On first attempts to reconcile quantum principles with gravity

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Abstract. In his first paper of 1916 on gravitational waves Einstein began to speculate on *interactions* between the principles of the old quantum theory and his theory of gravitation. With this contribution Einstein has stimulated a lot of similar speculations, during the dawn and the development of Quantum Mechanics. These speculations have culminated with the first try to quantize the gravitational field, that was provided by Rosenfeld in 1930. In this paper we briefly explain why this period (1916-1930) should be inserted into the history of Quantum Gravity and then we focus on Klein's approach to the problem of reconciling Wave Mechanics with gravity, during the two-years period 1926-1927. His attempt should be looked as the prehistory of Quantum Field Theory in a curved background.

1. Introduction

The term Quantum Gravity (QG) is often associated to the idea that the gravitational field must be quantized. Up to the present, we do not know how to construct this theory in a consistent way. From the birth of General Relativity (GR) in 1915 until today, many approaches tried to face, in a broad sense, the problem of harmonizing the quantum principles and GR.

In 1916, Einstein [1] was the first to argue that quantum effects must modify his general theory. In fact he had in mind the Bohr's principle of stationary orbits that had already modified the classical idea of the atomic collapse in the case of the energy loss due to electromagnetic wave emission and that seemed to suggest that a similar solution is needed to avoid the energy loss caused by gravitational wave emission. Presumably he did not know, at that time, that this kind of atomic collapse is characterized by a time of the order of 10^{37}s, that is an enormous lack of time compared with the life of our Universe[1] [3].

The first try to quantize the gravitational field appeared in 1930. In this year Rosenfeld published two papers. In his first paper[2] [5] Rosenfeld considers a Lagrangian that includes the electromagnetic field, the Dirac field and the gravitational field. Concerning the latter, he writes the Einstein-Hilbert action using the tetrad formalism[3]. In his approach, quantization takes place when classical variables, the tetrads, and their conjugate momenta are substituted with hermitian operators[4]. With this paper the *canonical quantization approach* enters into the

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[1] Recent results [2] give approximately $(4.354 \pm 0.012) \times 10^{17}$s.
[2] See [4] for a commented English translation.
[3] Rosenfeld refers to this formalism as "the one-body theory proposed by Weyl" [6].
[4] In those years physicists were used to talk about *c-numbers* (classical) and *q-numbers* (quantum).

history of QG. In a second paper [7], the linearization procedure for the metric $g_{\mu\nu}$, introduced by Einstein[5][1], is used: $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{\chi}h_{\mu\nu}$, where χ is proportional to the Newton constant and $h_{\mu\nu}$ is the perturbation of the Minkowski metric $\eta_{\mu\nu}$. In this article Rosenfeld writes the linearized Lagrangian and chooses, as field variables, the perturbation of the metric, that he rewrites in terms of annihilation and creation operators. This splitting procedure will be adopted also later on by Gupta [8] and Feynman [9], the fathers of the *covariant quantization approach*.

Now we want to raise the following question: should the endeavours *preceding* Rosenfeld's work be considered part of the history of QG? To answer the question we point out the following facts. First of all, the idea that *every field can be quantized* appeared for the first time in 1929, thanks to Heisenberg and Pauli [10]. Despite this, Einstein had already speculated on possible interactions between the principles of the old quantum theory[6] and his theory of gravitation. Likewise, Klein[7] and other researchers, during the early years of Quantum Mechanics (QM), tried to find out a unifying framework that could harmonize Wave Mechanics[8] with gravity. It was a failed[9] attempt to modify GR for reconciling it with quantum principles and some authors have already underlined this fact [13]-[15]. For this reason we decided to study more in detail Klein's program from the perspective of the history of QG. To answer the question we posed, another important fact to consider is the role played by the semi-classical methods born after Rosenfeld papers, like e.g. Quantum Field Theory (QFT) in curved space-times. Even though they are not directly connected with QG problems, they revealed new phenomena and raised new problems that will find a solution in a consistent theory of QG. We have in mind the black hole radiation [16] and the *information loss paradox* [17], discovered by Hawking in the seventies. Concerning the latter, it will be avoided by a unitary description of the black hole radiation process in the framework of a consistent theory of QG. From a modern point of view, QFT in curved backgrounds is a *semi-classical* approach, because *quantum* matter fields live in a *classical* curved background: we can consider this approach as a way to reconcile quantum principles with gravity at an energy scale where the quantum effects of the gravitational field are negligible and for this reason we believe that the development of these methods should belong to the history of QG. And last but not least, let us consider String Theory, one of the most promising approach to QG. It combines together various features coming from different areas, like e.g. Klein's idea of extra dimensions or QFT in two dimensions, without quantizing the gravitational field directly. In fact String Theory offers a picture where QG *emerges* from the quantum theory of strings [18].

