Journal of Physics: Conference Series **465** (2013) 012008 doi:10.1088/1742-6596/465/1/012008

Trajectory control around non-spherical bodies modelled by parallelepipeds

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Abstract. This work is a study of the dynamics around bodies with non-spherical shapes. The gravitational field of a body with non-uniform mass distribution is not central, resulting on orbits around this body that behave different from a keplerian orbit, generating a perturbation on the gravitational field. Irregular objects can be modelled by combining several geometric figures like parallelepipeds. This model can be applied to asteroids, which are objects with non-spherical shapes. The disturbing force generated by these bodies can then be obtained as the sum of the force on each cube. The present paper makes a first study in this problem by showing the evolution of the keplerian elements of orbits around a cube. It also considers a closed loop control system for the trajectory, required to compensate the perturbations due to the irregular shape of the body with the use of a low thrust engine. The main goal is to show that it is possible to use this type of control to keep the orbit keplerian all the time and the details on how to do that.

1. Objectives

The main goal of this work is to study the problem of maneuvering a space vehicle around a body with asymmetric distribution of mass using continuous low thrust in closed loop form for the trajectory.

Some important texts about the formulation for the potential of rectangular objects were developed in the early 90s [1] and continued being developed decades after that [2] [3]. More recently, some progresses on this subject were made [4] [5] [6]. Studies of the property of orbits around elongated rectangular objects were also developed [7].

In this work, the simulator must model the gravitational field of a body following the equations for the potential of a cube, which have a gravitational field that acts different from a spherical body.

Therefore, knowing the deviation on the orbital velocity due to the disturbance of the gravitational field, it is possible to apply the necessary corrections to obtain the desired orbit.

2. Irregular shaped bodies

Asteroids are small bodies orbiting the Sun, with irregular shapes and no atmosphere. Most of the asteroids are located in the Asteroids belt, between the orbits of Mars and Jupiter. A big part of them are carbonaceous, followed by the silicates type, and a smaller part is metallic.

Asteroids with perihelion below 1.3 AU are called Near-Earth Asteroids (NEAs) [8]. Those whose minimum orbit intersection distance with Earth are 0.05 AU, and depending on its absolute magnitude, are considered potentially hazardous asteroids (PHAs). Important data of asteroids were obtained using radar images [9], especially information about the shapes [10].

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The gravitational field of an asteroid is not very strong, but when getting closer to the body the perturbation generated by the irregularity of the shape has a stronger effect. For missions planned to analyze closely an asteroid, stay in orbit around a body like this, or even land on it, the study of the gravitational field of the body is essential.

3. Methodology

The gravitational field of the body is modelled as the field generated by a cubic shaped object, assumed to have a constant density. Depending on the distribution of mass and the general shape on an asteroid, it can be represented as the sum of mass concentrations [11][12], or even several geometric shapes, like polyhedrons [13], cubes and other geometric figures that suits best for the case. In this work the cube was chosen for the simulations, using the formulation of its gravitational force previously developed by MacMillan [1]. The reason to choose the cube is that this shape has already several studies in the literature regarding its potential and, since the sizes and the number of cubes used to represent an asteroid can vary arbitrarily, almost all the asteroids can be modelled by this shape. So, as an initial study of this problem, a study regarding the orbital evolution of orbits around one cube is made, as well as a study of the control technique used to keep this orbit keplerian. The technique used here can be applied to asteroids formed by the addition of several cubes.

To solve this problem, several steps need to be made. Given an initial state in Cartesian coordinates, this state can be transformed to keplerian elements and then propagated for the interval of the simulation step. Therefore, the new keplerian elements can be converted again to a new state expressed in Cartesian coordinates.

The algorithm implemented on the simulator should be capable of solving the equations for the transformation of the keplerian elements to the Cartesian state, so it is possible to maneuver the vehicle around a non-spherical body by applying the necessary corrections. The simulator used here is a tool developed by Rocco [14][15][16]. It has several capabilities, like simulating trajectories around spherical and irregular bodies, perform maneuvers of large and small amplitudes, which are correction and transfer maneuvers, besides other applications.

