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Powered Swing-By Maneuvers around the Moon

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Abstract. A Swing-By maneuver occurs when a satellite approaches a celestial body to gain or lose energy from its gravitational field. The present work studies Swing-By maneuvers that are combined with the use of an impulsive thrust in different directions during the passage of the spacecraft by the periapsis of its trajectory around the Moon. The main objective of this type of maneuver is the fuel economy for orbital transfers. From the results, it is visible that the best direction to apply the impulse is not the direction to the motion of the spacecraft, as might be expected. In fact, by using a different direction, it is possible to maximize the effects of the Swing-By by decreasing the periapsis distance and/or increasing the turning angle of the maneuver, that are the key parameters to specify the variation of energy due to the Swing-By. The changes in the periapsis distance and turning angle cause modifications in the geometry of the original Swing-By, generating a maneuver with new parameters. This new Swing-By compensates for the loss of energy transfer that results of applying the impulse in a in a non-tangential direction.

1. Introduction

The maneuver that makes a spacecraft to gain or lose energy from the gravity field of a celestial body due to a close approach is called Swing-By. Several space missions used this technique and there are many papers in the literature studying this type of problem. An interesting one is written by Longuski and Strange [1], which developed a graphical method to study Swing-By trajectories with the goal of reducing the mass when starting the mission and/or the flight time. The literature also shows applications of successive Swing-Bys, like in Dunham and Davis [2], that studied multiple swing-bys around the Moon. Prado and Broucke [3] simulated and classified Swing-By maneuvers using the planet Jupiter as the body for the close approach and calculated the effects of these Swing-Bys. The goal was to check which ones of those orbits have a passage near the Earth. It was shown which one of those trajectories have a potential use for missions involving departures and/or arrivals in the planet Earth. Prado [4] studied the Swing-By trajectories under the model given by the elliptic restricted three-body problem and classified them in four groups: elliptic direct, elliptic retrograde, hyperbolic direct and hyperbolic retrograde. Prado [4] also showed the effects of the eccentricity of the primaries and of the position of the secondary body in the results. Several families of orbits have been found. The results obtained can be used to find optimal parameters that solve several types of problems. McConaghy et al [5] combines Swing-Bys maneuvers with low thrust, in order to obtain optimal trajectories. Gomes and Prado [6] extended this type of study to cover the situation where the Swing-

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By is made by a cloud of particles, instead of a single one, that can happen when an explosion of a spacecraft occurs. Other examples of missions can be seen in references [7] and [8].

This present research studies the Swing-By maneuver combined with the application of an impulse, which is applied exactly when the spacecraft passes by the periapsis of its orbit around the Moon. This impulse can be applied in different directions. These conditions were studied in the Earth-Moon-Spacecraft system. This work is a generalization of the paper by Prado [9], where similar studies were made. The idea is to find maneuvers that can be later used by the ASTER mission [10], which is a mission planned to visit a triple Near-Earth Asteroid (NEA) called 2001 SN263.

There are three parameters that describe the Swing-By maneuver: V_∞, the magnitude of the approach velocity of the spacecraft with respect to M_2, that is the Moon; $\mathbf{r_p}$, the periapsis distance for the Swing-By, which is the vector connecting the Moon to the spacecraft; and ψ, that is the approach angle, the angle between the vector $\mathbf{r_p}$ and the line M_1-M_2 (Earth-Moon). The vectors are represented in bold.

For the case of the powered Swing-By, that is a Swing-By maneuver combined with the application of an impulse, there are two additional parameters to define the impulse: δV, the magnitude of the impulse, given in km/s and α, the angle between the velocity vector of the satellite in the first orbit and the impulse applied. This angle α defines the direction of the impulse applied.

2. Dynamical System

In this work we used the model given by the restricted three-body problem. Numerical integrations were performed to obtain the results. M_1, the Earth, is the primary body, with the largest mass, M_2, the Moon, is the secondary body, orbiting M_1, and M_3 is the particle, with negligible mass, that orbits the Earth and makes a Swing-By with the Moon. The canonical system of units is used, adopting the unit of distance as being the distance between the Earth and the Moon, the unit of mass is defined as the mass of the Earth added to the mass of the Moon and the unit of time is chosen such that the period of the motion of the primaries is 2π. In this way, the equations of motion in the rotational system (in this system M_1 and M_2 are in fixed positions) are [11]:

$$\ddot{x} - 2\dot{y} = \frac{\partial\Omega}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial\Omega}{\partial y} \tag{2}$$

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{3}$$

The potential Ω, that includes the gravitational and the centrifugal forces, depends of r_1 and r_2, which are the distances Earth-spacecraft and Moon-spacecraft, respectively, in canonical units. It is clear that, when r_1 and r_2 are small or near zero, the numerical integration has accuracy problems. To avoid this situation the method given by the Lemaître Regularization [12] is used. This method avoids singularities by using a substitution of variables.