For these reasons we think that *every* endeavour to reconcile quantum principles with gravity should be inserted in the history of QG, even though in the approach considered the gravitational field is treated classically. We imagine the history of QG as a chain of islands; on every island there is a farm, with physicists at work, and every island corresponds to a different approach to the problem of reconciling quantum postulates with the theory of gravitation.

Our paper aims to consider a particular farm, that tried to unify gravity, Maxwell's theory and Wave Mechanics, with the main focus on Klein's work in the years 1926 and 1927. As far as we know, we note for the first time that Klein's contribution should be inserted into the *prehistory* of QFT in curved backgrounds.

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[5] We use the signature $(-, +, +, +)$ and μ, ν run from 0 to 3.

[6] This term refers, as usual, to all models developed to explain new phenomena that arose between 1900 and 1925, using quantized classical quantities (Planck's black body theory, Einstein-Bohr-Sommerfeld quantization rules etc.).

[7] We refer to Oskar Klein, not to be confused with Felix Klein.

[8] In the old quantum theory there were two main approaches [11]: the *particle* approach and the *wave* approach. They led to the birth of Matrix Mechanics in 1925, developed by Heisenberg, Born and Jordan, and to the birth of Wave Mechanics at the beginning of 1926, developed by Schrödinger.

[9] Klein himself admitted that his program failed because "as Dirac may well say, my main trouble came from trying to solve too many problems at a time!" [12].

2. Klein's contribution

Maxwell's legacy has shown that unifying different areas of physics could lead to a better understanding of our world. Following this idea, some researchers started to search for a *unified theory*, that could describe both GR and Maxwell's theory in a unified framework. Among these, some physicists assumed that we live in a *five-dimensional* world: in addition to the usual four space-time dimensions, there is a fifth space-like dimension[10], x^5, that is compactified on a circle[11] of radius l. In this approach, the five-dimensional metric $\gamma_{\bar{\mu}\bar{\nu}}$ $\bar{\mu}, \bar{\nu} = 0, 1, 2, 3, 5$, once decomposed in its low-dimensional components, can describe gravity and Maxwell's theory. In fact $\gamma_{\mu\nu}$ and $\gamma_{5\mu}$ behave like a four-dimensional symmetric tensor and a four-dimensional vector respectively and then they could play the role of the metric and of the electromagnetic potential. Kaluza and Klein are known as the fathers of this approach[12].

Kaluza made the assumption of cilindricity in 1921 [19]. In the last sentence of his paper he declares to be afraid that quantum theory would be a menace for his theory[13]. Klein found out the same approach later and independently in 1926. On the contrary he was convinced that quantum principles should play a fundamental role in his version of a five-dimensional unified theory.

Klein's program can be summarized in the following three steps: writing a five-dimensional wave equation[14] for a massive particle (electron/proton); embedding it into a curved space-time; getting a conservation principle. In his first paper [20], he introduces two five-dimensional metrics, related to each other and depending on the four-dimensional x^μ only. The first line element is $d\sigma^2 = \gamma_{\bar{\mu}\bar{\nu}}(x^\mu)dx^{\bar{\mu}}dx^{\bar{\nu}}$ that is decomposed in the following way:

$$d\sigma^2 = \gamma_{55}(dx^5)^2 + 2\gamma_{5\mu}dx^5 dx^\mu + \gamma_{\mu\nu}dx^\mu dx^\nu = \gamma_{55}\left(dx^5 + \frac{\gamma_{5\mu}}{\gamma_{55}}dx^\mu\right)^2 + \left(\gamma_{\mu\nu} - \frac{\gamma_{5\mu}\gamma_{5\nu}}{\gamma_{55}}\right)dx^\mu dx^\nu$$

$$= \alpha\left(d\vartheta\right)^2 + g_{\mu\nu}dx^\mu dx^\nu = \alpha d\vartheta^2 + ds^2.$$

