

Radiation Damping in a Non-Abelian Strongly-Coupled Gauge Theory

Mariano Chernicoff[†], J. Antonio García[★] and Alberto Güijosa[★]

[†] Departament de Física Fonamental, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain, Email: mchernicoff@ub.edu

[★] Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares Universidad Nacional Autónoma de México, Apdo. Postal 70-543, México D.F. 04510, México Email: [garcia](mailto:garcia@nucleares.unam.mx), alberto@nucleares.unam.mx

Abstract. We study a ‘composite’ or ‘dressed’ quark in strongly-coupled $\mathcal{N} = 4$ super-Yang-Mills (SYM), making use of the AdS/CFT correspondence. We show that the standard string dynamics nicely captures the physics of the quark and its surrounding non-Abelian field configuration, making it possible to derive a relativistic equation of motion that incorporates the effects of radiation damping. From this equation one can deduce a non-standard dispersion relation for the composite quark, as well as a Lorentz covariant formula for its rate of radiation.

1. Motivation

A considerable effort has been invested in the study of strongly-coupled thermal non-Abelian plasmas by means of the AdS/CFT correspondence [1, 2]. The jet quenching observed at RHIC indicates that partons crossing the quark-gluon plasma experience significant energy loss. This question has been studied in a strongly-coupled $\mathcal{N} = 4$ SYM plasma for a variety of probes, including quarks [3], mesons [4], baryons [5, 6], gluons [5, 7] and k -quarks [5]. Here we will analyze quark energy loss *in vacuum*, which for a strongly-coupled non-Abelian gauge theory is interesting already in itself, and is in addition helpful for developing intuition that might extend to the finite temperature case [8, 9].

We expect an accelerating quark to radiate, and to experience a damping force due to the emitted radiation. In the context of classical electrodynamics, and for a non-relativistic electron modeled as a vanishingly small charge distribution, this effect is incorporated in the classic Abraham-Lorentz equation [10, 11]

$$m \left(\frac{d^2 \vec{x}}{dt^2} - t_e \frac{d^3 \vec{x}}{dt^3} \right) = \vec{F} . \quad (1)$$

The second term in the left-hand side is the damping term, which is seen to be associated with a characteristic timescale $t_e \equiv 2e^2/3mc^3$, set by the classical electron radius. The search for a Lorentz-covariant version of (1) led to the (Abraham-)Lorentz-Dirac equation [12],

$$m \left(\frac{d^2 x^\mu}{d\tau^2} - t_e \left[\frac{d^3 x^\mu}{d\tau^3} - \frac{1}{c^2} \frac{d^2 x_\nu}{d\tau^2} \frac{d^2 x^\nu}{d\tau^2} \frac{dx^\mu}{d\tau} \right] \right) = \mathcal{F}^\mu , \quad (2)$$



with τ the proper time and $\mathcal{F}^\mu \equiv \gamma(\vec{F} \cdot \vec{v}/c, \vec{F})$ the 4-force. The second term within the square brackets is the negative of the rate at which 4-momentum is carried away from the charge by radiation (as given by the covariant Lienard formula), so it is only this term that can properly be called radiation reaction. The first term within the square brackets, usually called the Schott term, and whose spatial part yields the damping force of (1) in the non-relativistic limit, is known to arise from the effect of the charge's 'near' (as opposed to radiation) field [13, 14].

The appearance of a third-order term in (1) and (2) leads to unphysical behavior, including pre-accelerating and self-accelerating (or 'runaway') solutions. These deficiencies are known to originate from the assumption that the charge is pointlike. For a non-relativistic charge distribution of small but finite size ℓ , (1) is corrected by an infinite number of higher-derivative terms,

$$m \left(\frac{d^2 \vec{x}}{dt^2} - t_e \left[\frac{d^3 \vec{x}}{dt^3} - \sum_{n=1}^{\infty} b_n \frac{\ell^n}{c^n} \frac{d^{3+n} \vec{x}}{dt^{3+n}} \right] \right) = \vec{F} \quad (3)$$