The formulation for the potential, with the keplerian term and a perturbation due to the cubic shape of the object, is given by [1]:

$$U = \frac{Gm}{r} - \frac{7a^4Gm}{30r^9}[x^4 + y^4 + z^4 - 3(x^2y^2 + x^2z^2 + y^2z^2)] \tag{1}$$

The components of the force (Fx, Fy and Fz) for the perturbation are derived from the formulation for the potential equation (1). They are:

$$Fx = \frac{7a4Gm(x^4 - 5x^2(y^2 + z^2) + (y^4 - y^2z^2 + z^4))}{6(x^2 + y^2 + z^2)^{11/2}} \tag{2}$$

$$Fy = \frac{7a4Gmy(3x^4 + y^4 - 5y^2z^2 + 3z^4 - x^2(5y^2 + 3z^2))}{6(x^2 + y^2 + z^2)^{11/2}} \tag{3}$$

$$Fz = \frac{7a4Gmz(3x^4 + 3y^4 - 5y^2z^2 + z^4 - x^2(3y^2 + 5z^2))}{6(x^2 + y^2 + z^2)^{11/2}} \tag{4}$$

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The terms *x*, *y*, *z* are the coordinates for the position of a particle that is travelling around the asteroid, *G* is the gravitational constant, *a* and *m* are the edge and the mass of the cube, respectively.

The total perturbation force (*Ft*) is given by equation (5):

$$Ft = \sqrt{Fx^2 + Fy^2 + Fz^2} \tag{5}$$

Using the equations of the forces, the components of the velocity change ΔVx, ΔVy, and ΔVz can be obtained. These results make it possible to apply the corrections due to the perturbation. The total variation of velocity to be applied to the satellite (ΔVT) is given by:

$$\Delta VT = \sqrt{\Delta Vx^2 + \Delta Vy^2 + \Delta Vz^2} \tag{6}$$

4. Results

Simulations were made considering the gravitational field of a cube. The initial orbital elements used for the simulations are: semi-major axis of 86 km, eccentricity of 0.13, inclination of 80^o, right ascension of the ascending node of 160^o, and argument of periapsis of 72^o.

The graphics that will be presented were generated for two different cases:

- *Case I:* trajectories with no control, which shows a comparison of the perturbed orbit and a kleperian orbit of reference. The idea is to show the evolution of the orbital elements and how they differ from a keplerian orbit, when no control is applied to the spacecraft;
- *Case II:* using propulsion, considering the correction of the perturbation of the orbit, so that the vehicle stays in a trajectory close to the keplerian orbit of reference.

Figure 1 shows the orbit around the cube using the initial orbital elements described above. Figures 2 to 11 show the evolution of the orbital elements along the time for case I and case II. Case I considers the orbits showing the gravitational perturbation, and case II using propulsion to correct the effects of the perturbation on the orbit of the spacecraft. The reference orbit is represented in magenta, and the perturbed orbit in blue. The technique used to control the trajectory was to determine the current orbit, considering the perturbation force at each instant of time, and the reference keplerian orbit. Then, by the difference between the current and the reference orbits an error signal was generated. This error signal was used to calculate the necessary thrust force to correct the trajectory in such way that the two forces (perturbation and thrust) have the same magnitude and opposite directions, compensating each other and making the orbit stays close to the keplerian orbit all the time.

Figure 12 shows the magnitude of the thrust applied. Figure 13 is the perturbation graphic, where it is possible to notice that the force of the propulsion is obtained from figure 12, but it is acting contrary to the perturbation, as expected.

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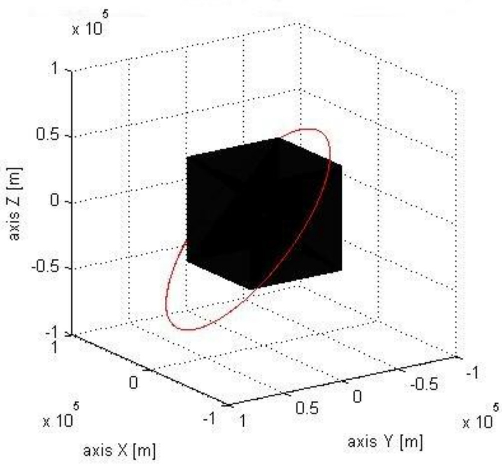

Figure 1. Orbit around a cube.

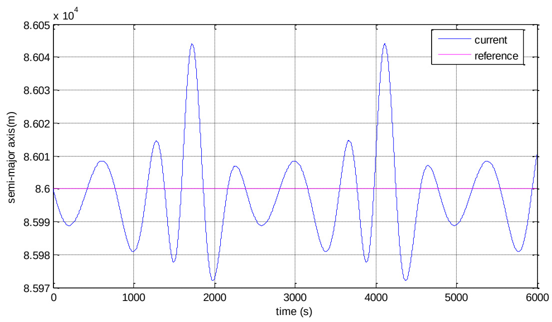

Figure 2. Semi-major axis, case I.

