On low-energy effective action in three-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric electrodynamics

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Abstract. We discuss general structure of low-energy effective actions in $\mathcal{N} = 2$ and $\mathcal{N} = 4$ three-dimensional supersymmetric electrodynamics (SQED) in gauge superfield sector. There are specific terms in the effective action having no four-dimensional analogs. Some of these terms are responsible for the moduli space metric in the Coulomb branch of the theory. We find two-loop quantum corrections to the moduli space metric in the $\mathcal{N} = 2$ SQED and show that in the $\mathcal{N} = 4$ SQED the moduli space does not receive two-loop quantum corrections.

1. Introduction and conclusions

Three-dimensional supersymmetric gauge theories with extended supersymmetry have attracted considerable attention recently [1, 2]. On the one hand, they exhibit reach duality properties [3, 4, 5, 6, 7, 8, 9, 10, 11], which restrict their low-energy dynamics severely. On the other hand, they have deep relations to the dynamics of multiple M2 branes described in terms of the BLG [12, 13, 14, 15, 16, 17, 18] and ABJM [19, 20] models.

In the present paper we study low-energy dynamics in $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SQEDs without Chern-Simons term. When the Chern-Simons therm vanishes, the Coulomb branch exists and the low-energy effective action is given by supersymmetric Euler-Heisenberg effective action for the gauge superfield. In four-dimensional $\mathcal{N} = 1$ and $\mathcal{N} = 2$ SQEDs the Euler-Heisenberg effective action was studied in [21, 22, 23, 24, 25, 26]. It is interesting to compare the forms of Euler-Heisenberg effective actions among three- and four-dimensional SQEDs.

Recall that in $\mathcal{N} = 1$, $d = 4$ supersymmetric gauge theories the gauge superfield V has Grassmann-odd superfield strengths W_α and $\bar{W}_{\dot\alpha}$ which carry spinorial indices. In three-dimensional case these two Grassmann-odd superfield strengths remain, but there is also a Grassmann-even superfield G which is a scalar with respect to the Lorentz group. To study the Euler-Heisenberg effective action it is sufficient to consider constant on-shell superfield strengths $\partial_m G = \partial_m W_\alpha = \partial_m \bar{W}_\alpha$ constrained by $D^2 W_\alpha = \bar{D}^2 \bar{W}_\alpha = 0$. [1] Hence, the most general action with the superfields W_α, $\bar{W}_\alpha$ and G subject to these constraints reads

$$\int d^3x\, d^4\theta \left[A(G, D_\gamma W_\delta) + W^\alpha \bar{W}^\beta B_{\alpha\beta}(G, D_\gamma W_\delta) + W^2 \bar{W}^2 C(G, D_\gamma W_\delta) \right], \qquad (1.1)$$

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[1] In components, these constraints correspond, in particular, to constant Maxwell field strength, $\partial_m F_{np} = 0$.

where A, B and C are some functions. It is important to note that the first two terms in (1.1) have no four-dimensional analogs and are specific for three-dimensional gauge theories in $\mathcal{N} = 2$, $d = 3$ superspace.

In the present paper we will discuss only the first term in (1.1) under additional constraints, $W_\alpha = \bar{W}_\alpha = 0$, $G = const$. In this case (1.1) reduces to a simple expression

$$\int d^3x d^4\theta \, f(G) \,, \tag{1.2}$$

where $f(G)$ is a real function of one real argument which we refer to as the effective potential. This function has good geometrical meaning [7]: it is responsible for the moduli space metric for the $\mathcal{N} = 2$, $d = 3$ gauge multiplet. In particular, the classical action for the gauge superfield V implies $f(G) = \frac{1}{e^2}G^2$, where e is the gauge coupling constant. This quadratic function corresponds to flat moduli space metric while loop quantum corrections deform the flat moduli space. For a long time only one-loop quantum corrections to $f(G)$ were known [3, 4]. In our recent publication [27] we found two-loop effective potential in $\mathcal{N} = 2$ SQED:

$$f(G) = \frac{1}{g^2}G^2 + \frac{1}{2\pi}(G\ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2}) - \frac{e^2}{8\pi^2}\ln(G^2 + m^2) \,. \tag{1.3}$$

Here m is the mass of the chiral matter superfield. The corresponding moduli space metric reads

$$ds^2 = \frac{1}{2}g(r)dr^2 + \frac{1}{2}\frac{1}{g(r)}d\sigma^2 \,, \quad g(r) = \frac{1}{e^2} + \frac{1}{4\pi}\frac{1}{\sqrt{r^2 + m^2}} + \frac{e^2}{8\pi^2}\frac{r^2 - m^2}{(r^2 + m^2)^2} \,. \tag{1.4}$$

Equations (1.3) and (1.4) represent one of the main results of [27] which will be reviewed in the present contribution.