We call θ the true anomaly of the point where the impulse is applied, and, in general, it is obtained through the scalar product of $\mathbf{r_p}$ and $\mathbf{r}$, where $\mathbf{r}$ is the position vector of this point with respect to M_2. In this case, the study was done with the impulse applied at the periapsis, so $\mathbf{r_p}$ and $\mathbf{r}$ are coincident ($\mathbf{r} = \mathbf{r_p}$), then $\theta = 0°$ in Figure 1, that shows the geometry of the problem.

The steps of the algorithm to solve this problem are: 1) From the point for the application of the impulse, which in this case is $\theta = 0$ °, integrates the orbit backwards in time [13] to get the information of the first orbit, before the close approach; 2) At the point P, shown in Figure 1, the impulse is applied, with magnitude δV and making a direction α with respect to the velocity of the spacecraft; 3) From the application of the impulse, the integration of the orbit is performed forward in time to get the information from the new orbit, after the passage; 4) To finalize, the calculation of the variation of energy is made, by making the difference between the energies of the orbits after and before the close approach.

It is important to emphasize that the impulse is applied inside of the Moons' sphere of influence [14] and that the direction of this impulse is defined by the angle α, with respect to the velocity of the spacecraft. Then, by varying the values of the angle α, it is possible to find the geometry that leads to the maximum variation of energy.

3. Results

For all cases studied in this paper we used, as initial condition, r_p = 1.1 radius of the Moon. The goal is to get the best direction to apply the impulse, in order to have the maximum variation of energy.

We studied the cases where θ = 0°, α varying from -180° to 180°, in steps of 0.1. The values that were used for ψ, are ψ = 90°, ψ = 180°, ψ = 225°, ψ = 270° and ψ = 315°.

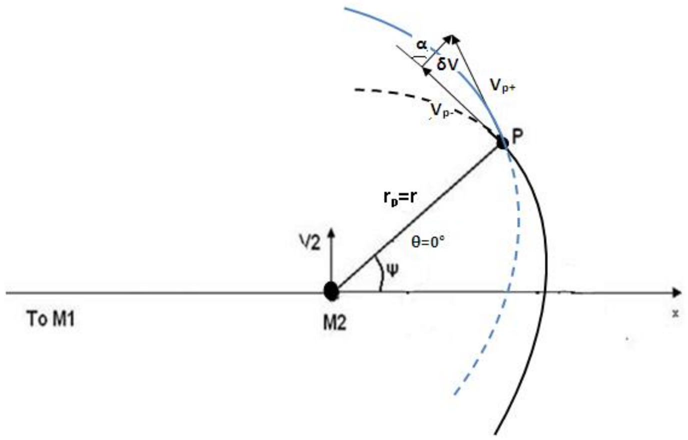

Figure 1 – Geometry of the Swing-By when applying an impulse at the periapsis of the orbit.

For the cases where ψ = 90°, the maximum variation of energy occurred when α = 0°, for all values of the magnitude of the impulse. This fact happens because, when ψ = 90°, the maneuver has its maximum energy loss due to the Swing-By (see reference [9]). This conclusion comes from Equation (4) [9], that show the energy variation due to the Swing-By, where V_2 is the velocity of the Moon around the Earth and δ is half of the deflection angle.

$$\Delta E = -2V_2 V_\infty \sin(\delta)\sin(\psi) \tag{4}$$

In this situation it is necessary to avoid directions of impulses that can lower the periapsis distance, so increasing the effects of the Swing-By. It is necessary to take into account that, when applying the impulse in the vehicle, the magnitude of r_p may change. If α is negative, r_p tends to decrease, because the spacecraft is directed to the Moon. The point where the periapsis of the first orbit is located is also part of the second orbit, so it is impossible to reduce the value of r_p. Therefore, the best solution is to keep the original value of r_p, and the value of the angle α to reach this situation is zero. Figures 2 and 3 show the results for the case ψ = 90°. Those results are different from Prado [9], because this research considered the problem of the maximization of the magnitude of the variation of energy. In the preset work the magnitude is not taken and the variation of energy itself has to be maximized. So, in situations where the Swing-By decreases the energy of the spacecraft, the goal is to minimize this variation to reduce the loss. Those figures show the angle α, in degrees, in the horizontal axis and the variation of energy, in dimensionless units, in the vertical axis. It is visible the presence of a maximum variation for the energy.

