Some New Results in AdS_4/CFT_3 Duality[1]

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Abstract. In this talk I will present two new results related to the duality of AdS_4 backgrounds of M theory or Type IIA string theory and three-dimensional Chern-Simons-Matter theories. The first result is a $1/\lambda$ correction to the supergravity/CFT relation between R and λ in the original ABJM and ABJ models. This part is based on work with Shinji Hirano, that appeared in [1]. The second result concerns the generalization of the ABJM model to unequal CS levels, which has been conjectured to be dual to an AdS_4 solution of massive Type IIA supergravity. I will present a simple brane construction that supports this conjecture. This part is based on work with Gilad Lifschytz, that appeared in [2].

1. Introduction

The AdS/CFT correspondence relates quantum gravity in an Anti-de-Sitter (AdS) spacetime background to a conformal field theory in one lower dimension, which one can regard as living at the spatial boundary of the AdS spacetime. The examples that are known explicitly are derived from string theory using D-branes, which, in a specific large N and low energy limit, can simultaneously be described either in terms of the supergravity background they create, or in terms of the gauge field theory on their worldvolume. In its most symmetric form, this duality involves an AdS spacetime and a conformal field theory (CFT). The simplest example comes from D3-branes in Type IIB string theory, which lead to a duality between the maximally supersymmetric ($\mathcal{N} = 4$) Yang-Mills theory in four dimensions and the $AdS_5 \times S^5$ solution of Type IIB supergravity [3]. The parameters N and $\lambda = g_{YM}^2 N$ of the gauge theory translate, respectively, into the inverse coupling constant and the curvature radius of the gravity theory. This means that the low energy supergravity description is valid when the gauge theory is strongly coupled, and a perturbative gauge theory description is valid when the background is highly curved. This correspondence has been generalized and tested in many ways, and there is now an impressive collection of AdS_5/CFT_4 dual pairs derived from D3-branes in various non-trivial backgrounds.

The M2-brane and M5-brane of M theory also give rise to AdS backgrounds, however the corresponding CFTs cannot be derived directly, since the underlying degrees of freedom of M theory are not known. M2-branes in flat space, in particular, lead to an $AdS_4 \times S^7$ solution, and should therefore be described by a three-dimensional CFT with $\mathcal{N} = 8$ supersymmetry and

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an $SO(8)$ global R symmetry. A formal definition of this CFT is provided by the reduction to Type IIA string theory; it can be identified as the IR fixed point of three-dimensional $\mathcal{N} = 8$ Super-Yang-Mills theory, corresponding to the worldvolume theory on a collection of D2-branes. However, the fixed point is strongly coupled, and this does not lead to a Lagrangian description. Faced with the failure to derive the required CFT, an obvious alternative is to guess its form. In [4], an attempt was made using an $SU(N)$ gauge field with a pure Chern-Simons (CS) action together with matter fields. The CS action is classically conformal invariant, and with a suitable matter content could be made into a superconformal theory. This did not quite work, since the maximal supersymmetry in this case is $\mathcal{N} = 3$, but it pointed in the right direction. The first breakthrough was made by Bagger and Lambert, who presented an explicit $\mathcal{N} = 8$ CFT which had the correct degrees of freedom to describe M2-branes [5]. Initially this model was formulated as a new kind of gauge theory based on a 3-algebra, but subsequently it was reformulated as an ordinary CS gauge theory with an $SU(2) \times SU(2)$ gauge group with opposite CS coefficients k and $-k$, and with matter in the bi-fundamental representation [6]. This theory was conjectured to describe two M2-branes in certain orbifold backgrounds of M theory [7]. However, it left a number of questions open, most importantly how to describe M2-branes in flat space, and how to generalize to N M2-branes, and thereby make contact with the supergravity dual.