(1)

Klein decided to set $\alpha = \frac{16\pi G}{e^2 c^2}$, where G, c and e are the Newton constant, the speed of light and the electric charge respectively. Using these Ansätze, Klein proved that the variational principle applied to an Einstein-Hilbert action in five dimensions leads to a set of equations that can be identified with the four-dimensional Einstein's equations and the Maxwell's equations[15]:

$$\delta\int d^5 x\sqrt{-\gamma}R^{(5)} = 0 \quad \rightarrow \quad R^{(4)\mu\nu} - \frac{1}{2}g^{\mu\nu}R^{(4)} = \frac{8\pi G}{c^4}T^{\mu\nu} \quad , \quad \partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0$$

This happens because Klein identifies the $g_{\mu\nu}$ of equation 1 with the four-dimensional metric and because he defines the electromagnetic four-potential A_μ, whose field strength is $F^{\mu\nu}$, in the following way:

$$A_\mu = \frac{c}{e}\left(\frac{\gamma_{5\mu}}{\gamma_{55}}\right) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The second line element that he introduces is $d\hat{\sigma}^2 = a_{\bar{\mu}\bar{\nu}}dx^{\bar{\mu}}dx^{\bar{\nu}}$ defined by:

$$d\hat{\sigma}^2 = \hat{k}d\vartheta^2 + ds^2 = d\sigma^2 + \left(\hat{k} - \alpha\right)d\vartheta^2$$

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[10] Because of this assumption, from now on, we will call this approach *the five-dimensional farm*.

[11] This Ansatz is often known as the hypothesis of *cilindricity*.

[12] The γ_{55} component behaves like a scalar field and it is today known as the *dilaton*. Its role was not fully understood at that time: it was set as a constant by these authors.

[13] "After all, what threatens all the Ansatz, which demand universal validity, is the sphinx of modern physics - quantum theory"

[14] As we will see, the wave equation he used is the *Klein-Gordon* (K-G) *equation*.

[15] In this sense, Klein's theory unifies gravitational and electromagnetic forces. $T^{\mu\nu}$ is the usual electromagnetic energy-momentum tensor.

As stated above, in 1925 Klein was convinced that a starting point to reconcile QM with GR was to introduce a relativistic version of the Schrödinger equation in a curved background. As a first try, Klein followed the analogy with light[16], then he decided to set $\hat{k} = \frac{1}{m^2 c^2}$, where m represents the electron mass, to hide it into the previous metric and finally he wrote a five-dimensional *massless* wave equation[17]:

$$a^{\bar{\mu}\bar{\nu}} \left(\frac{\partial}{\partial x^{\bar{\mu}}} + \Gamma^{\bar{\sigma}}_{\bar{\mu}\bar{\nu}} \right) \partial_{\bar{\sigma}} \Psi = a^{\bar{\mu}\bar{\nu}} D_{\bar{\mu}} \partial_{\bar{\nu}} \Psi = 0. \tag{2}$$

The *covariant derivative* $D_{\bar{\mu}}$ tells us that the dynamic is on a curved manifold[18] and the equation reproduces, in the optical geometric limit, an Hamilton-Jacobi equation for a light-like wave. From a modern point of view, this is the wave equation for a massless scalar field.

Using again the metric $a_{\bar{\mu}\bar{\nu}}$ Klein introduces a Lagrangian, that represents the dynamic of a *massless* particle in five-dimensions:

$$L = \frac{1}{2} a_{\bar{\mu}\bar{\nu}} \frac{dx^{\bar{\mu}}}{d\lambda} \frac{dx^{\bar{\nu}}}{d\lambda} = \frac{1}{2} \left[\hat{k} \left(\frac{d\vartheta}{d\lambda} \right)^2 + \left(\frac{ds}{d\lambda} \right)^2 \right]$$

where λ is the affine parameter that parametrizes the geodesics. The conjugated momenta $p^{\bar{\mu}}$, defined as usual by $p^{\bar{\mu}} = \frac{\partial L}{\partial(dx^{\bar{\mu}}/d\lambda)}$, produce a conservation principle, $\frac{dp^5}{d\lambda} = 0$ because the metric does not depend on x^5, and the four-dimensional geodesic equation for a *charged electron* moving in a gravitational and electromagnetic field.

