practical (K,Na)NbO_3 based energy harvester as a verification.

2. Theoretical solution of output power

Figure 1 shows a schematic illustration of the cantilever based piezoelectric harvester. In the harvester, induced charge in the piezoelectric film can be calculated from a following equation,

$$Q = b\int_0^L e_{31}S_x + \varepsilon_{33}E_z dx = \frac{bhe_{31}}{2}\frac{\partial^2 F}{\partial x^2}\bigg|_{x=L} - bL\varepsilon_{33}\frac{V}{d} \quad (1),$$

where, S_x is strain in x-direction, E_z is electric field in z-direction, and F is a mode function of cantilever. Assuming a fundamental mode, the mode function F is as below:

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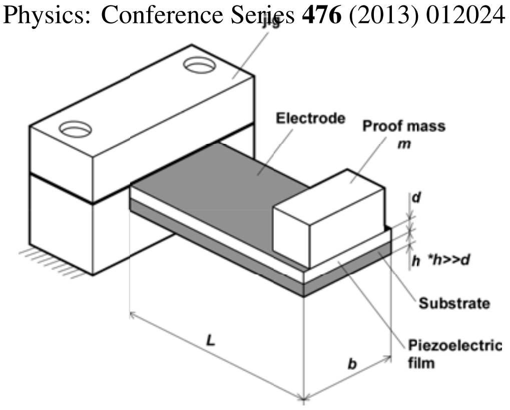


Figure 1. Schematic of cantilever based piezoelectric energy harvester

$$F(x) = C_0\left[K\left(\cosh\frac{\lambda}{L}x - \cos\frac{\lambda}{L}x\right) + \left(\sinh\frac{\lambda}{L}x - \sin\frac{\lambda}{L}x\right)\right] \quad (2),$$

where, K and λ are -1.3622 and 1.875, respectively. C_0 is also constant determined with vibration conditions. In this equation, a term of time $e^{-j\omega t}$ was omitted.

When the harvester is connected to the external load R, output power P can be obtained using following relation:

$$P = RI^2 = R\left(\frac{\partial Q}{\partial t}\right)^2 \quad (3).$$

Substituting (1) and (2) into (3) and using following relations

$$\text{Ohm's low: } V = RI, \quad \text{Matching condition: } R = \frac{1}{\omega C} = \frac{d}{\omega\varepsilon_{33}bL} \quad (4)(5),$$

P is derived as

$$P = \frac{1}{32}\frac{\omega bdh^2e_{31}^2}{\varepsilon_{33}L}C_0^2\left[K\frac{\lambda}{L}(\sinh\lambda + \sin\lambda) + \frac{\lambda}{L}(\cosh\lambda + \cos\lambda)\right]^2 \quad (6).$$

3. Introduction of correction coefficients

Table 1 shows a performance of cantilever based energy harvester reported from other researchers [4-9]. The constant C_0 and displacement of the top of cantilever $F(L)$ were calculated from (2) and (6). In this figure, C_0 and $F(L)$ were overestimated by using Eqs. (6). This overestimation was noticeable in the smaller harvester. This is interpreted as that Bernoulli-Euler hypothesis was broken by deformation of the mode function with large displacement amplitude in the micro device. We must reconstruct the output power equations. However, it is complicated and not so easy to derive the strain distribution under the large displacement amplitude without Bernoulli-Euler hypothesis.

In this study, we try to empirically expect the output power. In Fig. 2, the C_0 was normalized with vibration condition and plotted for the flexure rigidity EI. From this figure, it was found that plotted points were ordered on one line. Using this linear relation, Eqs. (6) can be modified as below:

$$P = \frac{1}{32}\frac{\omega bdh^2e_{31}^2}{\varepsilon_{33}L}\left\{m_{eq}aL^3A(EI)^B\right\}^2\left[K\frac{\lambda}{L}(\sinh\lambda + \sin\lambda) + \frac{\lambda}{L}(\cosh\lambda + \cos\lambda)\right]^2 \quad (7),$$

where, A and B are approximation coefficients obtained from Fig. 2, a and m_{eq} are a vibration acceleration and an equivalent mass, respectively.