(with b_n some numerical coefficients), and is physically sound as long as $\ell > ct_e$ [14]. Upon shifting attention to the quantum case, one intuitively expects the pointlike 'bare' electron to acquire an effective size of order the Compton wavelength $\lambda_C \equiv \hbar/mc$, due to its surrounding cloud of virtual particles. Given that $\lambda_C \gg ct_e$, there should then be no room for unphysical behavior. Indeed, in [15] it was shown that nonrelativistic QED leads to

$$m \left(\frac{d^2 \vec{x}}{dt^2} - t_e \left[\frac{d^3 \vec{x}}{dt^3} - \sum_{n=1}^{\infty} d_n \frac{\lambda_C^n}{c^n} \frac{d^{3+n} \vec{x}}{dt^{3+n}} \right] \right) = \vec{F} , \quad (4)$$

which has no runaway solutions, and shows that the charge develops a characteristic size $\ell = \lambda_C$.

Going further to the quantum non-Abelian case is a serious challenge. Nevertheless, we will show here that the AdS/CFT correspondence [1] allows us to address this question rather easily in certain strongly-coupled non-Abelian gauge theories [16, 17]. We expect the basic story we will uncover to apply generally to all instances of the gauge/string duality (including cases with finite temperature or chemical potentials), but for simplicity we will concentrate on the specific example of $\mathcal{N} = 4$ SYM. Besides the gauge field, this maximally supersymmetric and conformally invariant theory (CFT) contains 6 real scalar fields and 4 Weyl fermions, all in the *adjoint* representation of the gauge group. We will be able to derive a Lorentz covariant equation summarizing the dynamics of a quark in this strongly-coupled theory, which will turn out to be a nonlinear generalization of the Lorentz-Dirac equation (2), with a higher-derivative structure somewhat similar to (4).

2. Basic Setup

From this point on we work in natural units, $c = \hbar = 1$. It is by now well-known that $\mathcal{N} = 4$ $SU(N_c)$ SYM with coupling g_{YM} is, despite appearances, completely equivalent [1] to Type IIB string theory on a background that asymptotically approaches the five-dimensional anti-de Sitter (AdS) geometry

$$ds^2 = G_{MN} dx^M dx^N = \frac{R^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right) , \quad (5)$$

where $R^4/l_s^4 = g_{YM}^2 N_c \equiv \lambda$ denotes the 't Hooft coupling, and l_s is the string length. The radial direction z is mapped holographically into a variable length scale in the gauge theory, in such a way that $z \rightarrow 0$ and $z \rightarrow \infty$ are respectively the ultraviolet and infrared limits. The directions $x^\mu \equiv (t, \vec{x})$ are parallel to the AdS boundary $z = 0$ and are directly identified with the gauge theory directions. The state of IIB string theory described by the unperturbed metric (5) corresponds to the vacuum of the $\mathcal{N} = 4$ SYM theory, and the closed string sector

describing (small or large) fluctuations on top of it fully captures the gluonic (+ adjoint scalar and fermionic) physics. The string theory description is under calculational control only for small string coupling and low curvatures, which translates into $N_c \gg 1$, $\lambda \gg 1$.

It is also known that one can add to SYM N_f flavors of matter in the *fundamental* representation of the $SU(N_c)$ gauge group by introducing in the string theory setup an open string sector associated with a stack of N_f D7-branes [18]. We will refer to these degrees of freedom as ‘quarks,’ even though, being $\mathcal{N} = 2$ supersymmetric, they include both spin 1/2 and spin 0 fields. The D7-brane embedding is chosen to be translationally invariant along the gauge theory directions x^μ , and extend in the radial direction from the boundary of AdS at $z = 0$ up to a location

$$z_m = \frac{\sqrt{\lambda}}{2\pi m} \quad (6)$$

determined by the mass m of the quarks. For $N_f \ll N_c$, the backreaction of the D7-branes on the geometry can be neglected; in the field theory this corresponds to a ‘quenched’ approximation.