The $\mathcal{N} = 4$, $d = 3$ electrodynamics, as compared with $\mathcal{N} = 2$ SQED, has extra propagating chiral superfield Φ which, together with V, forms the $\mathcal{N} = 4$ gauge multiplet. Therefore, an extension of (1.2) to the $\mathcal{N} = 4$ supersymmetric case reads

$$\int d^3x d^4\theta \, f(G, \Phi\bar{\Phi}) \,. \tag{1.5}$$

This effective potential in known to be one-loop exact in $\mathcal{N} = 4$ supersymmetric gauge theories [3, 7]. For $\mathcal{N} = 4$ SQED, one-loop quantum contributions to (1.5) were computed in [28],

$$f(G, \Phi\bar{\Phi}) = \frac{1}{e^2}(G^2 - \frac{1}{2}\bar{\Phi}\Phi) + \frac{1}{2\pi}\Big[G\ln(G + \sqrt{G^2 + \bar{\Phi}\Phi}) - \sqrt{G^2 + \Phi\bar{\Phi}}\Big]. \tag{1.6}$$

In the present paper, by analyzing the structure of Feynman graphs in $\mathcal{N} = 2$, $d = 3$ superspace, we show that two-loop quantum contributions to (1.5) vanish, confirming the non-renormalizability of the effective potential beyond one loop. The moduli space metric corresponding to the effective potential (1.6) is the well-known Taub-NUT metric [29].

It is natural to generalize the present results to the non-Abelian case and to find two-loop quantum correction to the moduli space metric (1.4) in $\mathcal{N} = 2$ supersymmetric Yang-Mills models with different matter superfields. Another interesting extension could be to find all-loop quantum contributions to (1.5) and derive the effective potential $f(G)$ in a closed form. Such an effective potential would give non-perturbative expression for the moduli space metric in $\mathcal{N} = 2$ SQED. Another tempting problem is to find effective Kähler potential in three-dimensional gauge theories in $\mathcal{N} = 2$, $d = 3$ superspace.

The present contribution is based essentially on the results of our recent work [27]. Here we will employ superspace notations adopted in this paper.

2. $\mathcal{N} = 2$ supersymmetric electrodynamics

2.1. Classical action

The classical action of the $\mathcal{N} = 2$, $d = 3$ supersymmetric electrodynamics reads

$$S_{\mathcal{N}=2} = \frac{1}{e^2} \int d^7 z \, G^2 - \int d^7 z \, \left(\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_- \right) - \left(m \int d^5 z \, Q_+ Q_- + c.c. \right), \quad (2.1)$$

where $Q_\pm$ are chiral superfields with opposite charges with respect to the gauge superfield V. G is the superfield strength for the gauge superfield V,

$$G = \frac{i}{2} \bar{D}^\alpha D_\alpha V \,. \tag{2.2}$$

This superfield is real $G^* = G$ and linear,

$$D^2 G = \bar{D}^2 G = 0 \,. \tag{2.3}$$

The action (2.1) appears by virtue of the dimensional reduction from the $\mathcal{N} = 1$, $d = 4$ electrodynamics [30, 31].

We are interested in the part of low-energy effective action which depends on the gauge superfield only, $\Gamma = \Gamma[V]$, while the chiral superfields $Q_\pm$ are integrated out. For this problem the background field method in the $\mathcal{N} = 2$, $d = 3$ superspace [32] appears to be useful. We split the gauge superfield V into the background V and quantum v parts

$$V \to V + e\,v \,. \tag{2.4}$$

Upon this splitting the Maxwell term in (2.1) changes as

$$\frac{1}{e^2} \int d^7 z \, G^2 \to \frac{1}{e^2} \int d^7 z \, G^2 + \frac{i}{e} \int d^7 z \, v D^\alpha W_\alpha + \frac{1}{8} \int d^7 z \, v D^\alpha \bar{D}^2 D_\alpha v \,. \tag{2.5}$$

The operator $D^\alpha \bar{D}^2 D_\alpha$ in the last term is degenerate and requires gauge fixing. In particular, the Fermi-Feynman gauge is implemented by the following gauge-fixing term

$$S_{\text{gf}} = -\frac{1}{16} \int d^7 z \, v \{ D^2, \bar{D}^2 \} v \,. \tag{2.6}$$

