

Reconstruction of 3D Radar Targets from Profile Functions in Arbitrary Directions with Level-set

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Abstract. The profile function of an object is defined as its transverse cross-sectional area versus distance along the observing direction and it is used by the ramp response technique to identify radar targets. Existing reconstruction algorithms have good performance with profile functions from 3 mutually orthogonal directions, while they give distorted results otherwise. To solve this inverse problem, we use the level set method, which iteratively deforms the shape of the target under a velocity field. An appropriate velocity is used and satisfactory reconstructed 3D images are presented for arbitrary directions.

1. Introduction

Radar imaging from profile functions using the ramp response technique [1] can be used for generating 3-dimensional images of radar targets (even stealthy or buried) so as to identify them. With existing reconstruction algorithms [2] [3], reconstructed 3D images are quite accurate for mutually orthogonal observing angles, while distorted results are obtained for non-orthogonal directions. Nevertheless, due to the limited view range of remote sensing radar equipments, in practice, it is almost impossible to guarantee 3 mutually orthogonal observing directions. Therefore, in this paper, we are concerned with image reconstruction from profile functions in arbitrary directions.

The level set method, devised by Osher and Sethian [4] [5], represents the evolving object as the zero level of an implicit higher dimensional function. This implicit representation keeps the object binary and evolves the indirect parameter, i.e. the level set function, instead of the parameterized shapes, which is completely adapted to our problem. Moreover, it has been effectively illustrated in the inverse scattering problem [6] [7]. Consequently, we apply this method to obtain an optimal shape of the target with profile functions in arbitrary directions.

2. Radar imaging from profile functions

As presented in [1], the ramp response of a radar target $h_r(t)$ is approximately proportional to its exact cross-sectional area perpendicular to the observing direction, noted the “geometrical” profile functions, $A_g(x)$:

$$h_r(t) \approx -\frac{1}{\pi c^2} A_g(x) \quad \text{with} \quad x = \frac{ct}{2} \quad (1)$$

where c is the speed of light in freespace, t the time variable, and x the space variable.



For applications and experiments using the ramp response technique, the profile function, defined as the “physical” profile function $A_p(x)$, which is an estimate of the geometrical profile function, is calculated from its transient ramp response by:

$$A_p(x) = -\pi^2 h_r(t) \approx A_g(x) \quad \text{with} \quad x = \frac{ct}{2} \quad (2)$$

The initial reconstruction algorithm from profile functions proposed by Young [2], named “approximate limiting surface”, uses a set of hyperbolic surfaces limiting the contour of the object for each of 3 observing directions and it is limited to single convex objects. On the contrary, the reconstruction algorithm presented in [3] uses the product of profile functions in 3 directions as a weighting function to account for the probability that one point belongs to the object. It exploits the information of profile functions more efficiently, and therefore extends to non-convex and separated objects. Unfortunately, both algorithms generate distorted results in non-orthogonal directions. Therefore, we aim to find a method for reconstruction in arbitrary directions.

3. Reconstruction using the level-set method

We attempt to use an iterative process to obtain an optimal estimate of the target by minimizing the mismatch between the data, i.e. profile functions of the unknown object, and the profile functions of the updated object. Our problem can be formalized in the form of

$$M^d u = A^d \quad (3)$$

where A^d is the observed data, the profile function observed in direction d , u the vector representing the unknown binary object we are looking for, M^d the observing matrix (or mapping matrix) and d the index number of directions. $d = 1, 2, 3$ in our case with 3 observing directions.

Most of iterative methods evolve the explicit function of object shape during the iteration. On the contrary, the level set method [4] represents the shape as the zero level of a higher order level set function φ , which is negative inside the object and positive outside, so as to keep the evolving object binary and to perform shape deformation in an implicit way.

The deformation of the object is formalized as a Hamilton-Jacobi equation for the level set function φ under a velocity field [7]:

$$\frac{\partial \varphi}{\partial t} + V |\nabla \varphi| = 0 \quad (4)$$

where φ is the level set function, $|\nabla \varphi|$ the gradient of φ , V the velocity.

Following the formulation in [7] and the desirable properties of the velocity [8], we propose a velocity adapted to our case:

$$V = -\sum_d (M^d)^T (M^d u - A^d) \quad (5)$$

where $(X)^T$ is the transpose of a matrix X .

