

Preliminary Attitude Control Studies for the ASTER Mission

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Abstract. This work discusses an attitude control study for the ASTER mission, the first Brazilian mission to the deep space. The study is part of a larger scenario that is the development of optimal trajectories to navigate in the 2001 SN263 asteroid system, together with the generation of orbit and attitude controllers for autonomous operation. The spacecraft attitude is defined from the orientation of the body reference system to the Local Vertical Local Horizontal (LVLH) of a circular orbit around the Alpha asteroid. The rotational equations of motion involve the dynamic equations, where the three angular speeds are generated from a set of three reaction wheels and the gravitational torque. The rotational kinematics is represented in the Euler angles format. The controller is developed via the linear quadratic regulator approach with output feedback. It involves the generation of a stability augmentation (SAS) loop and a tracking outer loop, with a compensator of desired structure. It was chosen the feedback of the p, q and r angular speeds in the SAS, one for each reaction wheel. In the outer loop, it was chosen a proportional integral compensator. The parameters are tuned using a numerical minimization that represents a linear quadratic cost, with weightings in the tracking error and controls. Simulations are performed with the nonlinear model. For small angle manoeuvres, the linear results with reaction wheels or thrusters are reasonable, but, for larger manoeuvres, nonlinear control techniques shall be applied, for example, the sliding mode control.

1. Introduction

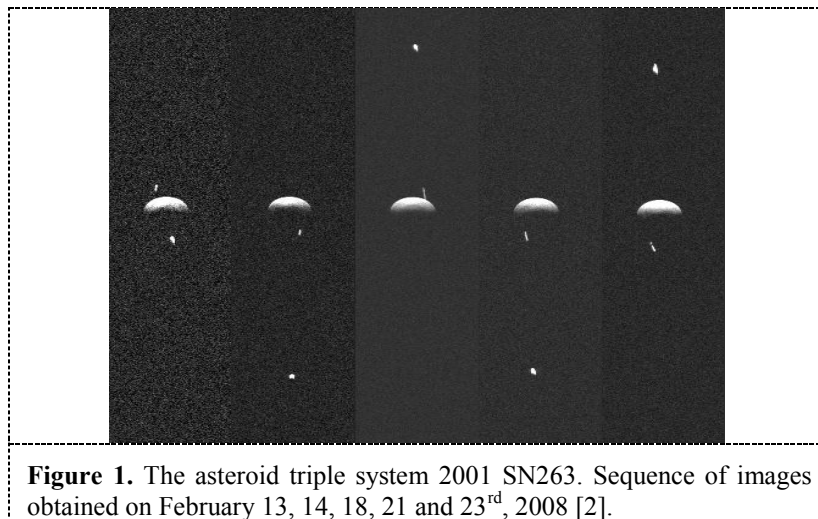
ASTER is the first Brazilian mission to the deep space. It will study the triple asteroid system 2001 SN263. The attitude control study presented here is part of a broader scenario of development of orbit and attitude controllers for autonomous operation. They involve the generation of optimal trajectories to navigate inside the asteroid system, which were already developed in [1]. The 2001 SN263 is composed of three bodies: Alpha (central), Gamma (inner orbit) and Beta (outer orbit) [2]. The objective of this paper is to develop a controller to stabilize and track attitude commands for a spacecraft in a circular orbit around Alpha, which suffers perturbations from Beta and Gamma. The ephemeris of Beta and Gamma around Alpha was previously determined using numerical integration, from physical information of the system given from former studies [3]. The rotational kinematics is represented in the Euler angles format. The controller is developed via the linear quadratic regulator approach with output feedback [4]; it involves the generation of a stability augmentation (SAS) loop, together with a tracker in an outer loop composed by a compensator of desired structure. The feedback of the angular speeds was chosen in the SAS, while a proportional integral compensator was set in the



outer loop. The control parameters are tuned using a numerical minimization that represents a linear quadratic cost, with weighting in the tracking errors and controls [5]. The compensator design is performed with a linearized attitude model for the circular orbit around Alpha, but the simulations are performed with the nonlinear model. The possibilities of use reaction wheels and thrusters as actuators are compared.

2. 2001 SN263

The 2001 SN263 system was discovered by the Lincoln Near-Earth Asteroid Research (LINEAR) project in 2001. Further observations made on 2008 revealed that what was believed to be one single asteroid was in fact a system of asteroids. Observations made during 16 days on February 2008 by the radio astronomers of the Arecibo Observatory in Porto Rico, showed that the 2001 SN263 is in fact a triple system composed by a central body and two satellites (figure 1), Alpha, Beta and Gamma [2]. The components of the system have diameters of about 2.8 km (Alpha), 1.2 km (Beta) and 0.5 km (Gamma). The two smaller bodies orbit Alpha with a semi-major axis of about 17 km (period of 147hrs) and a semi-major axis of about 4 km (period of 46hrs) [3].