Adding this term to (2.1) we get

$$S_{\text{quantum}} \;\;=\;\; S_2 + S_{\text{int}} \,, \tag{2.7}$$

$$S_2 \;\;=\;\; -\int d^7 z \left(v \Box v + \bar{\mathcal{Q}}_+ \mathcal{Q}_+ + \bar{\mathcal{Q}}_- \mathcal{Q}_- \right) - \left(m \int d^5 z \, \mathcal{Q}_+ \mathcal{Q}_- + c.c. \right), \tag{2.8}$$

$$S_{\text{int}} \;\;=\;\; -2 \int d^7 z \left[e \left(\bar{\mathcal{Q}}_+ \mathcal{Q}_+ - \bar{\mathcal{Q}}_- \mathcal{Q}_- \right) v + e^2 \left(\bar{\mathcal{Q}}_+ \mathcal{Q}_+ + \bar{\mathcal{Q}}_- \mathcal{Q}_- \right) v^2 \right] + O(e^3), \tag{2.9}$$

where $\mathcal{Q}_\pm$ and $\bar{\mathcal{Q}}_\pm$ are covariantly (anti)chiral superfields with respect to the background gauge superfield,

$$\bar{\mathcal{Q}}_+ = \bar{Q}_+ e^{2V}, \quad \mathcal{Q}_+ = Q_+, \quad \bar{\mathcal{Q}}_- = \bar{Q}_- e^{-2V}, \quad \mathcal{Q}_- = Q_- \,. \tag{2.10}$$

2.2. Propagators and heat kernels
The action S_2 (2.8) is responsible for the propagators,

$$
\begin{aligned}
i\langle \mathcal{Q}_+(z)\mathcal{Q}_-(z')\rangle &= -mG_+(z,z')\,, \\
i\langle \bar{\mathcal{Q}}_+(z)\bar{\mathcal{Q}}_-(z')\rangle &= mG_-(z',z)\,, \\
i\langle \mathcal{Q}_+(z)\bar{\mathcal{Q}}_+(z')\rangle &= G_{+-}(z,z') = G_{-+}(z',z)\,, \\
i\langle \bar{\mathcal{Q}}_-(z)\mathcal{Q}_-(z')\rangle &= G_{-+}(z,z')\,,
\end{aligned}
\tag{2.11}
$$

where the Green's functions G_+ and G_{+-} obey the equations

$$
(\Box_\pm + m^2)G_\pm(z,z') = -\delta_\pm(z,z')\,,
\tag{2.12}
$$

$$
\frac{1}{4}\nabla^2 G_{+-}(z,z') + m^2 G_-(z,z') = -\delta_-(z,z')\,,
\tag{2.13}
$$

$$
\frac{1}{4}\bar{\nabla}^2 G_{-+}(z,z') + m^2 G_+(z,z') = -\delta_+(z,z')\,.
\tag{2.14}
$$

Here $\delta_\pm(z,z')$ are (anti)chiral delta-functions and $\Box_\pm$ are d'Alembertians acting in the space of (anti)chiral superfields,

$$
\Box_+ = \nabla^m\nabla_m + G^2 + \frac{i}{2}(\nabla^\alpha W_\alpha) + iW^\alpha\nabla_\alpha\,,
\tag{2.15}
$$

$$
\Box_- = \nabla^m\nabla_m + G^2 - \frac{i}{2}(\bar{\nabla}^\alpha\bar{W}_\alpha) - i\bar{W}^\alpha\bar{\nabla}_\alpha\,.
\tag{2.16}
$$

The Green's functions (2.12)–(2.14) can be expressed in terms of corresponding heat kernels,

$$
G_\pm(z,z') = i\int_0^\infty ds\, K_\pm(z,z'|s)e^{is(m^2+i\epsilon)}\,,
\tag{2.17}
$$

$$
G_{+-}(z,z') = i\int_0^\infty ds\, K_{+-}(z,z'|s)e^{is(m^2+i\epsilon)}\,,
\tag{2.18}
$$

where the standard $\epsilon \to +0$ prescription is assumed. These heat kernels were computed exactly for the on-shell gauge superfield background $D^\alpha W_\alpha = 0$ subject to $\partial_m W_\alpha = 0$, [27]

$$
K_+(z,z'|s) = \frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\rho^m\rho^n - \frac{1}{2}\bar{\zeta}^\beta\rho_{\beta\gamma}W^\gamma}\zeta^2 I(z,z')
\tag{2.19}
$$