Figures 4 and 5 show the results for the case $\psi = 270°$. In these cases, the maximum variation of energy occurred when α is negative. According to Prado [9], this fact happens because there is an increase in the deflection angle. Since the value of α is negative, the radial velocity gets a negative component from the impulse, and so the spacecraft is directed to the Moon. This makes the turning angle to increase and, consequently, the variation of energy is increased. Another reason to increase the performance of the Swing-By is that the periapsis distance is also decreased by the impulse with negative values of α, so a closer passage by the Moon is reached. There is a loss of energy due to the fact that the impulse is not applied in the direction of the motion of the spacecraft, but the net result of these gains and losses is positive, and then the gain due to the more efficient Swing-By compensates the loss of energy transfer that occurs due to the fact that the impulse vector is not aligned to the direction of the velocity vector of the spacecraft. This explanation covers all cases with $180° < \psi < 360°$. Table 1 shows more details, emphasizing the values of α that result in maximum variations of energy for the cases where $\psi = 225°$, $\psi = 270°$ and $\psi = 315°$. The symbol ΔE_{max} represents the maximum variation of energy and it is shown in the canonical system of units.

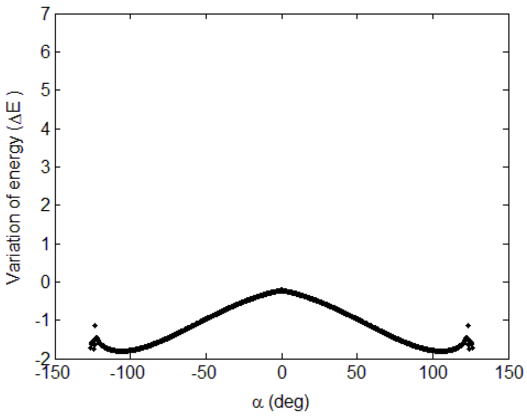

Figure 2 – Variation of energy (canonical system of units) for $\theta = 0°$, $\psi = 90°$ and $\delta V = 0.5$ km/s.

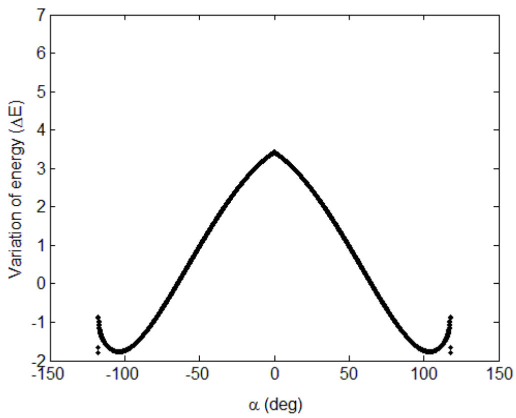

Figure 3 – Variation of energy (canonical system of units) for $\theta = 0°$, $\psi = 90°$ and $\delta V = 1.5$ km/s.

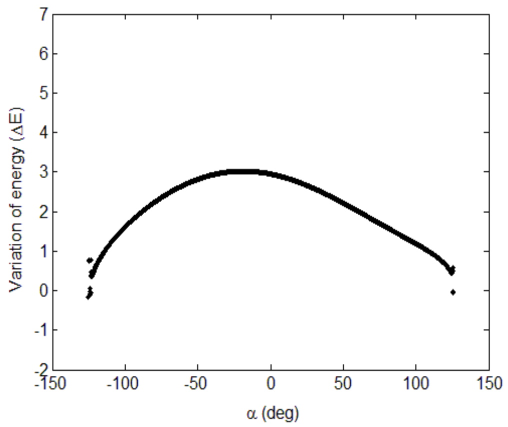

Figure 4 – Variation of energy (canonical system of units) for $\theta = 0°$, $\psi = 270°$ and $\delta V = 0.5$ km/s.

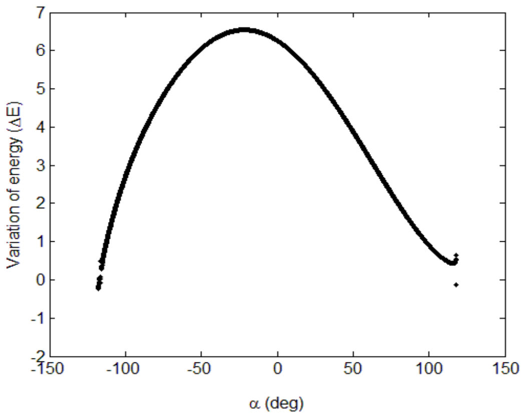

Figure 5 – Variation of energy (canonical system of units) for $\theta = 0°$, $\psi = 270°$ and $\delta V = 1.5$ km/s.