The evolution of Beta and Gamma around Alpha were first studied at [3] and no official accurate model has been derived yet. In order to obtain the ephemeris for the period of the mission, the initial conditions extracted from [3] were propagated using an Adams integrator for 20 days from June 1st of 2019 until June 20th of the same year.

3. Equation of motion

Some of the spacecraft's characteristics relevant to this work are: an initial wet mass of 152 to 157 kg with 66 to 71 kg of propellant at the time of launch. In this sense, it is assumed that the spacecraft will reach the system with approximately 100 kg of total mass. Also, the consideration is made that the mass does not vary during the control scenario of this paper.

The model used for control is composed by attitude equations of motion of a rigid spacecraft (equation (1)) with three reaction wheels placed in the body's principal axis of inertia (equations (3), (4) and (5)), or with clusters of thrusters which can deliver a pure rotational acceleration at each of the principal axes (equations (7), (8) and (9)). Both models include perturbation originated from the gravity gradient torque of the three asteroids present in the system (equation (2)). The deduction of these equations is commonly found in the attitude dynamics literature.

The spacecraft angular velocity propagation is presented by equation (1); where, I represents the inertia tensor that, when referenced to the principal axes of inertia, results in a diagonal matrix $I =$

diag(I_1, I_2, I_3), ω is the angular velocity given by p, q and r in each axis, $\vec{H}$ is the angular momentum and $\vec{M}$, the external torque.

$$\frac{d\vec{H}}{dt} = I \cdot \dot{\vec{\omega}} + \vec{\omega} \times \vec{H} = \vec{M} \quad (1)$$

The system's gravity gradient torque is represented by equation (2); where, $c_{i,3}$ is the 3rd column of the transformation matrix from the inertial frame to the body-fixed frame, R_j is the distance between the spacecraft's centre of mass and the asteroid's centre of mass, j represents the number of the body: Alpha is 1, Beta 2 and Gamma 3.

$$\{\vec{g}_c\}_\beta = \sum_{j=1}^n 3 \left(\frac{GM_j}{R_j^3} \right) c_{i,3} \times I c_{i,3} \rightarrow \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \sum_{j=1}^n 3 \frac{GM_j}{R_j^3} \begin{bmatrix} (I_3 - I_2) c_{23,j} c_{33,j} \\ (I_1 - I_3) c_{33,j} c_{13,j} \\ (I_2 - I_1) c_{13,j} c_{23,j} \end{bmatrix} \quad (2)$$

The equations (3) to (6) describe the spacecraft angular acceleration due to reaction wheels; where, h_i represents the momentum stored in each reaction wheel, J_a is the inertia of the reaction wheel (common to the three elements) and u_i is the torque of the wheel applied to the axis of the internal rotor, which is assumed to be equal to the temporal variation of the rotor's angular momentum.

$$\dot{p} = \frac{(-qr + M_x)(x-1)y + (h_3q - h_2r + u_1)(1+xy)}{-1 + J_a + x + J_a xy} \quad (3)$$

$$\dot{q} = \frac{(-pr + M_y)(x+y) + (-h_3p + h_1r + u_2)(1+xy)}{(J_a - 1)(1+xy)} \quad (4)$$

$$\dot{r} = \frac{(pq - M_z)(1+y)x + (h_2p - h_1q + u_3)(1+xy)}{J_a(1+xy) - 1 - y} \quad (5)$$

$$x \triangleq \frac{I_2 - I_1}{I_3}; \quad y \triangleq \frac{I_3 - I_2}{I_1} \quad (6)$$

Finally, equations (7) to (9) present the spacecraft angular acceleration due to thrusters; where u_i is the external torque around applied in the axis i , they are the controlled total thrust force (two anti parallel vectors) that include the thrust force and distance from the spacecraft's centre of mass.