These questions were answered by the ABJM model [8]. This model describes a collection of N coincident M2-branes in M theory at the singularity of an orbifold $\mathbb{C}^4/\mathbb{Z}_k$. The field theory in this case is an $\mathcal{N} = 6$ $U(N)_k \times U(N)_{-k}$ CS theory, with matter consisting of two hyper-multiplets in the bi-fundamental representation. At large N, the effective coupling constant is the 't Hooft coupling $\lambda = N/k$, so the field theory is weakly coupled if $k \gg N$. If $k \ll N$, there is a weakly curved dual supergravity description, given either by M theory on $AdS_4 \times S^7/\mathbb{Z}_k$ with N units of 4-form flux in AdS_4 if $k \ll N^{1/5}$, or by Type IIA string theory on $AdS_4 \times \mathbb{C}P^3$ with N units of RR 4-form flux in AdS_4 and k units of RR 2-form flux in $\mathbb{C}P^1 \subset \mathbb{C}P^3$ if $k \gg N^{1/5}$. For $k = 1$ this theory describes M2-branes in flat space, with the M theory dual $AdS_4 \times S^7$. In this case (and also for $k = 2$) the supersymmetry is enhanced to $\mathcal{N} = 8$. However, both the field theory and the Type IIA dual are strongly coupled in this case, and the enhancement of the supersymmetry is due to non-perturbative states [8, 9].

A simple extension of this model can be made by considering gauge groups with unequal ranks $U(N + l)_k \times U(N)_{-k}$, corresponding to adding l "fractional" M2-branes [10]. For $l \leq k$, the theory remains an $\mathcal{N} = 6$ CFT, with the same dual geometry, but with an additional flux turned on. In the M theory picture this is described by "discrete torsion", which is just a discrete holonomy of the C field on the torsion 3-cycle $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$, $\oint C_3/(2\pi) = l/k$. In the Type IIA description, this becomes a B field holonomy on $\mathbb{C}P^1 \subset \mathbb{C}P^3$, $b = \oint B_2/(2\pi)^2 = l/k$. This also implies an additional RR flux $\int_{\mathbb{C}P^2} F_4/(2\pi) = l$, corresponding to l D4-branes wrapped on the 2-cycle $\mathbb{C}P^1 \subset \mathbb{C}P^3$.

There has been a considerable amount of activity generalizing this model to theories with less supersymmetry, and there are now many explicit examples of AdS_4/CFT_3 dual pairs. In this talk I will describe two new results related to AdS_4/CFT_3 duality. The first is a correction to the duality relation due to higher curvature effects. The second result is related to the deformation of the model to unequal CS levels, which has been conjectured to be dual to a massive Type IIA supergravity AdS_4 background. I will describe a brane configuration that supports this conjecture, and show explicitly how it leads to different CS levels for the two gauge groups.

2. Anomalous corrections in the ABJ(M) model

In the $AdS_5 \times S^5$ case, the duality dictionary is exact to all orders in g_s and α' [11]. In particular, the relation between the parameters $R_{IIB}^4/(\alpha')^2 = 4\pi\lambda$ does not receive corrections beyond the low energy classical supergravity approximation. However, as we will now demonstrate, the analogous relation for $AdS_4 \times S^7/\mathbb{Z}_k$, $R_{IIA}^4/(\alpha')^2 = R_M^6/(k l_M^6) = 32\pi^2\lambda$, receives corrections

due to higher curvature and flux couplings. In particular, these corrections become relevant at two loops in the string worldsheet sigma model, and are important for the strong coupling test of the all loop Bethe ansatz proposed in [12].

2.1. M theory description
In the M theory description, these corrections correspond to additional contributions to the M2-brane charge from the gravitational anomaly term and the Chern-Simons term in the low energy eleven dimensional supergravity theory,

$$S_{11} = \frac{1}{2\kappa_{11}^2} \left[\int d^{11}x \sqrt{-G} \left(R - \frac{1}{2}|G_4|^2 \right) - \frac{1}{6} \int C_3 \wedge G_4 \wedge G_4 + (2\pi)^2 \int C_3 \wedge I_8 \right]. \quad (1)$$

The 8-form anomaly polynomial I_8 is related to the Euler class of the space transverse to the M2-brane, such that $\int I_8 = -\chi_{bulk}/24$, where χ_{bulk} is the contribution to the Euler characteristic from the bulk [13]. There are therefore three potential contributions to the M2-brane charge [14],

$$Q_{M2} = N - \frac{1}{2(2\pi)^2} \int_{\mathcal{M}_8} G_4 \wedge G_4 - \frac{\chi_{bulk}}{24} , \quad (2)$$

corresponding, respectively, to sources, flux, and the geometry of the transverse space.