It is also visible from Table 1 that, when $\psi = 180°$, the maximum variation of energy occurs when α has a small negative value. This happens because when $\psi = 180°$, the effect of the Swing-By in the maneuver is null and, if the impulse has a negative component, the spacecraft is deviated to get closer to the Moon, so generating a trajectory that has an angle of approach greater than 180°, so increasing the energy of the spacecraft. At the same time the periapsis distance is reduced, what increases the gains due to the Swing-By. A larger deflection angle is also reached. Those facts added together generates an energy gain that would not exist with the application of the impulse at $\psi = 180°$.

Looking now at the results for $\psi = 225°$, $\psi = 270°$ and $\psi = 315°$, it is visible that the value of α is always negative, for the reasons already explained. But the magnitude of the values are larger for $\psi = 225°$, because the orbits are still not located at the maximum value for the variation in energy, so the system has more flexibility to search for this optimal condition. After passing by the value $\psi = 270°$, the impulse is applied later in the passage by the periapsis, so there is less opportunity to influence the Swing-By. So the value of α is smaller, going close to zero, to take benefit of the larger transfer of energy due to the alignment of the impulse and the velocity of the spacecraft. The magnitude is smaller, but the value is still negative to cause the effects on the Swing-By already explained above.

Table 1. Maximum variation of energy for $r_p = 1.1$ Moon's radius.

θ	ψ	δV (km/s)	ΔE_{max}	α
0º	90º	2.0	5.6036	0º
0º	90º	2.5	8.0233	0º
0º	90º	3.0	10.6834	0º
0º	90º	3.5	13.5860	0º
0º	90º	4.0	16.7327	0º
0º	180º	2.0	3.8303	-6.5º
0º	180º	2.5	5.6045	-5.0º
0º	180º	3.0	7.6491	-4.0º
0º	180º	3.5	9.9574	-3.2º
0º	180º	4.0	12.5253	-2.7º
0º	225º	2.0	6.0703	-27.3º
0º	225º	2.5	8.0473	-26.2º
0º	225º	3.0	10.2934	-25.4º
0º	225º	3.5	12.8034	-24.8º
0º	225º	4.0	15.5739	-24.4º
0º	270º	2.0	8.6913	-22.0º
0º	270º	2.5	11.1007	-22.1º
0º	270º	3.0	13.7686	-22.1º
0º	270º	3.5	16.6932	-22.2º
0º	270º	4.0	19.8732	-22.2º
0º	315º	2.0	10.2319	-10.2º
0º	315º	2.5	13.0096	-10.7º
0º	315º	3.0	16.0312	-11.1º
0º	315º	3.5	19.2992	-11.5º
0º	315º	4.0	22.8151	-11.6º

4. Conclusions

With this study it was possible to analyze the importance of the direction of the impulse applied in a powered Swing-By maneuver, which is a maneuver that combines a close approach to a celestial body with the application of an impulsive thrust. The goal was to search for geometries that allow a maximum variation of the energy. The angle α helps to determine the curvature of the orbit of the spacecraft after the impulse, so reducing r_p and increasing the effects of the Swing-By. It was confirmed that the direction of the impulse is an important parameter in the powered Swing-By and it has strong influences in the behavior of the spacecraft. The results obtained also showed that, in most cases, captures occurred for values of α near the borders. That fact happened because when α is near -180° or 180°, there is a component of the impulse opposite to the motion of the spacecraft, that makes it to brake, thus reducing the size of the orbit and increasing the chance of capture.

The main conclusion is that the best way to apply the impulse in the orbit of the spacecraft is when it is applied out of the direction of the motion of the spacecraft, i.e., $\alpha \neq 0°$, usually making a negative angle and having a magnitude that depends on the angle of approach. The goal is to deviate the trajectory of the spacecraft to send it to pass closer to the Moon. This magnitude decreases with the angle of approach, been around 20 degrees for the situation where $\psi = 270°$. The reason for this decrease is that the spacecraft changes more its trajectory when it still did not pass by the geometry of maximum variation and so it tries to reach this situation. After passing by that, it changes its trajectory in smaller variation. When this situation happens, the maneuver gets less energy from the impulse, but it gains more energy from the Swing-By.

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