An isolated quark of mass m is dual to an open string that has one endpoint on the D7-branes at $z = z_m$, and extends radially all the way to the AdS horizon at $z \rightarrow \infty$. The string dynamics is governed by the Nambu-Goto action

$$S_{NG} = -\frac{1}{2\pi l_s^2} \int d^2\sigma \sqrt{-\det g_{ab}} \equiv \int d^2\sigma \mathcal{L}_{NG} , \quad (7)$$

where $g_{ab} \equiv \partial_a X^M \partial_b X^N G_{MN}(X)$ ($a, b = 0, 1$) denotes the induced metric on the worldsheet. We can exert an external force \vec{F} on the string endpoint by turning on an electric field $F_{0i} = F_i$ on the D7-branes. This amounts to adding to (7) the usual minimal coupling, which in terms of the endpoint/quark worldline $x^\mu(\tau) \equiv X^\mu(\tau, z_m)$ reads

$$S_F = \int d\tau A_\mu(x(\tau)) \frac{dx^\mu(\tau)}{d\tau} . \quad (8)$$

Variation of $S_{NG} + S_F$ implies the standard Nambu-Goto equation of motion for all interior points of the string, plus the boundary condition

$$\Pi_\mu^z(\tau)|_{z=z_m} = \mathcal{F}_\mu(\tau) \quad \forall \tau , \quad (9)$$

where $\Pi_\mu^z \equiv \partial \mathcal{L}_{NG} / \partial(\partial_z X^\mu)$ is the worldsheet (Noether) current associated with spacetime momentum, and $\mathcal{F}_\mu = -F_{\nu\mu} \partial_\tau x^\nu = (-\gamma \vec{F} \cdot \vec{v}, \gamma \vec{F})$ the Lorentz four-force.

Notice that the string is being described (as is customary) in first-quantized language, and, as long as it is sufficiently heavy, we are allowed to treat it semiclassically. In gauge theory language, then, we are coupling a first-quantized quark to the gluonic (+ other SYM) field(s), and then carrying out the full path integral over the strongly-coupled field(s) (the result of which is codified by the AdS spacetime), but treating the path integral over the quark trajectory $x^\mu(\tau)$ in a saddle-point approximation.¹

In more detail, it is really the endpoint of the string that corresponds to the quark, while the body of the string codifies the profile of the (near and radiation) gluonic (+ other SYM) field(s) set up by the quark.² The latter can be mapped out explicitly by computing one-point functions of local operators ($\langle \text{tr } F^2 \rangle, \langle T_{\mu\nu} \rangle, \dots$) in the presence of the quark, which, via the standard GKPW recipe [2], requires a determination of the near-boundary profile of the closed string fields ($\phi, h_{\mu\nu}, \dots$) generated by the macroscopic string.

¹ For a study of quantum fluctuations about this classical string configuration, see the very recent work [19].

² In other words, the string here is the $\mathcal{N} = 4$ SYM analog of the ‘QCD string’, with the surprising twist that it lives not in 4 but in 5 (+5) dimensions.

For the interpretation of our results, it will be crucial to keep in mind that the quark described by this string is not ‘bare’ but ‘composite’ or ‘dressed’. This can be seen most clearly by working out the expectation value of the gluonic field surrounding a static quark located at the origin [20],

$$\frac{1}{4g_{YM}^2} \langle \text{tr } F^2(x) \rangle = \frac{\sqrt{\lambda}}{16\pi^2 |\vec{x}|^4} \left[1 - \frac{1 + \frac{5}{2} \left(\frac{2\pi m |\vec{x}|}{\sqrt{\lambda}} \right)^2}{\left(1 + \left(\frac{2\pi m |\vec{x}|}{\sqrt{\lambda}} \right)^2 \right)^{5/2}} \right]. \quad (10)$$

For $m \rightarrow \infty$ ($z_m \rightarrow 0$), this is just the Coulombic field expected (by conformal invariance) for a pointlike charge. For finite m , the profile is still Coulombic far away from the origin, but in fact becomes non-singular at the location of the quark,

$$\frac{1}{4g_{YM}^2} \langle \text{tr } F^2(x) \rangle = \frac{\sqrt{\lambda}}{128\pi^2} \left[15 \left(\frac{2\pi m}{\sqrt{\lambda}} \right)^4 - \frac{35}{|\vec{x}|^4} \left(\frac{2\pi m |\vec{x}|}{\sqrt{\lambda}} \right)^6 + \dots \right] \quad \text{for } |\vec{x}| < \frac{\sqrt{\lambda}}{2\pi m}. \quad (11)$$

As seen in these equations, the characteristic thickness of this non-Abelian charge distribution is precisely the length scale z_m defined in (6). This is then the size of the gluonic cloud that surrounds the quark, or in other words, the analog of the Compton wavelength for our non-Abelian source.