Table 1. Performance of cantilever based piezoelectric energy harvester

Author	*Material*	f_r (kHz)	a (m/s^2)	m_{eq} (mg)	P (μW)	L (mm)	EI (μNm^2)	C_0	$F(L)$ (mm)
M. Marzencki *et. al.* [4]	AlN/Si	1500	19.6	0.48	0.45	0.52	1.08×10^{-3}	1.48×10^{-5}	106
H. Fang *et. al.* [5]	PZT/Si	609	9.8	1.61	2.16	1.7	3.79×10^{-2}	1.22×10^{-5}	26.9
D. Shen *et. al.*[6]	PZT/Si	463	19.6	1.37	2.15	3.2	0.186	2.90×10^{-5}	34.0
R. Elfrink *et. al.*[7]	AlN/Si	573	9.8	35.4	30.0	3.5	4.94	1.69×10^{-4}	181
Z Cao *et. al.*[8]	AlN /SUS	74.1	9.8	1010	5.13	4.2	10.1	1.94×10^{-4}	173
I. Kanno *et. al.*[9]	KNN/Si	1036	10	6.16	1.1	16.5×10^3	295	5.45×10^{-5}	2.77
	PZT/Si	982	10	6.54	1.0	17.5×10^3	295	7.74×10^{-6}	1.66

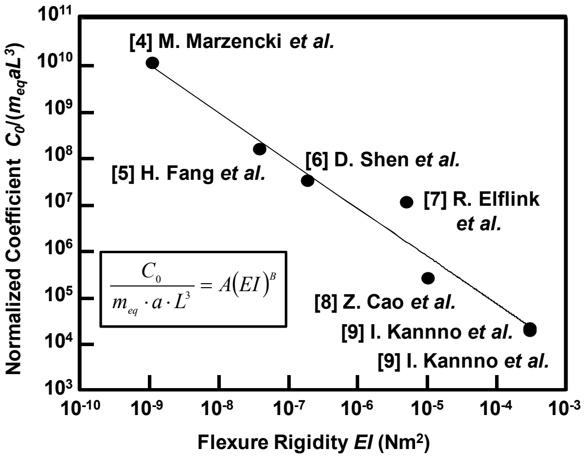
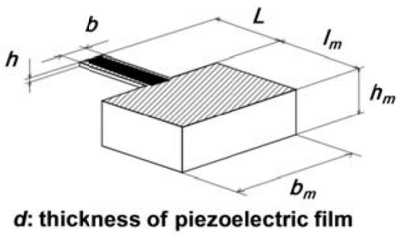


Figure 2. Relationship between the C_0 and flexure rigidity in the cantilever based piezoelectric energy harvester

4. Design and fabrication of the cantilever based energy harvester

Using derived equation, we practically design the cantilever based energy harvester to drive the communication chip based on IEEE 8002.15.4 standard. Table 2 shows a size of designed harvester. EI and C_0 were 2.11×10^{-2} μNm^2 and 2.93×10^{-6}, respectively. Output power was designed to 980 nW. According to the specifications of the communication chips [10][11], it was enough to drive them that the power was 50 mW at transmission/receiving mode and 3 μW at sleeping mode. When communication time and interval were respectively set to 5 msec. and 1 min., the power required for the harvester was about 7 μW. By connecting 10 harvesters in series, the energy to drive the

Table 2. Size of the cantilever based energy harvester



<Cantilever>	
Material	Si
b	210 μm
h	28 μm
L	1000 μm
<Proof mass>	
Material	Si
b_m	1000 μm
h_m	500 μm
l_m	600 μm
<Piezo film>	
Material	(K,Na)NbO$_3$
d	2 μm

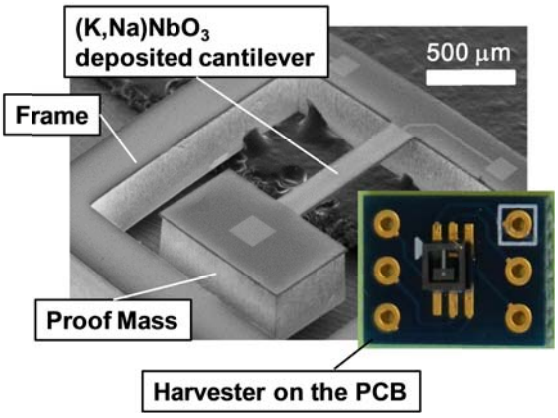


Figure 3. SEM image of the cantilever based energy harvester and photograph of the harvester mounted on the PCB for evaluation.

communication chip can be obtained sufficiently. Figure 3 shows a scanning electron micrograph of the energy harvester fabricated based on the design. This harvester was fabricated with fully Si bulk-micromachining [12].