$$\dot{p} = y \left[-qr + M_x - \frac{u_1}{I_2} \right] \quad (7)$$

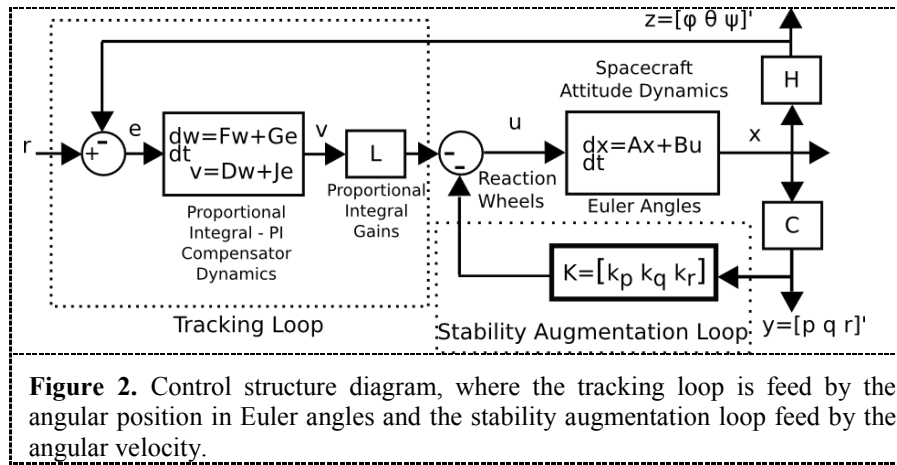
$$\dot{q} = \frac{x+y}{1+xy} \left[pr - M_y - \frac{u_2}{I_2} \right] \quad (8)$$

$$\dot{r} = x \left[-pq + M_z - \frac{u_3}{I_2} \right] \quad (9)$$

4. Control structure and design

The attitude control is based on two loops with distinct function, the first loop is responsible for the system stabilization based on the angular velocity (SAS) and the second loop is responsible for tracking the commands based on the attitude of the spacecraft in Euler angles. Figure 2 presents the diagram of the control structure. It depicts the two loops and their respective gains, as well as the attitude dynamics and the control input, u .

The linear quadratic regulator technique is used to compute the controllers' gains, as suggested in [4]. It involves the generation of a stability augmentation (SAS) loop with p, q and r feedback and a tracking outer loop, with a proportional integral (PI) compensator. The parameters are adjusted using a numerical minimization that represents a linear quadratic cost, with weighting matrixes in the tracking error and controls [4], [5].



The dynamics for the plant and compensator are defined as [5]:

$$\dot{x}_a = A_a x_a + B_a u + G_a r, y_a = C_a x_a + F_a r, z = H_a x_a \quad (10)$$

$$x_a = \begin{bmatrix} x \\ w \end{bmatrix}, y_a = \begin{bmatrix} y \\ v \end{bmatrix} \quad (11)$$

$$A_a = \begin{bmatrix} A & 0 \\ -GH & F \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, G_a = \begin{bmatrix} 0 \\ G \end{bmatrix}, C_a = \begin{bmatrix} C & 0 \\ -JH & D \end{bmatrix}, F_a = \begin{bmatrix} 0 \\ J \end{bmatrix}, H_a = \begin{bmatrix} H & 0 \end{bmatrix} \quad (12)$$

The variables in the above equations are defined in figure (2). The control can be given by the augmented equation $u = -[K \ L] \begin{bmatrix} y \\ v \end{bmatrix} = -K_a y_a$. The control gains are determined from the minimization of the quadratic index $J = \frac{1}{2} \int_0^\infty \tilde{e}^T \tilde{e} + \tilde{u}^T R \tilde{u} dt$; where, $\tilde{e} = e - \bar{e}$, $\tilde{u} = u - \bar{u}$ are the deviations of the tracking error and control input with respect to their steady state values $\bar{e}$ and $\bar{u}$. In reference [4], the problem of minimize the *functional* in the quadratic index J is converted to a problem of *function* minimization. In this sense, the performance index is given by $J = \frac{1}{2} \text{tr}(PX)$; where 'tr' is the trace operator and the matrices P and X are given by:

$$A_c^T P + P A_c + Q + C_a^T K_a^T R K_a C_a = 0 \quad (13)$$

$$A_c = A_a - B_a K_a C_a, B_c = G_a - B_a K_a F_a \quad (14)$$

In equation (14), A_c and B_c are the closed loop matrices. The equation (13) is of Lyapunov's type, it generates the symmetric positive definite matrix P . The existence and uniqueness of P requires that A_c is stable; a stable matrix in this context means that it is associated with a stable closed loop system, i.e. all their eigenvalues shall have negative real parts. The matrix Q is the weighting in the state, given from the output matrix H_a : $Q = H_a^T H_a$. The matrix X in the performance index is a quadratic function of the steady state $\bar{x}$: $X = \bar{x} \bar{x}^T = A_c^{-1} B_c r_0 r_0^T B_c^T A_c^{-T}$; where $A_c^{-T} = (A_c^T)^{-1}$ and r_0 is the amplitude of a step function applied as a reference command. Hence, the design is performed under the *hypothesis of step reference commands*.