The unusual connection between the *massless* wave equation and the *massive* electron was noted by de Broglie at the beginning of 1927 [21] and in his last paper [22] Klein rewrites equation 2 in an equivalent form, using the inverse of the *first* metric we introduced, $\gamma^{\bar{\mu}\bar{\nu}}$:

$$\gamma^{\bar{\mu}\bar{\nu}} D_{\bar{\mu}} \partial_{\bar{\nu}} \Psi = \mathcal{I}^2 \Psi \qquad \mathcal{I}^2 = \frac{1}{\hbar^2} \left(m^2 c^2 - \frac{e^2 c^2}{16\pi G} \right). \tag{3}$$

From our point of view, this is the K-G equation for a *massive* scalar field living in a curved background.

In particular, in this equation it appears the Planck constant[19] $\hbar$. To understand this fact, we start noting that the momentum p^5 does not have the usual dimension, because α, and x^5 as a consequence, is not dimensionless[20]. This means that the fifth coordinate having the right length dimension is $\tilde{x}^5 = \sqrt{\alpha} x^5$. Relaxing completely the analogy with light, Klein introduces another Lagrangian [23] using the *first* metric:

$$\tilde{L} = \frac{1}{2} m \gamma_{\bar{\mu}\bar{\nu}} \frac{dx^{\bar{\mu}}}{d\lambda} \frac{dx^{\bar{\nu}}}{d\lambda}.$$

This is the Lagrangian for a *massive* particle in five dimensions and the fifth component of the momentum associated to $\tilde{x}^5$ is conserved again: $\frac{d\tilde{p}^5}{d\lambda} = 0$. This means that $\tilde{p}^5$ is a constant to be determined. As before, if we ask to the four-momentum p^{μ} to reproduce the geodesic equation, the following identity must be satisfied: $\sqrt{\alpha} \tilde{p}^5 = 1$. Using the Ansatz of cilindricity, Klein writes $\tilde{p}^5 = \frac{nh}{l}$, as usual in QM, and then he sets $n = 1$. The wave function Ψ must be periodic in the

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[16] The analogy's tool has already guided Klein in introducing the five-dimensional space-time [12].

[17] For scalar functions a covariant derivative is equivalent to an ordinary derivative.

[18] The Cristoffel symbols $\Gamma^{\bar{\sigma}}_{\bar{\mu}\bar{\nu}}$ are related to the metric $\gamma^{\bar{\mu}\bar{\nu}}$.

[19] De Broglie was proud of the fact that this equation contains *all* known constants of Nature.

[20] See equation 1

fifth coordinate and equating both expression for $\tilde{p}^5$ we get the period: $l = \sqrt{\alpha}h$. Inserting this conditions into equation 2 makes the Planck constant to appear. From Klein's point of view, these equalities give a connection between the quantization rules and the quantization of the electric charge. But the only consequence of that is an estimation for the radius of the fifth dimension: the Planck length. Klein did not realize that setting $n = 1$ implies to have a particle with mass equal to the Planck mass, as it happens today in String Theory [18].

In his last paper of the two-year period [22], Klein follows his program again, but looking for a more general approach. In fact the starting point is the following invariant, that is proportional to the action for a massive scalar field[21]:

$$\mathcal{S} = \int \sqrt{-\gamma}\,\mathcal{L}\,d^5x = \int \sqrt{-\gamma}\,\frac{\hbar^2}{m}\left[\frac{1}{2}\partial_{\bar{\mu}}\Psi\partial^{\bar{\mu}}\Psi + \frac{1}{2}\mathcal{I}^2\Psi^2\right]d^5x. \tag{4}$$