4. Numerical results

In this section, illustrative results, which are reconstructed using the level set method, are presented. It is important to note that geometrical profile functions are used to validate this method. Hence, the mapping matrix M describes the relationship between the object and the geometrical profile function. The algorithm to construct the geometrical mapping matrix M and to calculate the geometrical profile function of a 3D object is presented in [9].

To show the performance in arbitrary directions, we choose 2 sets of observing directions: 3 mutually orthogonal directions ($d1 = x$ ($90^\circ, 0^\circ$), $d2 = y$, $d3 = z$) and 3 non-orthogonal directions ($d1 = (90^\circ,$

75°), $d2 = y$, $d3 = z$). Actually, in [8], the implementation of the velocity was incorrect and it required a large number of memory space. Therefore, a very simple cubic object with as few as $N^3=16^3$ pixels was tested. Here, 4 cases of various objects are considered: a sphere, an asymmetric object, two separated objects (a sphere and a cone) and a square step-cylinder, which are shown in figure 1 (a) – (d), respectively. For each object, a cubic computational domain equal to 1.5 times the object maximum dimension is used. Each computational domain is distributed into $N^3=64^3$ cells.

To start the iterative process, an initial guess is required. It is somewhat arbitrary, only should be inside the computational domain. Therefore, for each object, we choose a small cube of 1/8 of the object dimension as the initial estimate. Once the initial contour is given, the initial level set function φ_0 is defined as the signed distance between each point and the contour, negative inside and positive outside [7]. The gradient of the level set function, $|\nabla\varphi|$, is approximated by a Hamiltonian scheme [7]. The velocity of the evolution is given by (5). At each iteration k , the normalized residual is chosen as:

$$r_k = \frac{1}{3} \sum_{d=1}^3 \frac{\|A^d - M^d u_k\|_{L^2}}{\|A^d\|_{L^2}} \quad (6)$$

where $\|X\|_{L^2}$ is the Euclidean norm of X .

The residual decreases fast when a large time step δt is applied. But for the convergence and the stability of the Hamilton equation (4), the choice of the time step δt should satisfy the Courant–Friedrichs–Lewy condition (CFL condition) [10]:

$$\delta t \sum_i \frac{V_{x_i}}{\Delta x_i} \leq C_{\max} \quad i = 1 \cdots n \quad (7)$$

where n is 3 for three-dimensional case, Δx_i the spatial interval and $C_{\max} = 1$ is typically used for an explicit problem.

To stop the iteration when an acceptable residual is achieved, we set the tolerance of the residual r_k to $\varepsilon = 10^{-2}$ and a maximum number of iterations $k_{\max} = 80$. To show the performance of level-set method, for each object in figure 1 (a) – (d), we compare the image reconstructed from orthogonal directions (ii) and that reconstructed from non-orthogonal directions (iii). As can be seen in figure 1 (ii), with geometrical profile functions from 3 mutually orthogonal directions, 3D images reconstructed by the level-set method are in good agreement with the original shapes. In the non-orthogonal case, figure 1 (iii), the two directions, $d1 = (90^\circ, 75^\circ)$ and $d2 = y$, are very close to each other, with a small angle of 15° , therefore the reconstructed results are not as correct as those obtained in orthogonal case, but are still quite accurate to meet the need of identification. Comparing to the distorted images presented in [11], which are obtained by the algorithm of [3] from 3 non-orthogonal directions, the performance of reconstruction from profile functions is greatly improved. Although, the images reconstructed by the level-set method are not perfectly identical to the original shapes, they might be optimized by considering the regularization of the evolution, for example, smoothing contours.

The normalized residual r_k vs. the number of iterations k for both the orthogonal and non-orthogonal cases are compared in figure 2 for the corresponding objects in figure 1. For the objects in (a) – (c), the difference of the normalized residuals between the orthogonal and non-orthogonal cases are very small. The residuals decrease very rapidly during the first 20 iterations, and they remain stable when a certain small value is achieved. In figure 2, the residual value is given at the final iteration for the non-orthogonal case. Reconstructed images shown in figure 1 are obtained at the iteration $k_{\max} = 80$, except for the step cylinder (figure 2 (d)) in the non-orthogonal case, where the residual converges faster than that of the orthogonal case: Its evolution stops after 44 iterations with $r_k = 0.009$, which is less than the tolerance ε . Once again, figure 2 proves that reconstruction with the level-set method is sensitive neither to the relationship between the observing directions nor to the object shapes.