$$
K_{+-}(z,z'|s) = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\tilde{\rho}^m\tilde{\rho}^n + R(z,z')}I(z,z')\,,
\tag{2.20}
$$

where

$$
\zeta^\alpha = (\theta - \theta')^\alpha\,, \quad \bar{\zeta}^\alpha = (\bar{\theta} - \bar{\theta}')^\alpha\,, \quad \rho^m = (x - x')^m - i\gamma^m_{\alpha\beta}\zeta^\alpha\bar{\theta}'^\beta + i\gamma^m_{\alpha\beta}\theta'^\alpha\bar{\zeta}^\beta
\tag{2.21}
$$

are components of the supersymmetric interval and $I(z,z')$ is the parallel transport propagator [33] in $\mathcal{N}=2$, $d=3$ superspace [27]. Among its properties there are

$$
I(z,z) = 1\,, \qquad I(z,z')I(z',z) = 1\,,
\tag{2.22}
$$

which will be useful in quantum loop computations. The heat kernel (2.20) contains also a version of supersymmetric interval $\tilde{\rho}^m$ which is chiral with respect to the first argument and is antichiral with respect to the second one,

$$
\tilde{\rho}^m = \rho^m + i\zeta^\alpha\gamma^m_{\alpha\beta}\bar{\zeta}^\beta\,, \qquad D'_\alpha\tilde{\rho}^m = \bar{D}_\alpha\tilde{\rho}^m = 0\,.
\tag{2.23}
$$

The two-point function $R(z, z')$ in (2.20) was found in the form

$$
\begin{aligned}
R(z, z') \ = \ & -i\zeta\bar{\zeta}G + \frac{7i}{12}\bar{\zeta}^2\zeta W + \frac{i}{12}\zeta^2\bar{\zeta}\bar{W} - \frac{1}{2}\bar{\zeta}^\alpha\tilde{\rho}_{\alpha\beta}W^\beta - \frac{1}{2}\zeta^\alpha\tilde{\rho}_{\alpha\beta}\bar{W}^\beta \\
& + \frac{1}{12}\zeta^\alpha\bar{\zeta}^\beta[\tilde{\rho}_\beta^\gamma D_\alpha W_\gamma - 7\tilde{\rho}_\alpha^\gamma D_\gamma W_\beta]\,.
\end{aligned}
\tag{2.24}
$$

The heat kernels (2.19) and (2.20) contain also the operator

$$
\mathcal{O}(s) = e^{s(\bar{W}^\alpha\bar{\nabla}_\alpha - W^\alpha\nabla_\alpha)}\,,
\tag{2.25}
$$

and we use the following notations for derivatives of superfield strengths:

$$
N_{\alpha\beta} = D_{(\alpha}W_{\beta)}\,, \quad \bar{N}_{\alpha\beta} = \bar{D}_{(\alpha}\bar{W}_{\beta)}\,, \quad B^2 = \frac{1}{2}N_\alpha^\beta N_\beta^\alpha\,.
\tag{2.26}
$$

The propagator $G_0(z, z')$ for the gauge superfield v obeys the equation

$$
2i\langle v(z)\, v(z')\rangle = G_0(z, z')\,, \qquad \Box G_0(z, z') = -\delta^7(z - z')\,.
\tag{2.27}
$$

The corresponding heat kernel reads

$$
G_0(z, z') = i\int_0^\infty ds\, K_0(z, z'|s)\, e^{-s\epsilon}\,, \qquad K_0(z, z'|s) = \frac{1}{(4i\pi s)^{3/2}}e^{\frac{i\rho^m\rho_m}{4s}}\zeta^2\bar{\zeta}^2\,.
\tag{2.28}
$$

Here ρ^m, ζ^α and $\bar{\zeta}^\alpha$ are the components of supersymmetric interval (2.21).

2.3. Low-energy effective action

Consider loop expansion of the effective action in $\mathcal{N} = 2$, $d = 3$ SQED,

$$
\Gamma_{\mathcal{N}=2} = \Gamma_{\mathcal{N}=2}^{(1)} + \Gamma_{\mathcal{N}=2}^{(2)} + \dots\,,
\tag{2.29}
$$

where $\Gamma_{\mathcal{N}=2}^{(1)}$ and $\Gamma_{\mathcal{N}=2}^{(2)}$ are one- and two-loop quantum correction and dots correspond to higher-order terms. It is clear that the one-loop effective action is given by the loop of chiral superfields in the gauge superfield background,

$$
\Gamma_{\mathcal{N}=2}^{(1)} = i\,\mathrm{Tr}\ln(\Box_+ + m^2) = -i\int_0^\infty \frac{ds}{s}\int d^3x d^2\theta d^2\bar{\theta}\, K_+(z, z|s)\,,
\tag{2.30}
$$

where $K_+(z, z')$ is the chiral heat kernel (2.19). At coincident superspace points it reads

$$
K_+(z, z|s) = \frac{1}{8(i\pi s)^{3/2}}s^2 W^2 e^{isG^2}\frac{\tanh(sB/2)}{sB/2}\,.
\tag{2.31}
$$

