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Zero field Quantum Hall Effect in QED3

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Abstract. We study analytic structure of the fermion propagator in the Quantum Electrodynamics in 2+1 dimensions (QED3) in the Landau gauge, both in perturbation theory and nonperturbatively, by solving the corresponding Schwinger-Dyson equation in rainbow approximation. In the chiral limit, we found many nodal solutions, which could be interpreted as vacuum excitations. Armed with these solutions, we use the Kubo formula and calculate the filling factor for the zero field Quantum Hall Effect.

1. Introduction

One of the most remarkable phenomena in condensed matter physics is the Quantum Hall Effect (QHE), both integer [1] and fractionary [2]. Such phenomenon can be described entirely in terms of a relativistic theory such as quantum electrodynamics [3]. The transverse conductivity for relativistic fermions restricted to move in a plane at zero magnetic field becomes

$$\sigma_{xy} = -\nu \, \frac{e^2}{2\pi} \, , \tag{1}$$

in units where $c = \hbar = 1$. Here, e^2 is the electron charge and the number ν is the filling factor, which can be computed from the Kubo formula [4]

$$\nu = \frac{1}{6\pi^2}\epsilon^{\mu\nu\lambda} \int d^3p \, \mathrm{Tr}\left[S(p)\left(\partial_\mu S^{-1}(p)\right) S(p)\left(\partial_\nu S^{-1}(p)\right) S(p)\left(\partial_\lambda S^{-1}(p)\right)\right] \, , \tag{2}$$

where $S(p)$ is the electron propagator and $\partial_\mu = \partial/\partial p^\mu$. The fermion propagator is expressed in its most general form as

$$S(p) = \frac{F(p)}{\not{p} - M(p)} \, , \tag{3}$$

where $F(p)$ and $M(p)$ are the renormalization and mass functions respectively. It was shown by Acharya-Narayana [4] and Jellal [5] that, once wavefunction renormalization effects are neglected ($F(p) = 1$), the general expression for the filling factor acquires the form

$$\nu = \frac{1}{6\pi^2} \int d^3p \, \frac{3M(p) + 2p^2 M'(p)}{[p^2 + M^2(p)]^2} \, . \tag{4}$$

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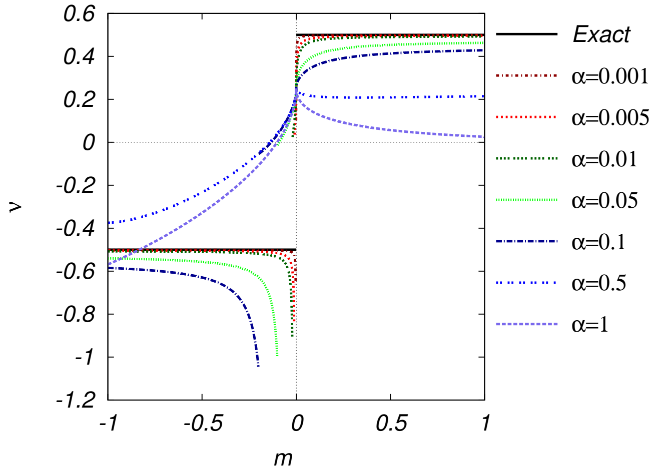

Figure 1. Filling factor ν at 1-loop, as a function of m for different values of the coupling α.

Our purpose is to check the validity of (4) using perturbation theory considering the free propagator and its 1-loop correction. Non-perturbatively, we solve the Schwinger-Dyson Equation (SDE) for the electron propagator in the rainbow truncation. Multiple nodal solutions are found. We calculate the filling factor ν associated to these solutions and interpret them as describing vacuum excitations.

2. Free Fermion Propagator

The inverse free fermion propagator is $S_0^{-1}(p) = \not{p} - m$, where m is the bare mass and we select the Dirac matrices in the form

$$\gamma^0 = \sigma^3 \ , \ \gamma^k = i\sigma^k \ , \tag{5}$$

with $k = 1, 2$ and σ_j are the Pauli matrices. The mass term violates parity [3], and therefore produces a non-trivial filling factor

$$\nu = \frac{1}{2}\frac{m}{|m|}. \tag{6}$$

This value corresponds to the filling factor of the zero field QHE [3]. In the massless limit, it is said that at zero magnetic field, vacuum is half-filled.