Our transverse space is $\mathbb{C}^4/\mathbb{Z}_k$, which has a total Euler characteristic $\chi(\mathbb{C}^4/\mathbb{Z}_k) = k$ [15]. This has a bulk contribution from the singularity, and a contribution from the boundary $S^7/\mathbb{Z}_k$. The boundary contribution is easily computed by realizing that $\mathbb{Z}_k$ acts freely on S^7, which is the boundary of $\mathbb{C}^4$, and by the fact that $\chi_{bnd}(\mathbb{C}^4) = \chi(\mathbb{C}^4) = \chi(point) = 1$. The $\mathbb{Z}_k$ action gives $\chi_{bnd}(\mathbb{C}^4/\mathbb{Z}_k) = 1/k$ and therefore $\chi_{bulk}(\mathbb{C}^4/\mathbb{Z}_k) = k - 1/k$. The fixed point of the orbifold therefore carries an M2-brane charge given by

$$Q_{M2}(\mathbb{C}^4/\mathbb{Z}_k) = -\frac{\chi_{bulk}}{24} = -\frac{1}{24} \left(k - \frac{1}{k} \right) . \quad (3)$$

This generalizes the known result for the so-called $OM2^-$ plane, $Q_{M2}(\mathbb{R}^8/\mathbb{Z}_2) = -1/16$ [16]. In this case there is an additional consistency check which comes from compactifying one of the coordinates of the $\mathbb{R}^8$, and reducing to Type IIA string theory. There are two $OM2^-$ planes in this case, that become a single orientifold plane $O2^-$ in Type IIA string theory. The D2-brane charge of the orientifold plane can be computed independently using string theory, and the result is $-1/8$, percisely twice the charge of the $OM2^-$ plane. There is no analogous simple Type IIA reduction for $k > 2$.

To compute the flux contribution in (2) we need to find a representative of the torsion class of G_4. We can do this by generalizing the construction of [16]. Start with the bundle $\mathcal{O}(-k)$ (the k-fold tensor product of the natural line bundle $\mathcal{O}(-1)$) over $\mathbb{C}P^3$. Taking a disk $|w| \leq 1$ in each fiber defines a smooth 8-dimensional manifold $\mathcal{M}$, whose boundary is $S^7/\mathbb{Z}_k$. The M2-brane charge can therefore be evaluated as

$$Q_{M2}(\text{flux}) = -\frac{1}{2} \int_{\mathcal{M}} \frac{G_4}{2\pi} \wedge \frac{G_4}{2\pi} . \quad (4)$$

The torsion class is represented by a 4-dimensional submanifold $\mathcal{W} \subset \mathcal{M}$, whose boundary is $S^3/\mathbb{Z}_k \subset S^7/\mathbb{Z}_k$:

$$\int_{\mathcal{W}} \frac{G_4}{2\pi} = \int_{S^3/\mathbb{Z}_k} \frac{C_3}{2\pi} = \frac{l}{k} . \quad (5)$$

A representative for the class of G_4 may then be constructed using the Poincare dual of the base $\mathbb{C}P^3$, which is a 2-form X satisfying

$$\int_{\mathcal{W}} X \wedge X = -k \ . \tag{6}$$

We can therefore identify $G_4/(2\pi) = -(l/k^2)X \wedge X$, and the the M2-brane charge is given by

$$Q_{M2}(\text{flux}) = -\frac{l^2}{2k^4} \int_{\mathcal{M}} X \wedge X \wedge X \wedge X = \frac{l^2}{2k} \ . \tag{7}$$

This again generalizes the $OM2$ plane case. In that case there is only one choice of discrete torsion $l = 1$, corresponding to an $OM2^+$ plane which carries an M2-brane charge of 3/16. Upon compactifying one of the $\mathbb{R}^8$ directions one then obtains one of the four variants of orientifold 2-planes in Type IIA string theory, $O2^-$, $O2^+$, $\widetilde{O2}^-$, or $\widetilde{O2}^+$, depending on which $OM2$-planes are placed at opposite points on the circle. Again, a similar consistency check by reduction to ten dimensions cannot be made for $k > 2$.