In fact, for computing the effective potential it is sufficient to consider the expression (2.31) for $B = 0$. Taking into account this simplification we calculate the integral over ds in (2.30),

$$
\Gamma_{\mathcal{N}=2}^{(1)} = \frac{1}{2\pi}\int d^7z\Big[G\ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2}\Big]\,.
\tag{2.32}
$$

Consider now the two-loop effective action

$$
\Gamma_{\mathcal{N}=2}^{(2)} = -2e^2\int d^7z\, d^7z'[G_{+-}(z, z')G_{+-}(z', z) + m^2 G_+(z, z')G_-(z, z')]G_0(z, z')\,.
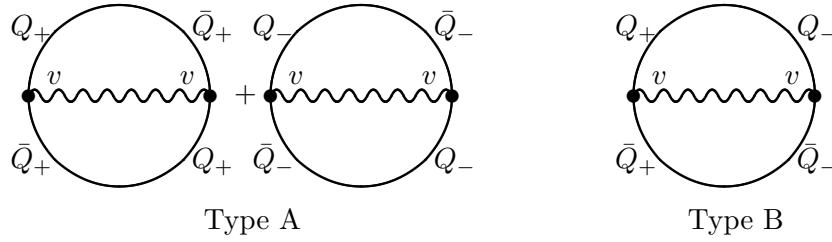\tag{2.33}
$$

Figure 1. Two-loop supergraphs in $\mathcal{N} = 2$ supersymmetric electrodynamics.

The two terms in r.h.s. correspond to the Feynman graphs of Type A and B in fig. 1, respectively. For obtaining two-loop quantum corrections to the effective potential it is sufficient to consider the propagators (2.19) and (2.20) with $W_\alpha = \bar{W}_\alpha = 0$, $F_{mn} = 0$, $N_{\alpha\beta} = 0$ and $B = 0$, leaving only the dependence on the superfield strengths G,

$$K_+(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} e^{isG^2} e^{\frac{i}{4s}\rho^m \rho_m} \zeta^2 I(z, z'), \tag{2.34}$$

$$K_{+-}(z, z'|s) = -\frac{1}{8(i\pi s)^{3/2}} e^{isG^2} e^{\frac{i}{4s}\tilde{\rho}^m \tilde{\rho}_m - i\zeta\bar{\zeta}G} I(z, z'). \tag{2.35}$$

Note that only the first term in (2.24) contributes to (2.35) in the case of such a simple background.

The heat kernel (2.28) for the photon propagator contains the delta-function for Grassmann variables $\zeta^2 \bar{\zeta}^2$. Therefore, it is sufficient to consider the heat kernels (2.34) and (2.35) at coincident Grassmann points,

$$K_+(z, z'|s)|_{\zeta=\bar{\zeta}=0} = 0, \tag{2.36}$$

$$K_{+-}(z, z'|s)|_{\zeta=\bar{\zeta}=0} = -\frac{1}{8(i\pi s)^{3/2}} e^{isG^2} e^{\frac{i}{4s}\rho^m \rho_m} I(z, z')|_{\zeta=\bar{\zeta}=0}. \tag{2.37}$$

Hence, only the diagrams of Type A in fig. 1 contribute to the effective potential. Substituting now (2.37) and (2.28) into (2.33) we find

$$\begin{aligned}
\Gamma^{(2)}_{\mathcal{N}=2} &= \frac{2ie^2}{(4i\pi)^{9/2}} \int d^7z\, d^3\rho \int_0^\infty \frac{ds\, dt\, du}{(stu)^{3/2}} e^{i(s+t)(G^2+m^2)} e^{\frac{i}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})\rho^2} \\
&= \frac{2e^2}{(4\pi)^3} \int d^7z \int_0^\infty \frac{ds\, dt\, du}{(st+su+tu)^{3/2}} e^{i(s+t)(G^2+m^2)}.
\end{aligned} \tag{2.38}$$