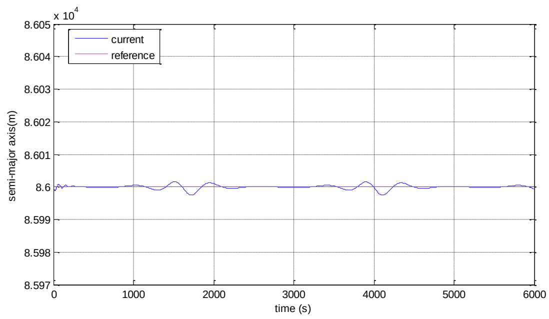

Figure 3. Semi-major axis, case II.

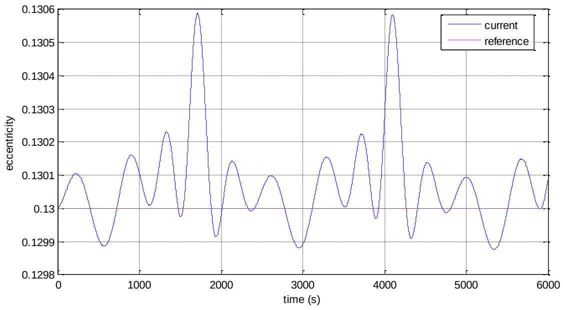

Figure 4. Eccentricity, case I.

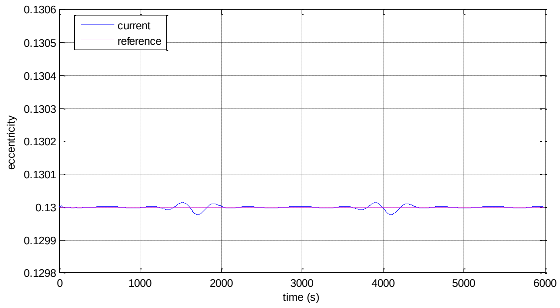

Figure 5. Eccentricity, case II.

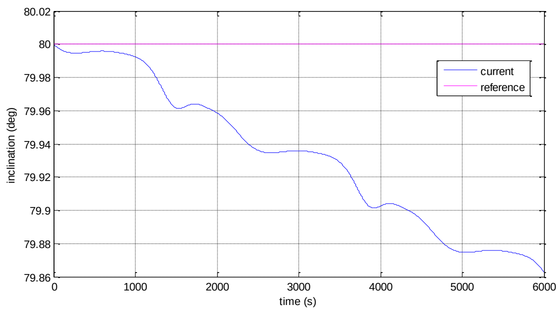

Figure 6. Inclination, case I.

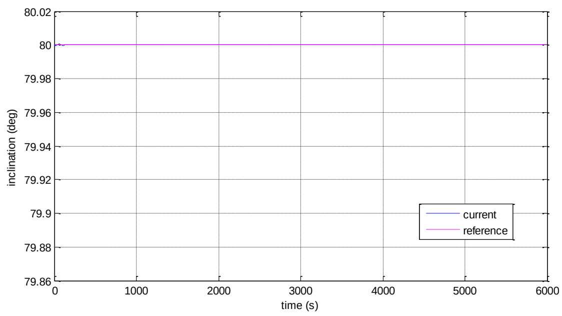

Figure 7. Inclination, case II.

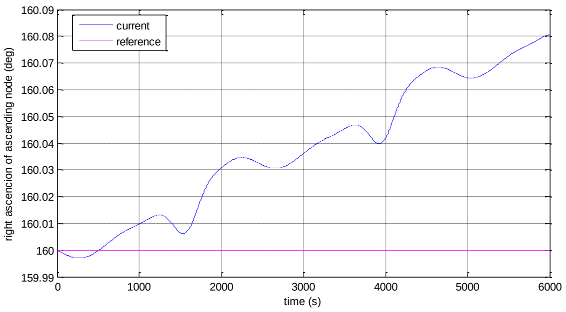

Figure 8. Right ascension of the ascending node, case I.

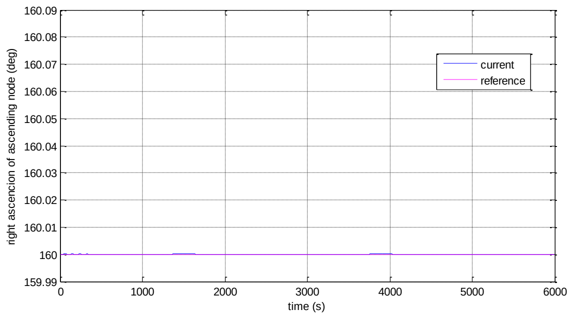

Figure 9. Right ascension of the ascending node, case II.