3. Generalized Lorentz-Dirac Equation for the Quark

By Lorentz invariance, given the description of the static quark, we know that a quark moving at constant velocity corresponds to a purely radial string, moving as a rigid vertical rod. When the quark accelerates, the body of the string will trail behind the endpoint, and will therefore exert a force on the latter. Remembering that the body of the string codifies the SYM fields sourced by the quark, we know that in the gauge theory this force is interpreted as the backreaction of the gluonic field on the quark. In other words, in the AdS/CFT context the quark has a ‘tail’, and it is this tail that is responsible for the damping effect we are after. This mechanism had been previously established in the computations of the drag force exerted on the quark by a thermal plasma, which is described in dual language in terms of a string living on a black hole geometry [21, 22]. Our analysis here will make it clear that the damping effect is equally present in the gauge theory vacuum [16, 17].

To flesh out this story, we need to determine the string profile corresponding to a given accelerated quark/endpoint trajectory, by solving the nonlinear equation of motion following from the Nambu-Goto action (7). Fortunately, for this task we can make use of the results obtained in a remarkable paper by Mikhailov [23], which we now briefly review (a more detailed explanation can be found in [8]). This author considered an infinitely massive quark ($z_m = 0$), and was able to solve the equation of motion for the dual string on AdS, for an *arbitrary* timelike trajectory of the string endpoint. In terms of the coordinates used in (5), his solution is

$$X^\mu(\tau, z) = z \frac{dx^\mu(\tau)}{d\tau} + x^\mu(\tau), \quad (12)$$

with $x^\mu(\tau)$ the worldline of the string endpoint at the AdS boundary— or, equivalently, the worldline of the dual, infinitely massive, quark— parametrized by its proper time τ . It is easy to check that the lines at constant τ are null with respect to the induced worldsheet metric, a fact that plays an important role in Mikhailov’s construction.

The solution (12) is retarded. To see this, note that, parametrizing the quark worldline by $x^0(\tau)$ instead of τ , and using $d\tau = \sqrt{1 - \vec{v}^2} dx^0$, where $\vec{v} \equiv d\vec{x}/dx^0$, the $\mu = 0$ component of (12) takes the form

$$t = z \frac{1}{\sqrt{1 - \vec{v}^2}} + t_{\text{ret}}, \quad (13)$$

where t_{ret} denotes the value of the quark/endpoint coordinate time $x^0(\tau)$ at the proper time τ relevant to the (t, z) segment of the string under consideration, and the endpoint velocity \vec{v} is meant to be evaluated at t_{ret} . In these same terms, the spatial components of (12) can be formulated as

$$\vec{X}(t, z) = z \frac{\vec{v}}{\sqrt{1 - \vec{v}^2}} + \vec{x}(t_{\text{ret}}) = (t - t_{\text{ret}})\vec{v} + \vec{x}(t_{\text{ret}}) . \quad (14)$$

We see here that the behavior at time $t = X^0(\tau, z)$ of the string segment located at radial position z (which, roughly speaking, codifies the gluonic field profile at length scale z and position $\vec{x} = \vec{X}(t, z)$ in the gauge theory) is completely determined by the behavior of the string endpoint at the *earlier* time $t_{\text{ret}}(t, z)$ determined from (13), i.e., by projecting back toward the boundary along the null line at fixed τ . (Note the analogy with the construction of the Lienard-Wiechert fields in classical electromagnetism.)