Here we did the Gaussian integration over $d^3\rho$,

$$\int d^3\rho\, e^{\frac{i}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})\rho^2} = -\left(\frac{4i\pi}{\frac{1}{s}+\frac{1}{t}+\frac{1}{u}}\right)^{3/2}. \tag{2.39}$$

Hence, after computing the integrals over the parameters s, t, u we get

$$\Gamma^{(2)}_{\mathcal{N}=2} = -\frac{e^2}{8\pi^2} \int d^7z\, \ln(G^2 + m^2). \tag{2.40}$$

Finally, putting together (2.32) and (2.40) we get the two-loop effective potential,

$$\Gamma_{\mathcal{N}=2} = \int d^7 z \, f(G) \,,$$

$$f(G) = \frac{1}{g^2} G^2 + \frac{1}{2\pi} (G \ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2}) - \frac{e^2}{8\pi^2} \ln(G^2 + m^2) \,. \quad (2.41)$$

Here the term $\frac{1}{e^2} G^2$ is a part of the classical action (2.1) in the gauge superfield sector. We stress that the effective potential (2.41) was known before only in the one-loop approximation [3, 4] while the two-loop correction to this result was found in our work [27].

2.4. Two-loop moduli space metric
The moduli space is an important object in gauge theories which allows one to study various issues of dualities [3, 4, 5, 6, 7, 8, 9, 10, 11] (see also [1, 2] for recent discussions of these problems). The perturbative quantum corrections to the moduli space metric in the $\mathcal{N} = 2$, $d = 3$ gauge theories are known only up the one-loop order [3, 4, 7]. The two-loop effective action (2.41) allows us to study two-loop moduli space in the $\mathcal{N} = 2$, $d = 3$ electrodynamics (2.1).

The moduli space in $\mathcal{N} = 2$, $d = 3$ SQED is a two-dimensional Kähler manifold. It can be parametrized by two real coordinates r and σ. The coordinate r is naturally identified with the vev of the scalar field ϕ which is a part of the $\mathcal{N} = 2$, $d = 3$ gauge multiplet, $r = \langle \phi \rangle$. This scalar is the lowest component of the superfield strength G,

$$G|_{\theta \to 0} = \phi \,. \quad (2.42)$$

Another scalar field a appears upon dualizing the Abelian vector A_m,

$$\partial_m a \propto \varepsilon_{mnp} F^{np} \,, \quad (2.43)$$

where F_{mn} is the Maxwell field strength corresponding to the Abelian vector field A_m. The coordinate σ corresponds to the vev of this scalar, $\sigma = \langle a \rangle$. The moduli space metric is parametrized by r and σ,

$$ds^2 = g_{rr}(r, \sigma) dr^2 + g_{\sigma\sigma}(r, \sigma) d\sigma^2 \,. \quad (2.44)$$

Our aim is to find the explicit form of the functions $g_{rr}(r, \sigma)$ and $g_{\sigma\sigma}(r, \sigma)$.

The procedure of deriving the metric (2.44) from the low-energy effective action is described in [7]: Given the function $f(G)$ one dualizes the linear superfield G into a chiral superfield Φ following standard procedure of [29]. The chiral superfield serves as the Lagrange multiplier for the linearity constraint (2.3),

$$S_{\text{low-energy}} = \int d^7 z \, [f(G) - G(\Phi + \bar{\Phi})] \,. \quad (2.45)$$

The superfield G is treated now as unconstrained. Varying (2.45) with respect to G we get

$$\Phi + \bar{\Phi} = f'(G) = \frac{2}{e^2} G + \frac{1}{2\pi} \ln(G + \sqrt{G^2 + m^2}) - \frac{e^2}{4\pi^2} \frac{G}{G^2 + m^2} \,. \quad (2.46)$$

From this equation the superfield G should be expressed in terms of $\Phi + \bar{\Phi}$ and substituted back to (2.45). This yields a sigma-model action,

$$S_{\text{low-energy}} = \int d^7 z \, K(\Phi + \bar{\Phi}) \,, \quad (2.47)$$

with some function $K(\Phi + \bar\Phi)$. We do not need the manifest expression for K since the Kähler metric is defined rather by its second derivative,

$$ds^2 = K'' \, d\Phi \, d\bar\Phi \,. \tag{2.48}$$

This metric should be expressed in terms of r and σ where $r = \langle G \rangle$ and σ can be identified with the imaginary part of Φ, $\sigma = \mathrm{Im}\,\Phi$. Using the fact that the inverse Legendre transform is a Legendre transform, we have

$$K'(\Phi + \bar\Phi) = r \,. \tag{2.49}$$

From this equation and from (2.46) we conclude

$$K''(\Phi + \bar\Phi) = \left(\frac{\partial(\Phi + \bar\Phi)}{\partial r} \right)^{-1} = \frac{1}{2} \frac{1}{g(r)} \,, \tag{2.50}$$

where

$$g(r) = \frac{1}{e^2} + \frac{1}{4\pi} \frac{1}{\sqrt{r^2 + m^2}} + \frac{e^2}{8\pi^2} \frac{r^2 - m^2}{(r^2 + m^2)^2} \,. \tag{2.51}$$