5. Verification

Figure 4 (a) shows output power for various load resistance when vibration acceleration was 9.8 m/s^2. Using load resistance of 90 kΩ, maximum output power of 730 nW was obtained. Figure 4 (b) shows the relationship between the C_0 and EI including the result of this study when the optimal lord resistance was applied. It was confirmed in Fig. 4 (b) that the result was plotted on the vicinity of approximation line. From these results, though this method is suitable for power estimation in the prototype fabrication, it can be mentioned that the accuracy was insufficient to optimize the output power. The relationship shown in Fig. 2 included the data of the harvesters using various piezoelectric materials in order to obtain the enough plotting points. If you can correct the data of the harvesters with same materials, you will obtain the more accurate approximation constants and be able to optimize the design using this method.

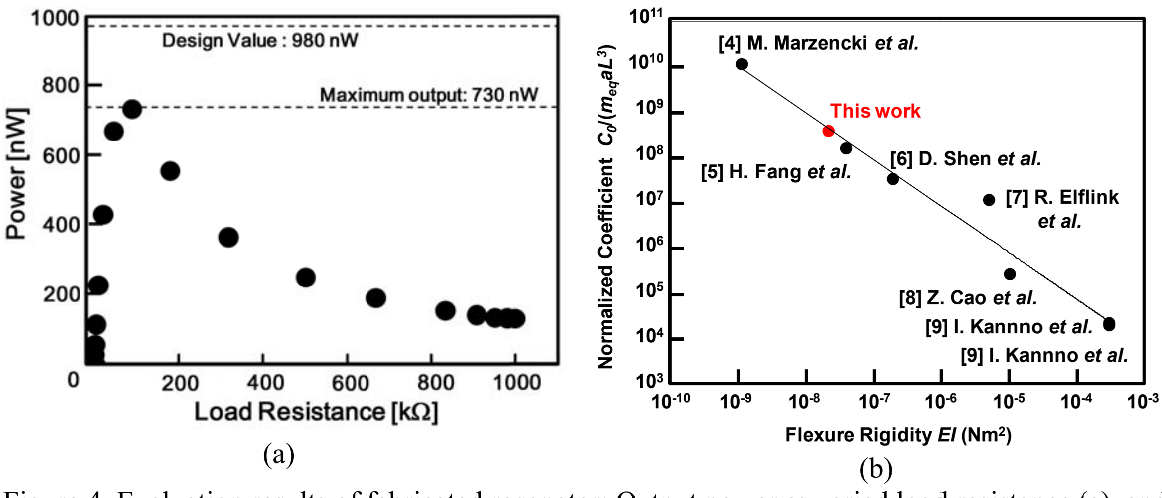


Figure 4. Evaluation results of fabricated resonator: Output power as varied load resistance (a), and relationship between the C_0 and EI (b)

6. Conclusion

A cantilever based piezoelectric energy harvesters is a key device for autonomous micro devices. The harvester must be design to satisfy many specifications such as size, resonant frequency, output power, and output impedance. However, it becomes complicated in the micro energy harvester to theoretically calculate the output power from the strain in the piezoelectric film since amplitude of vibration is too large to apply Bernoulli-Euler hypothesis. In this study, newly introducing the correction coefficient which is a function of flexure rigidity to the conventional theoretical solution, simple equation to estimate the output power was derived.

Then, this equation was applied to design a practical (K,Na)NbO_3 (KNN) based energy harvester. The designed harvester was fully fabricated with bulk micromachining. The harvester had the cantilever size of 210 μm × 28 μm × 1000 μm, the proof mass size of 1000 μm × 500 μm × 600 μm, and the thickness of KNN film of 2 μm. As an evaluation result, maximum output power of 730 nW was achieved when the load resistance, applied vibration frequency and the vibration acceleration were 90 Ω, 1.5 kHz and 9.8 m/s^2. And, the obtained output power was in good agreement with the design value.

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