In the design of the PI compensator, the following matrices where chosen:

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, J = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15)$$

$$K = \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_q & 0 \\ 0 & 0 & k_r \end{bmatrix}, L = \begin{bmatrix} k_{\phi i} & k_{\phi p} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{\theta i} & k_{\theta p} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{\psi i} & k_{\psi p} \end{bmatrix} \quad (16)$$

The matrices in equation (15) define the *structure of the compensator*, and equation (16), in turn, determines a *predefined structure* for the control gain matrices. There, k_s is the gain of the angular speed $s = p, q, r$; k_{ai} and k_{ap} are the integral and proportional gains of the tracking error in the angle $a = \phi, \theta, \psi$.

During the design of the controllers for the spacecraft, the matrices of the linearized equations were obtained from the truncation of first order terms in the Taylor series expansions. In this process, the equations were also simplified by removing the terms associated with the gravity gradient torque of the satellites Beta and Gamma. In this way, the linear model treats these elements as *external perturbations*.

The design parameter is the control weighting matrix R . In the determination of the control laws for booth actuators, thrusters or reaction wheels, this matrix is $R = \frac{1}{100} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The numerical implementation is performed in MATLAB. The minimization of the cost $J = \frac{1}{2} \text{tr}(PX)$ is carried out by the function *fmincon*. The process that determines the control gains also requires an *initial stabilizing guess*. This is done by a previous minimization where the cost is the spectral radius of matrix A_c (the spectral radius of a matrix is the largest real part of their eigenvalues).

5. Results

This section present closed loop simulations performed with the nonlinear model also considering the gravity gradient torques of the Beta and Gamma. Additionally, the results are determined with perturbations represented by initial conditions in angular velocity, $[p_0, q_0, r_0] = [-2^\circ/\text{s}, -3^\circ/\text{s}, 5^\circ/\text{s}]$, and angles, $[\phi_0, \theta_0, \psi_0] = [1^\circ, -2^\circ, 4^\circ]$. Figure 3 shows the response to a tracking command given by a step function of amplitude $[\phi_r, \theta_r, \psi_r] = [2^\circ, 1^\circ, -10^\circ]$.

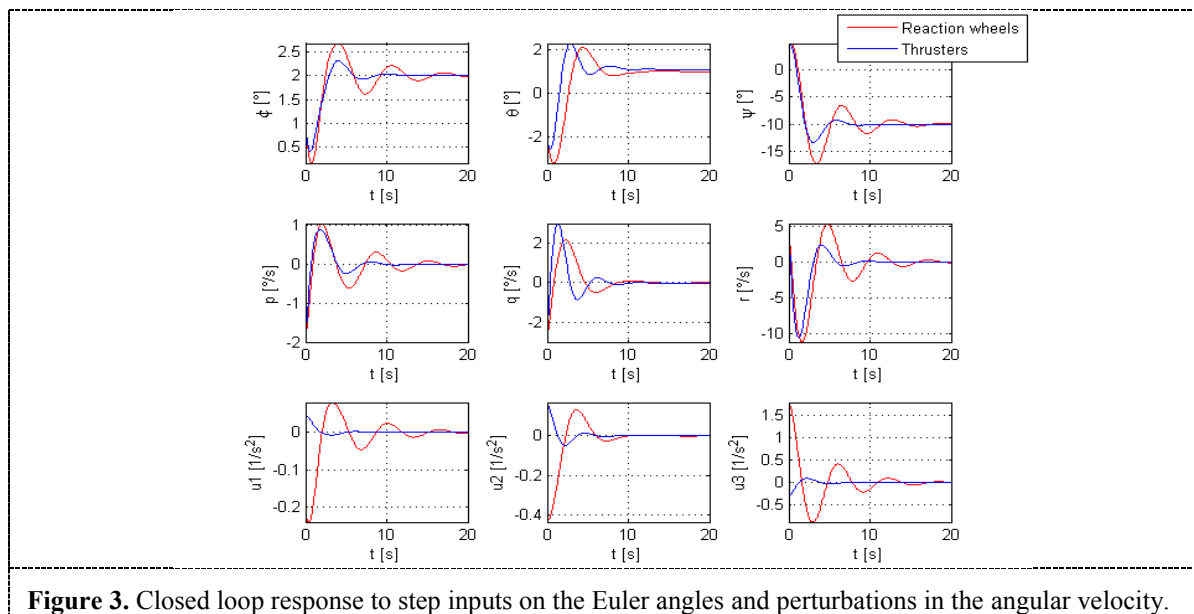


Figure 3. Closed loop response to step inputs on the Euler angles and perturbations in the angular velocity.