Finally, we note that (2.46) implies that

$$d\Phi = g(r)dr + id\sigma \,, \qquad d\bar\Phi = g(r)dr - id\sigma \,. \tag{2.52}$$

Substituting now (2.50) and the latter identities into (2.48) we find the moduli space metric in the form

$$ds^2 = \frac{1}{2} g(r) dr^2 + \frac{1}{2} \frac{1}{g(r)} d\sigma^2 \,. \tag{2.53}$$

In the massless limit the function $g(r)$ in (2.51) simplifies such that

$$ds^2|_{m=0} = \frac{1}{2} \left(\frac{1}{e^2} + \frac{1}{4\pi r} + \frac{e^2}{8\pi^2 r^2} \right) dr^2 + \frac{1}{2} \left(\frac{1}{e^2} + \frac{1}{4\pi r} + \frac{e^2}{8\pi^2 r^2} \right)^{-1} d\sigma^2 \,. \tag{2.54}$$

Equation (2.54) shows that the one-loop metric is corrected by the two-loop contribution $\frac{e^2}{8\pi^2 r^2}$. It is naturally to expect that the n-loop correction could be of the form $c_n \frac{1}{e^2} (\frac{e^2}{r})^n$, with some coefficient c_n. It is very tempting to compute such higher-loop coefficients c_n and to find a closed expression for all-loop moduli space metric both for the Abelian and non-Abelian $\mathcal{N} = 2$, $d = 3$ gauge theories.

3. $\mathcal{N} = 4$ supersymmetric electrodynamics
3.1. Classical action
$\mathcal{N} = 4$ supersymmetric gauge multiplet consists of the $\mathcal{N} = 2$ gauge superfield V and a chiral superfield Φ. The classical action of the $\mathcal{N} = 4$ supersymmetric electrodynamics reads

$$
\begin{aligned}
S_{\mathcal{N}=4} &= \frac{1}{e^2} \int d^7 z (G^2 - \frac{1}{2}\bar\Phi\Phi) + S_Q \,, \\
S_Q &= -\int d^7 z (\bar{\mathcal{Q}}_+ \mathcal{Q}_+ + \bar{\mathcal{Q}}_- \mathcal{Q}_-) - \int d^5 z \, \mathcal{Q}_+ \Phi \mathcal{Q}_- + \int d^5\bar z \, \bar{\mathcal{Q}}_+ \bar\Phi \bar{\mathcal{Q}}_- \,,
\end{aligned}
\tag{3.1}
$$

We are interested in the part of low-energy effective action in this model which depends both on V and Φ. Therefore, we make the background-quantum splitting for both these superfields,

$$V \to V + e\,v \,, \qquad \Phi \to \Phi + e\,\phi \,, \tag{3.2}$$
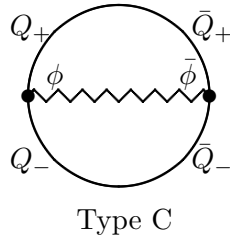
Type C

Figure 2. Two-loop supergraph in $\mathcal{N} = 4$ supersymmetric electrodynamics which involves chiral superfield propagator $\langle \phi \bar{\phi} \rangle$.

while the hypermultiplet $(\mathcal{Q}_+, \mathcal{Q}_-)$ is considered as the 'quantum' superfield which should be integrated out in the path integral. We impose the following constraints on the background superfield:

$$\partial_m G = 0, \quad W_\alpha = \bar{W}_\alpha = 0, \quad D_\alpha \Phi = 0. \tag{3.3}$$

These constraints are sufficient for obtaining the effective potential of the form (1.5).