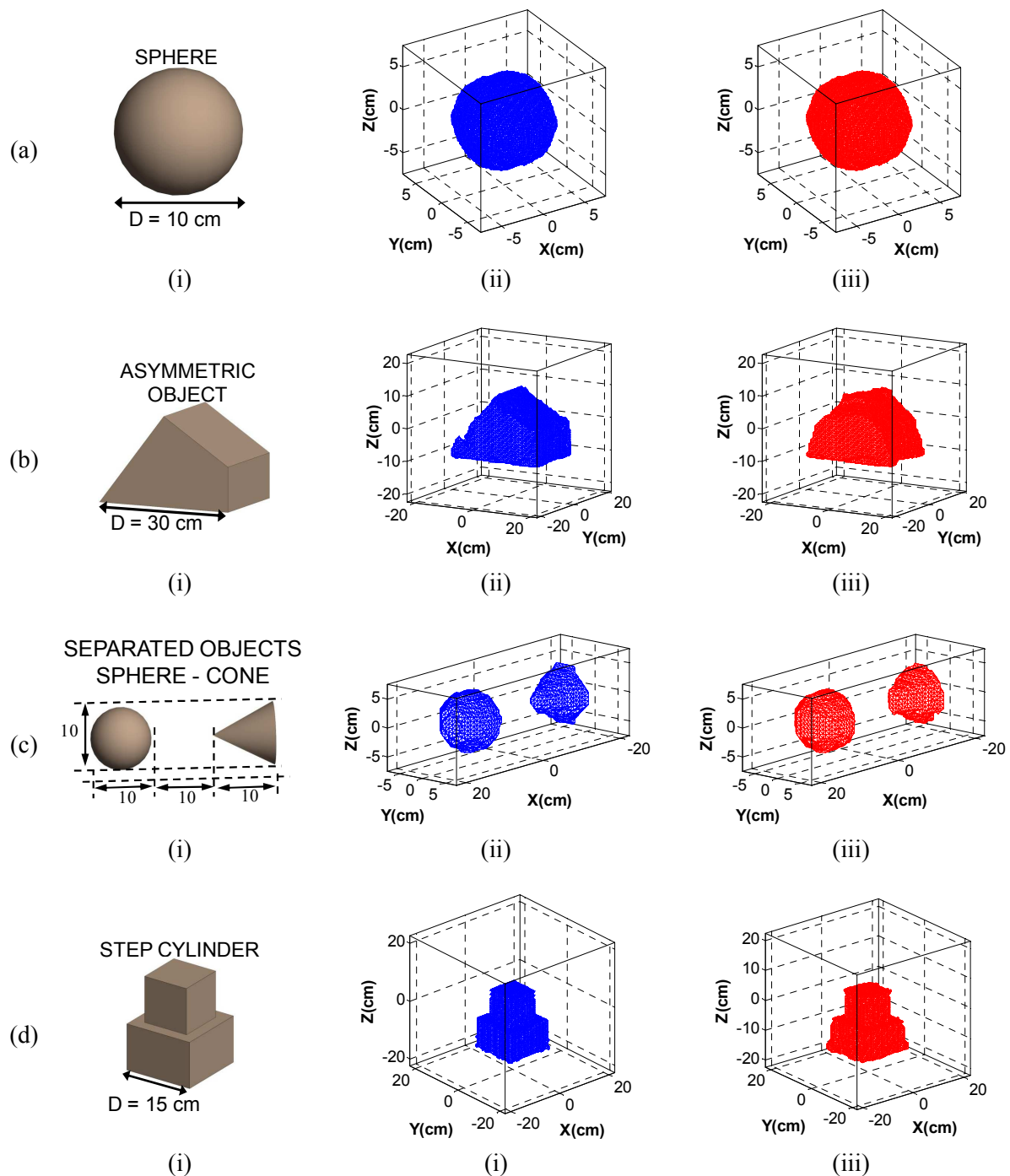


Figure 1. 3D reconstruction with level-set method for 4 objects (a) – (d) : (i) configuration of original object, (ii) reconstructed image obtained in orthogonal case, (iii) reconstructed image obtained in non-orthogonal case.