An analogous advanced solution built upon the same endpoint/quark trajectory can be obtained by reversing the sign of the first term in the right-hand side of (12). In gauge theory language, this choice of sign corresponds to the choice between a purely outgoing or purely ingoing boundary condition for the waves in the gluonic field at spatial infinity. Both on the string and the gauge theory sides, more general configurations should of course exist, but obtaining them explicitly is difficult due to the highly nonlinear character of the system. Henceforth we will focus solely on the retarded solutions, which are the ones that capture the physics of present interest, with influences propagating outward from the quark to infinity.

Using (13) and (14), Mikhailov was able to rewrite the total string energy in the form

$$E(t) = \frac{\sqrt{\lambda}}{2\pi} \int_{-\infty}^t dt_{\text{ret}} \frac{\vec{a}^2 - [\vec{v} \times \vec{a}]^2}{(1 - \vec{v}^2)^3} + E_q(\vec{v}(t)) , \quad (15)$$

where of course $\vec{a} \equiv d\vec{v}/dx^0$. The second term in the above equation arises from a total derivative on the string worldsheet, and gives the expected Lorentz-covariant expression for the energy intrinsic to the quark [8],

$$E_q(\vec{v}) = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{1}{\sqrt{1 - \vec{v}^2}} \frac{1}{z} \right) \Big|_{\infty}^{z_m=0} = \gamma m . \quad (16)$$

The first term in (15) must then represent the accumulated energy *lost* by the quark over all times prior to t . Surprisingly, the rate of energy loss is seen to have precisely the same form as the standard Lienard formula from classical electrodynamics. We therefore learn that in this non-Abelian, strongly-coupled theory, the energy loss of an infinitely massive (pointlike) quark depends *locally* on the quark worldline. For the spatial momentum, [23, 8] similarly find

$$\vec{P}(t) = \frac{\sqrt{\lambda}}{2\pi} \int_{-\infty}^t dt_{\text{ret}} \frac{\vec{a}^2 - [\vec{v} \times \vec{a}]^2}{(1 - \vec{v}^2)^3} \vec{v} + \vec{p}_q(\vec{v}(t)) , \quad (17)$$

with

$$\vec{p}_q = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{\vec{v}}{\sqrt{1 - \vec{v}^2}} \frac{1}{z} \right) \Big|_{\infty}^{z_m=0} = \gamma m \vec{v} . \quad (18)$$

We see then that, in spite of the non-linear nature of the system, Mikhailov's procedure leads to a clean separation between the tip and the tail of the string, i.e., between the quark (including its near field) and its gluonic radiation field.

In fact, using (12), the standard string dynamics reduces to standard particle dynamics at the level of the action: plugging (12) back into (7)+(8), we can explicitly carry out the integral

over z to obtain [17]

$$\begin{aligned} S_{\text{NG}} + S_{\text{F}} &= -\frac{R^2}{2\pi l_s^2} \int d\tau \int_{z_m \rightarrow 0}^{\infty} \frac{dz}{z^2} + \int d\tau A_{\mu}(x(\tau)) \frac{dx^{\mu}}{d\tau}(\tau) \\ &= -m \int d\tau + \int d\tau A_{\mu}(x(\tau)) \frac{dx^{\mu}}{d\tau}(\tau) , \end{aligned} \quad (19)$$

which is evidently the standard action for a pointlike externally forced relativistic particle (with mass $m \rightarrow \infty$). Notice that the associated equation of motion does *not* include a damping force, which is just as one would expect for an infinitely massive charge, because the coefficient $t_e \propto 1/m$ of the damping terms in (1) and (2) approaches zero as $m \rightarrow \infty$.