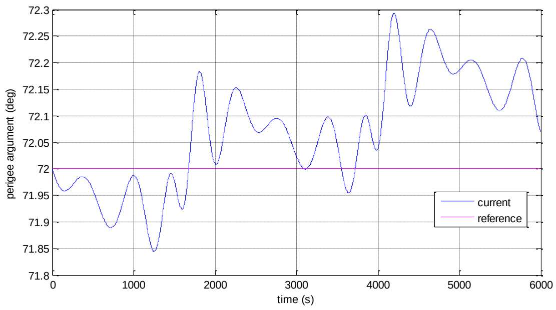

Figure 10. Argument of periapsis, case I.

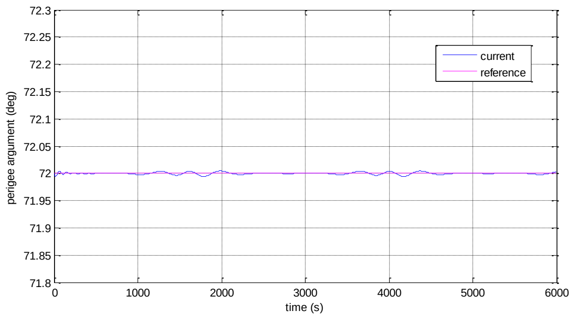

Figure 11. Argument of periapsis, case II.

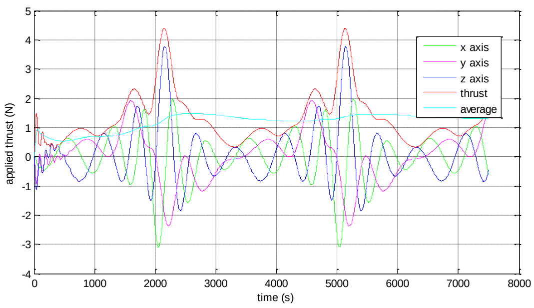

Figure 12. Applied thrust.

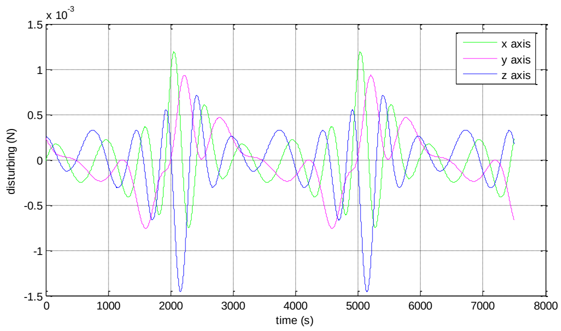

Figure 13. Perturbation.

Case I considers the situation where there is no propulsion applied to correct the perturbation, with the goal of showing the evolution of the orbital elements due to the shape of the object. Case II includes the application of a low thrust in order to control the orbit and to keep the keplerian orbit all the time. The initial inclination and eccentricity chosen, makes the vehicle pass close to the vertices, leading to peaks on the oscillations on the elements during the orbits. Those peaks occur because there are concentrations of extra mass, with respect to a spherical body inside the cube, near the vertices and so the gravitational perturbation near those positions are larger. Considering those peaks, the deviations of the uncontrolled orbit is of the order of 50 km in semi-major axis, 0.0006 in eccentricity, 0.14 degrees in inclination, 0.08 degrees in right ascension of the ascending node and 0.3 degrees in the argument of the periapsis. Simulations using different initial orbits for the spacecraft showed that the effects are stronger, both in terms of the deviations of the uncontrolled orbits or in terms of the fuel consumption to keep the keplerian orbit, when the orbits are closer to the cube.

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5. Conclusion

The work presented on this paper is a study of the dynamics of orbits around a non-spherical body, solving the problem of maneuvering a vehicle orbiting this body using continuous low thrust.

To test the capability of the simulator, examples of a cubic shaped object, with edges measuring 80 km, were used. Therefore, the orbit around this body did not behave as a keplerian orbit. Equations for the force generated by a cube were used on the simulations. The lower the orbit, the greater is the perturbation generated by the body.

The examples showed that the simulator is capable of modelling bodies with non-spherical shapes and also control the orbit around the body, proving that the control system is efficient and achieving the goals of the present research. Considerations for small and irregular bodies like asteroids can be created using cubes, and be modelled as the sum of cubes [17], according to the shape of the body.

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