Adding the two corrections, the total M2-brane charge in the orbifold background is given by

$$Q_{M2} = N - \frac{1}{24}\left(k - \frac{1}{k}\right) + \frac{l^2}{2k} \ . \tag{8}$$

In the near-horizon M theory dual description $R_M^6/l_M^6 = 32\pi^2 Q_{M2}$, and therefore

$$\frac{R_{IIA}^4}{(\alpha')^2} = \frac{R_M^6}{kl_M^6} = 32\pi^2\left(\lambda - \frac{1}{24}\left(1 - \frac{1}{k^2}\right) + \frac{l^2}{2k^2}\right) \ . \tag{9}$$

Note that the correction vanishes for $k = 1$, as it should. Note also that the correction is $\mathcal{O}(1/\lambda)$ relative to the classical low-energy supergravity result, which means it corresponds to a two-loop worldsheet sigma model effect.

2.2. Type IIA description and a puzzle

In the Type IIA description, the above correction corresponds to a shift in the D2-brane charge. We can compute this shift by considering the effects of other branes added to the near-horizon $AdS_4 \times \mathbb{C}P^3$ background. Specifically, we consider a D($2n$)-brane which forms a domain wall in AdS_4 and wraps $\mathbb{C}P^{n-1} \subset \mathbb{C}P^3$, with $n = 1, 2$ and 3. On the one hand, each such brane produces a jump in one of the field theory parameters N, l or k, and on the other hand, each can carry charges of lower dimensional branes due to its worldvolume CS and curvature couplings. By comparing these two effects one can derive a map between the field theory parameters and the D-brane charges. More precisely, this relates the field theory parameters to the D-brane *Page* charges, which are defined by the modified RR field strengths $\hat{F} = \tilde{F} \wedge e^{-B_2}$, where $\tilde{F}$ are the gauge-invariant field strengths. Since the modified field strengths satisfy the ordinary Bianchi identity, $d\hat{F} = 0$, Page charge is conserved and quantized, although it is not gauge invariant. However, for the domain wall D-branes, the Page charge is well-defined up to large gauge transformations that shift the B-field holonomy b by an integer. The explicit map is [17][2]:

$$Q_{D2}^P = N + \frac{k}{12} \ , \quad Q_{D4}^P = l - \frac{k}{2} \ , \quad Q_{D6}^P = k \ . \tag{10}$$

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[2] This was generalized in [2] to include D8-branes, and the associated parameter q corresponding to the difference between the two CS levels. This parameter will be discussed in the next section.

In particular, the shift in the D4-brane charge relative to l comes from the CS term on a D6-brane which wraps $\mathbb{C}P^2$. The quantization of the worldvolume magnetic flux is shifted by $1/2$ in this case due to an anomaly associated with non-spin manifolds [18]. Therefore each D6-brane domain wall, which shifts k by one, also changes the D4-brane charge by $1/2$. The shift in the D2-brane charge is a sum of the contribution from a CS term and a curvature term on the D6-brane domain wall.

While the Page charges are naturally related to the integer field theory parameters, the quantity that is naturally related to the curvature radius R_{IIA} is the Maxwell D2-brane charge, which is defined by the gauge invariant field strength $\tilde{F}_6$. This is related to the Page charges by

$$Q_{D2}^M = Q_{D2}^P + b Q_{D4}^P + \frac{1}{2} b^2 Q_{D6}^P. \tag{11}$$

The B-field holonomy b is given by

$$b = -\frac{l}{k} + \frac{1}{2}, \tag{12}$$

where the $1/2$ shift relative to the value claimed in [10] is again due to the anomaly of [18], this time applied to a D4-brane wrapped on $\mathbb{C}P^2$ [17]. The $1/2$ shift in the worldvolume flux must be supplemented by a $1/2$ shift in b to ensure an integer tadpole, which can be cancelled by including an integer number of strings. The sign convention for the B-field is also different here. Substituting (10) into (11) then gives

$$Q_{D2}^M = N + \frac{k}{12} - \frac{\left(l - \frac{k}{2}\right)^2}{2k}. \tag{13}$$

Note that this result is only valid at large k, where the weakly coupled Type IIA description holds. In particular, there could be an $\mathcal{O}(1/k)$ correction. Indeed, to get the known results for $k = 1$ and $k = 2$ requires adding an amount $1/(24k)$. We are led to conjecture that the total D2-brane Maxwell charge is

$$Q_{D2}^M = N - \frac{1}{24}\left(k - \frac{1}{k}\right) - \frac{l(l-k)}{2k}. \tag{14}$$