These results show that both actuators can cope with the disturbance; however, the torque generated by the thrusters has amplitudes with one order of magnitude smaller than the amplitudes for the reaction wheels torque; which is understandable since the acceleration provided by the thrusters can be much higher than the one provided by the reaction wheels. A better settling time can be noticed with the use of thrusters. They also generate responses with smaller overshoots. Nevertheless, the result given by using thrusters can also be reached with reaction wheels by properly adjusting the control weighting matrix. Moreover, the extra time gained with a faster convergence is negligible with respect to the orbital period. So, both actuators are a reasonable choice for this task. However, one

shall note that thrusters are a more expensive controller, because they use fuel that is stored in spacecraft, while the reaction wheel can be powered with electric energy obtained from solar panels.

In order to test the validity of the use of this linear technique for controlling the spacecraft attitude, figure (4) shows a simulation performed with step inputs of amplitude $[\phi_r, \theta_r, \psi_r] = [40^\circ, 40^\circ, 40^\circ]$. The performance of booth actuators is clearly degraded. Reaction wheels cannot generate a steady response in the observed time horizon. On the other hand, thrusters also generate a very oscillatory behaviour. This test suggests the necessity of use nonlinear control laws during the execution of large angle manoeuvres.

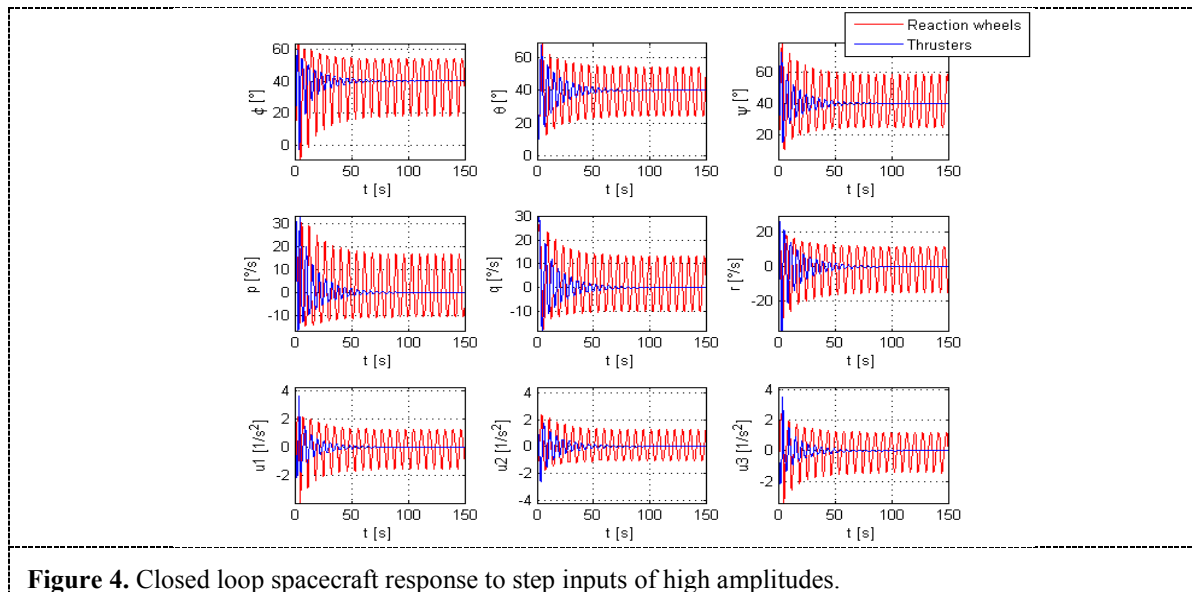


Figure 4. Closed loop spacecraft response to step inputs of high amplitudes.

6. Conclusions

In this work, an attitude controller for the spacecraft in ASTER mission was successfully developed using both reaction wheels and thrusters by means of two distinct loops for stabilization and command tracking with the controller based on Euler angles. A control methodology was used to generate the gains based on a linear quadratic regulator approach with output feedback. The linear model is given by a circular orbit around Alpha ignoring the perturbations due to the gravitational influence of the two remaining satellites: Beta and Gamma. The evaluation was performed via nonlinear simulations, the gravity gradient torque of all the three bodies in the asteroid system was considered. Under small amplitude commands, booth actuators, reaction wheels or thrusters, can provide similar performance. Some differences in settling time are negligible when compared with the orbital period. However, under the application of large amplitude commands, the performance of booth actuators is depreciated, which suggest that nonlinear control laws shall be used for large angle manoeuvres.

References

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