Let us now consider the more interesting case of a quark with finite mass, $z_m > 0$, where there should be a noticeable damping effect. As we emphasized at the end of the previous section, in this case our non-Abelian source is no longer pointlike but has size z_m . On the string theory side, the string endpoint is now at $z = z_m$, and we must again require it to follow the given quark trajectory, $x^{\mu}(\tau)$. As before, this condition by itself does not pick out a unique string embedding. Just like we discussed for the infinitely massive case, we additionally require the solution to be retarded, in order to focus on the gluonic field causally set up by the quark. As in [8], we can inherit this structure by truncating a suitably selected retarded Mikhailov solution. The embeddings of interest to us can thus be regarded as the $z \geq z_m$ portions of the solutions (12). These are parametrized by data at the AdS boundary $z = 0$, which are now merely auxiliary and will henceforth be denoted with tildes, to distinguish them from the actual physical data associated with the endpoint/quark at $z = z_m$. In this notation, the string embedding reads

$$X^{\mu}(\tilde{\tau}, z) = z \frac{d\tilde{x}^{\mu}(\tilde{\tau})}{d\tilde{\tau}} + \tilde{x}^{\mu}(\tilde{\tau}) . \quad (20)$$

Differentiation with respect to $\tilde{\tau}$ and evaluation at $z = z_m$ (where we can read off the quark trajectory $x^{\mu}(\tilde{\tau}) \equiv X^{\mu}(\tilde{\tau}, z_m)$) leads to

$$\frac{dx^{\mu}}{d\tilde{\tau}} = z_m \frac{d^2 \tilde{x}^{\mu}}{d\tilde{\tau}^2} + \frac{d\tilde{x}^{\mu}}{d\tilde{\tau}} , \quad (21)$$

which in turn implies

$$d\tau^2 \equiv -dx^{\mu} dx_{\mu} = d\tilde{\tau}^2 \left[1 - z_m^2 \left(\frac{d^2 \tilde{x}}{d\tilde{\tau}^2} \right)^2 \right] \quad (22)$$

and

$$\frac{d^2 x^{\mu}}{d\tilde{\tau}^2} = z_m \frac{d^3 \tilde{x}^{\mu}}{d\tilde{\tau}^3} + \frac{d^2 \tilde{x}^{\mu}}{d\tilde{\tau}^2} . \quad (23)$$

We are now finally ready to derive the desired equation of motion for the quark. This must simply be dual to the equation of motion satisfied by the string endpoint, which we know to be given by the standard boundary condition (9). For the embeddings (20), this condition reads

$$\Pi_{\mu}^z(\tau) = \frac{\sqrt{\lambda}}{2\pi} \frac{d\tilde{\tau}}{d\tau} \left[\frac{1}{z_m} \frac{d^2 \tilde{x}^{\mu}}{d\tilde{\tau}^2} + \left(\frac{d^2 \tilde{x}^{\mu}}{d\tilde{\tau}^2} \right)^2 \frac{d\tilde{x}^{\mu}}{d\tilde{\tau}} \right] = \mathcal{F}_{\mu} . \quad (24)$$

Using (22), (23) and carrying out some additional algebra (see [17] for details), this can be rewritten in the form

$$\frac{d}{d\tau} \left(\frac{m \frac{dx^{\mu}}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}^{\mu}}{\sqrt{1 - \frac{\lambda}{4\pi^2 m^4} \mathcal{F}^2}} \right) = \frac{\mathcal{F}^{\mu} - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^2 \frac{dx^{\mu}}{d\tau}}{1 - \frac{\lambda}{4\pi^2 m^4} \mathcal{F}^2} , \quad (25)$$

which is our main result [16, 17].

Notice that the characteristic length scale appearing in (25) is precisely $z_m = \sqrt{\lambda}/2\pi m$, which as discussed below (11), is the quark Compton wavelength. Let us now examine the behavior of a quark that is sufficiently heavy, or is forced sufficiently softly, that the condition $\sqrt{\lambda}|\mathcal{F}^2|/2\pi m^2 \ll 1$ holds. It is then natural to expand the equation of motion in a power series in this small parameter. To zeroth order in this expansion, we correctly reproduce the pointlike result $m\partial_\tau^2 x^\mu = \mathcal{F}^\mu$. If we instead keep terms up to first order, we find

$$m \frac{d}{d\tau} \left(\frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^\mu \right) = \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^2 \frac{dx^\mu}{d\tau}.$$