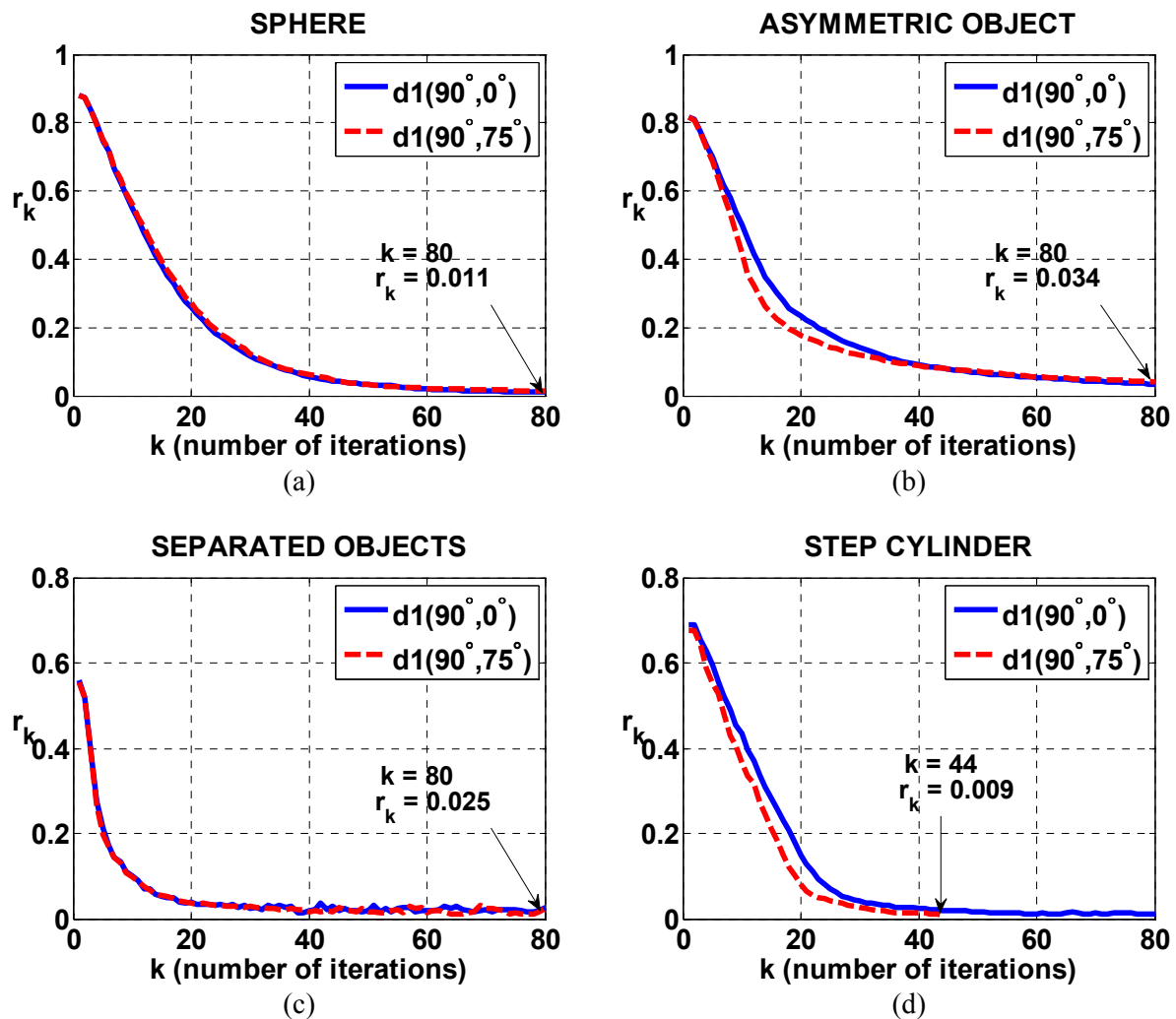


Figure 2. Comparison of the normalized residual r_k between the orthogonal (blue solid line) and non-orthogonal cases (red dash line) for the objects of figure 1.

5. Conclusion and perspectives

With the ramp response technique, radar imaging from profile functions only in 3 directions can produce an approximate contour for the target (even stealthy or buried). Previous algorithms for image reconstruction from profile functions work well in mutually orthogonal directions, but generate distorted estimates in non-orthogonal case. To optimize this performance, the level set method, which has been proved to be effective in shape optimization, is used. An adaptive velocity in our case is used and promising results of reconstruction for single and/or separated objects in arbitrary directions are obtained.

Until now, to validate the level-set method for reconstructions with profile functions, we only use the geometrical profile functions, namely the ideal case. The physical profile functions obtained from the real data will be considered. Moreover, to study the stability of the level-set method, some noise will be added in the data for the reconstruction process. The current mapping matrix we use is to depict the geometrical data of the original object and the estimate, therefore in the future, we are aim to find a mapping matrix adapted for the physical case.

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