Upon quantization in the Fermi-Feynman gauge (2.6), we end up with the following action for 'quantum' superfields,

$$S_{\text{quantum}} = S_2 + S_{\text{int}}, \tag{3.4}$$

$$S_2 = -\int d^7 z (v \Box v + \frac{1}{2} \bar{\phi}\phi + \bar{\mathcal{Q}}_+ \mathcal{Q}_+ + \bar{\mathcal{Q}}_- \mathcal{Q}_-) - \left(\int d^5 z \, \mathcal{Q}_+ \Phi \mathcal{Q}_- + c.c. \right), \tag{3.5}$$

$$S_{\text{int}} = -2 \int d^7 z \left[e \left(\bar{\mathcal{Q}}_+ \mathcal{Q}_+ - \bar{\mathcal{Q}}_- \mathcal{Q}_- \right) v + e^2 \left(\bar{\mathcal{Q}}_+ \mathcal{Q}_+ + \bar{\mathcal{Q}}_- \mathcal{Q}_- \right) v^2 \right]$$
$$- e \int d^5 z \, \mathcal{Q}_+ \phi \mathcal{Q}_- + e \int d^5 \bar{z} \, \bar{\mathcal{Q}}_+ \bar{\phi} \bar{\mathcal{Q}}_+ + O(e^3). \tag{3.6}$$

The propagators for the chiral matter superfields and for the gauge superfield V are the same as in the $\mathcal{N} = 2$ electrodynamics (2.11) and (2.27). However, the chiral superfield ϕ propagates now,

$$\langle \phi(z) \bar{\phi}(z') \rangle = -\frac{i}{8} \bar{D}^2 D^2 G_0(z, z'). \tag{3.7}$$

3.2. Low-energy effective action
The one-loop effective action $\Gamma^{(1)}_{\mathcal{N}=4}$ receives contributions only from the chiral matter superfields which are the same as in the $\mathcal{N} = 2$ electrodynamics (2.1). The only difference is that the chiral superfield is dynamical now while in the $\mathcal{N} = 2$ case it was 'frozen' to be equal to the mass parameter m. Therefore we can easily generalize the one-loop effective action (2.32) to the $\mathcal{N} = 4$ case,

$$\Gamma^{(1)}_{\mathcal{N}=4} = \frac{1}{2\pi} \int d^7 z \left[G \ln(G + \sqrt{G^2 + \bar{\Phi}\Phi}) - \sqrt{G^2 + \Phi\bar{\Phi}} \right]. \tag{3.8}$$

The two-loop effective action in the $\mathcal{N} = 4$ electrodynamics differs from the $\mathcal{N} = 2$ case because the chiral superfield Φ propagates now. Indeed, apart from the diagrams in fig. 1, there is an additional two-loop diagram with quantum $\langle \phi \bar{\phi} \rangle$ line, see fig. 2. This extra graph yields

additional contributions to the two-loop effective action which we denote by $\Gamma_{\rm C}$,

$$\Gamma_{\mathcal{N}=4}^{(2)} = \Gamma_{\rm A} + \Gamma_{\rm B} + \Gamma_{\rm C}\,, \tag{3.9}$$

$$\Gamma_{\rm A} = -2e^2 \int d^7z\, d^7z'\, G_{+-}(z,z')G_{+-}(z',z)G_0(z,z')\,, \tag{3.10}$$

$$\Gamma_{\rm B} = -2e^2 \int d^7z\, d^7z'\, \bar{\Phi}\Phi\, G_+(z,z')G_-(z,z')G_0(z,z')\,, \tag{3.11}$$

$$\Gamma_{\rm C} = 2e^2 \int d^7z\, d^7z'\, G_{+-}(z,z')G_{+-}(z,z')G_0(z,z')\,. \tag{3.12}$$

We are interested in the part of the low-energy effective action depending on constant background superfields G and Φ which we refer to as the effective potential. It is easy to see that the part of the low-energy effective action (3.11) does not contribute to the effective potential because of (2.36). Hence, it is necessary to consider only

$$\Gamma_{\rm A} + \Gamma_{\rm B} = 2e^2 \int d^7z\, d^7z'[G_{+-}(z,z')G_{+-}(z,z') - G_{+-}(z,z')G_{+-}(z',z)]G_0(z,z')\,. \tag{3.13}$$

The heat kernel $K_{+-}(z,z'|s)$ given by (2.37) is symmetric with respect to the superspace points. Hence, the two terms in the r.h.s. of (3.13) exactly cancel each other. Thus, we conclude that there are no two-loop quantum corrections to the effective potential (3.8). This result is a direct check of the general statement that the moduli space metric in the $\mathcal{N} = 4$, $d = 3$ gauge theories is one-loop exact [3, 7]. This metric is known as the Taub-NUT metric which was originally derived in [29] from geometrical principles.

Acknowledgments

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