In the $\mathcal{O}(\sqrt{\lambda})$ terms it is consistent, to this order, to replace \mathcal{F}^μ with its zeroth order value, thereby obtaining

$$m \left(\frac{d^2 x^\mu}{d\tau^2} - \frac{\sqrt{\lambda}}{2\pi m} \frac{d^3 x^\mu}{d\tau^3} \right) = \mathcal{F}^\mu - \frac{\sqrt{\lambda}}{2\pi} \frac{d^2 x^\nu}{d\tau^2} \frac{d^2 x_\nu}{d\tau^2} \frac{dx^\mu}{d\tau}. \quad (26)$$

Interestingly, this coincides *exactly* with the Lorentz-Dirac equation (2), with the Compton wavelength (6) playing the role of characteristic size t_e for the composite quark. This is indeed the natural quantum scale of the problem.

We can continue this expansion procedure to arbitrarily high order in $\sqrt{\lambda}|\mathcal{F}^2|/2\pi m^2$. Our full equation (25) is thus recognized as an extension of the Lorentz-Dirac equation that automatically incorporates the size z_m of our non-pointlike non-Abelian source. It is curious to note that (25), which incorporates the effect of radiation damping on the quark, has been obtained from (12), which does *not* include such damping for the string itself (the latter is $1/N_c^2$ suppressed).

The full physical content of (25) can be made transparent by rewriting it in the form

$$\frac{dP^\mu}{d\tau} \equiv \frac{dp_q^\mu}{d\tau} + \frac{dP_{rad}^\mu}{d\tau} = \mathcal{F}^\mu, \quad (27)$$

recognizing P^μ as the total string (= quark + radiation) four-momentum,

$$p_q^\mu = \frac{m \frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m} \mathcal{F}^\mu}{\sqrt{1 - \frac{\lambda}{4\pi^2 m^4} \mathcal{F}^2}} \quad (28)$$

as the intrinsic momentum of the quark including the contribution of the near-field sourced by it (or, in quantum mechanical language, of the gluonic cloud surrounding the quark), and

$$\frac{dP_{rad}^\mu}{d\tau} = \frac{\sqrt{\lambda} \mathcal{F}^2}{2\pi m^2} \left(\frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m^2} \mathcal{F}^\mu \right) \quad (29)$$

as the rate at which momentum is carried away from the quark by chromo-electromagnetic radiation. Both (28) and (29) diverge when $\mathcal{F}^2 \rightarrow \mathcal{F}_{crit}^2$, where $\mathcal{F}_{crit}^2 = 4\pi^2 m^4/\lambda$ is the critical value at which the force becomes strong enough to nucleate quark-antiquark pairs (or, in dual language, to create open strings) [24].

Unlike its classical electrodynamic counterpart (2), our dressed quark equation of motion has no self-accelerating (runaway) solutions: in the (continuous) absence of an external force, (25) uniquely predicts that the 4-acceleration of the quark must vanish. Interestingly, the converse

to this last statement is not true: constant 4-velocity does not uniquely imply a vanishing force. E.g., for motion purely along one dimension, (25) reads

$$ma = \frac{F(1-v^2)^{3/2}}{\sqrt{1 - \frac{\lambda}{4\pi^2 m^4} F^2}} + \frac{\frac{\sqrt{\lambda}}{2\pi m} \frac{dF}{dt} (1-v^2)}{1 - \frac{\lambda}{4\pi^2 m^4} F^2}. \quad (30)$$

For $a = 0$, we obtain a differential equation with solution $F(t) = \pm(2\pi m^2/\sqrt{\lambda}) \operatorname{sech}[2\pi m(t - t_0)/\gamma\sqrt{\lambda}]$, with t_0 an integration constant [17]. The energy provided to the system by this particular $F(t)$ does not translate into an increase of the quark/endpoint velocity, but into a continuous modification of the string tail, or, in gauge theory language, a change of the gluonic field profile. This is again a consequence of the extended, and hence deformable, nature of the quark.

All in all, then, we have in (25) a physically sensible and interesting description of the dynamics of a composite quark in SYM. It is truly remarkable that the AdS/CFT correspondence grants us such direct access to this piece of strongly-coupled non-Abelian physics.