3. Fermion Propagator in Perturbation Theory

At 1-loop order, the fermion propagator in covariant gauges is

$$\begin{aligned} \frac{1}{F_1(p)} &= 1 - \frac{\alpha\xi}{2\pi^2 p^2}\int d^3k \, \frac{(k^2+p^2)(k\cdot p) - 2k^2p^2}{q^4(k^2+m^2)} \ , \\ \frac{M_1(p)}{F_1(p)} &= m - \frac{\alpha(2+\xi)m}{2\pi^2}\int d^3k \, \frac{1}{q^2(k^2+m^2)} \ , \end{aligned} \tag{7}$$

where $\alpha = e^2/(4\pi)$, as usual and ξ is the covariant gauge parameter. Performing the integrations involved, we obtain [6]

$$\begin{aligned} \frac{1}{F_1(p)} &= 1 + \frac{\alpha\xi}{2p^2}[m - (m^2 - p^2)I(p;m)] \ , \\ \frac{M_1(p)}{F_1(p)} &= m[1 + \alpha(\xi+2)I(p;m)] \ , \end{aligned} \tag{8}$$

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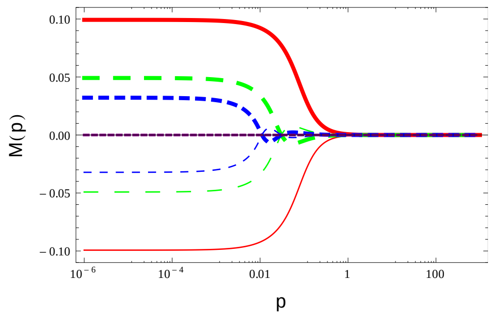

Figure 2. Multiple nodal solutions to the gap equation (11).

where $I(p;m) = (1/p)\arctan(p/m)$. In the Landau gauge ($\xi = 0$), again $F_1(p) = 1$, thus we can use the Kubo formula (4).

Inserting the mass function $M_1(p)$, we obtain the filling factor as function of the electron mass for various values of the coupling α, as shown in Fig. 1. It is observed that when $\alpha \ll 1$, there are no significant deviation from the half-filling value. Deviations are more evident when α is larger, but in this regime, the perturbative behavior is compromised.

4. Dynamical Mass Generation and the Filling Factor

In this section we study the non-perturbative structure of the fermion propagator through the corresponding SDE

$$S^{-1}(p) = S_0^{-1}(p) - 4\pi\alpha \int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k;p) S(k) \gamma^\nu \Delta_{\mu\nu}(q) \,, \tag{9}$$

where $q = k - p$, $\Gamma^\mu(k;p)$ is the fermion-photon vertex, which from now onward we consider bare, $\Gamma^\mu(k;p) = \gamma^\mu$, and the full photon propagator is given by

$$\Delta_{\mu\nu}(q) = -\left[\frac{G(q)}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - \xi \frac{q_\mu q_\nu}{q^4}\right] \,, \tag{10}$$

where $G(q) = 1/(1+\Pi(q))$ is the wavefunction renormalization function of the photon, with $\Pi(q)$ is the vacuum polarization scalar. $G(q) = 1$ corresponds to the bare photon propagator, which is our choice for the truncation of SDE. Dynamical mass generation with reducible fermions has been studied in [7]. It is assumed that along with chiral symmetry, parity is also dynamically broken. This view, however, has been challenged [8]. We stick to the interpretation of [7]. Working in Landau gauge, Eq. (9) reduces to

$$M(p) = 8\pi\alpha \int \frac{d^3k}{(2\pi)^3} \frac{M(k)}{q^2(k^2 + M^2(k))} \,, \tag{11}$$

in Euclidean space. We have found multiple nodal solutions to the above equation in the chiral limit [9], as shown in Fig. 2. These solutions posses a mirror image around the horizontal axis. Such solutions gradually disappear when the bare mass m is increased and it reaches a critical mass (m_{crit}). Above m_{crit}, only the non-nodal solution remains [9].

We insert these solutions into equation (4) and we obtain the filling factor ν as a function of the number of nodes. Results are displayed in Fig. 3. For the non-nodal solution, the filling factor is slightly smaller that $1/2$. The difference arises from the assumptions behind the rainbow

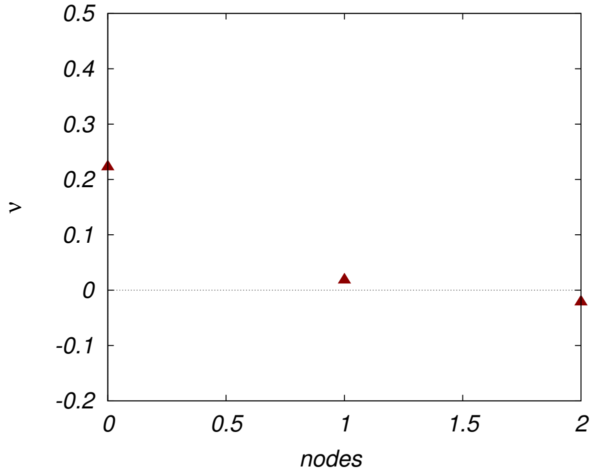

Figure 3. Filling factor ν for the many nodes solutions.

approximation. For higher nodes, we observe that the filling factor, in absolute value, diminishes. This allows to interpret the nodal solutions as vacuum excitations; there are less of those states populating the vacuum.

5. Conclusions

In this study, we arrived to the following conclusions:

- In perturbation theory, there are no significant corrections to the Kubo formula so long as the coupling remains small.
- Noded solutions show that there is a reduction in ν as the number of nodes increases. Therefore, these solutions could be interpreted as vacuum excitations.

All these things are being checked in detail [10].

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