Now, varying the action with respect to the dynamical variables Ψ and $\gamma_{\bar{\mu}\bar{\nu}}$, he obtains a new five-dimensional *conservation principle*. In fact we can write:

$$\delta\mathcal{S} = \delta_\gamma\mathcal{S} + \delta_\Psi\mathcal{S} = \int \sqrt{-\gamma}\left[\Theta_{\bar{\mu}\bar{\nu}}\delta\gamma^{\bar{\mu}\bar{\nu}} + \frac{\hbar^2}{m}\left(D_{\bar{\mu}}\partial^{\bar{\mu}}\Psi - \mathcal{I}^2\Psi\right)\delta\Psi\right]d^5x$$

where $\Theta_{\bar{\mu}\bar{\nu}}$ is one half of the energy-momentum tensor of the scalar field as we define it today [24]. Introducing an infinitesimal variation of the coordinates $x^{\bar{\mu}} \to x^{\bar{\mu}} + \xi^{\bar{\mu}}$ the fields vary as usual, $\delta\Psi = \partial_{\bar{\mu}}\Psi\xi^{\bar{\mu}}$ and $\delta\gamma_{\bar{\mu}\bar{\nu}} = D_{(\bar{\mu}}\xi_{\bar{\nu})}$, and integrating by parts we get:

$$\delta\mathcal{S} = \int \sqrt{-\gamma}\left[D^{\bar{\mu}}\Theta_{\bar{\mu}\bar{\nu}} + \frac{\hbar^2}{m}\left(D_{\bar{\mu}}\partial^{\bar{\mu}}\Psi - \mathcal{I}^2\Psi\right)\partial_{\bar{\nu}}\Psi\right]\xi^{\bar{\nu}}d^5x$$

At the end, imposing the variational principle and the wave equation for Ψ in the last equation, Klein obtains the covariant version of the energy-momentum conservation law[22]: $D^{\bar{\mu}}\Theta_{\bar{\mu}\bar{\nu}} = 0$.

We would like to conclude this section with some remarks.

As far as we know, the action for a scalar field, the related energy-momentum tensor and its conservation law, connected with the free equations, appear for the *first* time in Klein's papers. Klein never used the word *field* referring to the Ψ, he always explicitly considers it as the wave function of the electron. In spite of this, his approach is similar to the point of view that will be adopted by developers of QFT in curved backgrounds.

In 1927 Klein and Jordan [25] published the paper where they connect the Bose-Einstein statistics to the K-G equation. Despite this, at that time Klein considered Ψ like the wave function associated to the electron. Pais [26] pointed out that, in the Jordan-Klein paper, the authors treated the wave function as a field, in fact they write Ψ in terms of annihilation and creation operator. Nonetheless, in his subsequent paper on five-dimensional approach, the last we commented, Klein never use this decomposition.

We can appreciate how Klein changed his point of view, during the two-year period. In the Concluding Remarks of the first paper of the period [20], he presents as fundamental his introduction of the fifth dimension. In particular he states that it is possible to understand this radical modification "through quantum theory". At the end of the introduction of his last paper [22], Klein realizes that his approach is incomplete, but he is still convinced that his procedure is a natural starting point to construct a general theory of quantum fields[23].

[21] The action did not have the meaning that we give it today in field theory and an overall minus sign is missing in front of the action.

[22] He also applies this procedure to the free electromagnetic field case, where it works in a similar way.

[23] "Hierdurch und nach dem erwähnten Gesichtspunkt scheint sich diese fünfdimensionale Form der Relativitätstheorie als der natürliche Ausgangspunkt für eine allgemeine Quantenfeldtheorie darzubieten".

3. Conclusions

In the years 1926-1927 the five-dimensional farm was an active research area. All physicists belonging to the farm tried to modify GR to reconcile it with Wave Mechanics. From our modern point of view they lay in an intermediate stage between the first and the second quantization approach. In Klein's papers we find for the first time the action, the equations of motion and the energy-momentum tensor for a massive scalar field living in a curved background. His starting point will be shared by developers of QFT in curved backgrounds. Even though they are not directly connected with QG, these attempts should belong to the history of QG. In particular we suggest to insert Klein's contribution in the *prehistory* of QFT in curved backgrounds.

Acknowledgments

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