References

- [1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200]; O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].
- [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
- [3] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP **0607** (2006) 013 [arXiv:hep-th/0605158]; S. S. Gubser, Phys. Rev. D **74** (2006) 126005 [arXiv:hep-th/0605182]; J. Casalderrey-Solana and D. Teaney, Phys. Rev. D **74** (2006) 085012 [arXiv:hep-ph/0605199]; H. Liu, K. Rajagopal and U. A. Wiedemann, Phys. Rev. Lett. **97** (2006) 182301 [arXiv:hep-ph/0605178].
- [4] K. Peeters, J. Sonnenschein and M. Zamaklar, Phys. Rev. D **74**, 106008 (2006) [arXiv:hep-th/0606195]; H. Liu, K. Rajagopal and U. A. Wiedemann, Phys. Rev. Lett. **98**, 182301 (2007) [arXiv:hep-ph/0607062]; M. Chernoiff, J. A. García and A. Güijosa, JHEP **0609**, 068 (2006) [arXiv:hep-th/0607089]; K. Dusling, J. Erdmenger, M. Kaminski, F. Rust, D. Teaney and C. Young, JHEP **0810**, 098 (2008) [arXiv:0808.0957 [hep-th]].
- [5] M. Chernoiff and A. Güijosa, JHEP **0702** (2007) 084 [arXiv:hep-th/0611155].
- [6] C. Athanasiou, H. Liu and K. Rajagopal, JHEP **0805**, 083 (2008) [arXiv:0801.1117 [hep-th]].
- [7] S. S. Gubser, D. R. Gulotta, S. S. Pufu and F. D. Rocha, “Gluon energy loss in the gauge-string duality,” JHEP **0810**, 052 (2008) [arXiv:0803.1470 [hep-th]].
- [8] M. Chernoiff and A. Güijosa, JHEP **0806**, 005 (2008) [arXiv:0803.3070 [hep-th]].
- [9] Y. Hatta, E. Iancu and A. H. Mueller, arXiv:0803.2481 [hep-th]; K. B. Fadafan, H. Liu, K. Rajagopal and U. A. Wiedemann, Eur. Phys. J. C **61**, 553 (2009) [arXiv:0809.2869 [hep-ph]].
- [10] M. Abraham, Ann. Physik **10** (1903) 105.
- [11] H. A. Lorentz, *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat*, 2nd ed., (Dover, 1952).
- [12] P. A. M. Dirac, Proc. Roy. Soc. Lond. A **167** (1938) 148.
- [13] C. Teitelboim, Phys. Rev. D **1** (1970) 1572 [Erratum-ibid. D **2** (1970) 1763].
- [14] F. Rohrlich, *Classical Charged Particles*, 2nd. ed. (Addison Wesley, Redwood City, California, 1990); Am. J. Phys. **65** (1997) 1051.
- [15] E. J. Moniz and D. H. Sharp, Phys. Rev. D **15** (1977) 2850.
- [16] M. Chernoiff, J. A. García and A. Güijosa, Phys. Rev. Lett. **102** (2009) 241601 [arXiv:0903.2047 [hep-th]].
- [17] M. Chernoiff, J. A. García and A. Güijosa, JHEP **0909** (2009) 080 [arXiv:0906.1592 [hep-th]].
- [18] A. Karch and E. Katz, JHEP **0206** (2002) 043 [arXiv:hep-th/0205236].
- [19] E. Cáceres, M. Chernoiff, A. Güijosa and J. F. Pedraza, arXiv:1003.5332 [hep-th].
- [20] J. L. Hovdebo, M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, Int. J. Mod. Phys. A **20** (2005) 3428.
- [21] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP **0607** (2006) 013 [arXiv:hep-th/0605158].
- [22] S. S. Gubser, Phys. Rev. D **74** (2006) 126005 [arXiv:hep-th/0605182].
- [23] A. Mikhailov, arXiv:hep-th/0305196.
- [24] J. Casalderrey-Solana and D. Teaney, JHEP **0704** (2007) 039 [arXiv:hep-th/0701123].