However, this raises a puzzle. Although both (8) and (14) reproduce the same known results for $k = 1$ and $k = 2$, they differ for $k > 2$. The resolution of this discrepency requires more work, but there is an indication that something is missing in the M theory calculation. Namely, the shifted quantization of Page charge for D4-branes wrapping $\mathbb{C}P^1$ requires a similar shift in the quantization of charge for M5-branes wrapping $S^3/\mathbb{Z}_k$. However, the mechanism for the shift must be different here, since the anomaly of [18] doesn't apply. We would like to argue that the M5-brane charge quantization is shifted due to an M2-brane parity anomaly, of the type described in [19]. In the usual argument for Dirac quantization we consider a Euclidean M2-brane wrapped on an S^3 in the S^4 surrounding an M5-brane. The M2-brane path integral acquires a phase $\exp(i \int_{S^3} C_3)$, which is single valued only if $\exp(i \int_{S^4} G_4) = 1$. The period of G_4 would therefore have to be an integer multiple of 2π. However, there is another phase factor coming from the path integral over the worldvolume fermions of the M2-brane. This is the parity anomaly. The condition of single-valuedness is shifted to

$$(-1)^{P_1(N(S^4))/2} e^{i \int_{S^4} G_4} = 1, \tag{15}$$

where $N(S^4)$ is the normal bundle to S^4 in the total space. In our case the relevant part of the space is $S^7/\mathbb{Z}_k$, which can be described as an $SU(2)/\mathbb{Z}_k$ instanton bundle over S^4. Therefore $P_1(N(S^4)) = -2k$, and we obtain the required M5-brane charge shift by $-k/2$. This explains the shift from l to $l - k$ in comparing (8) with (14), but it does not completely resolve their discrepency.

3. Branes and massive Type IIA duals

Massive Type IIA supergravity is a variant of Type IIA supergravity, in which the NSNS 2-form eats the RR 1-form and becomes massive [20]. This also gives rise to a cosmological constant given by the mass-squared. This theory was subsequently interpreted in terms of a background RR 0-form field strength F_0, which is associated with D8-branes in Type IIA string theory [21]. However it is not yet known whether D8-branes lift to M theory, and therefore whether massive Type IIA supergravity is part of M theory. On the other hand, a variety of supersymmetric $AdS_4 \times M_6$ solutions with $F_0 \neq 0$ are known [22, 23, 24], and this raises the question of what their dual 3d CFTs are.

In a number of simple cases, corresponding to deformations of the $\mathcal{N} = 6$ solution with $F_0 = 0$, it was argued that the dual CFTs are deformations of the ABJM model to $U(N)_{k_1} \times U(N)_{k_2}$ Chern-Simons-Matter (CSM) theories with $k_1 + k_2 = F_0$, and additional superpotential terms for the matter fields [25]. The simplest example is a non-supersymmetric deformation of the field theory that preserves the $SO(6)$ global symmetry, in which only the sum of the CS levels is changed. This $\mathcal{N} = 0$ CFT was conjectured to be dual to a non-supersymmetric $AdS_4 \times \mathbb{C}P^3$ solution of massive Type IIA supergravity with the $SO(6)$ invariant metric on $\mathbb{C}P^3$. The main evidence for this conjecture comes from considering the properties of D-branes in the $F_0 \neq 0$ background. For example, tadpole cancellation on the D0-brane requires F_0 strings to end on it, which agrees with the fact that the dual di-monopole operator of the field theory has extra gauge indices when $k_1 \neq -k_2$, which must be saturated by $|k_1 + k_2|$ semi-infinite Wilson lines.

In principle, the condition $k_1 + k_2 = F_0$ can be satisfied in many ways. However there should be a unique field theory dual to a given background. One of the questions we would like to address is how precisely the F_0 background affects each of the two CS levels. To answer this question, and also to gain more insight into the AdS_4/CFT_3 duality in the massive IIA case, we will construct a brane realization of these theories.

We start with Type IIA string theory in the $\mathcal{N} = 6$ $AdS_4 \times \mathbb{C}P^3$ background. We then deform this background by nucleating a D8-brane that fills AdS_4, and wraps a 5-dimensional cycle in $\mathbb{C}P^3$. We can do this because the cycle is topologically trivial, and therefore the D8-brane carries no conserved charge. On the other hand it forms a domain wall in $\mathbb{C}P^3$, across which F_0 changes by one unit (Fig. 1a). Since this configuration breaks all the supersymmetry, the deformation is not flat, but we can still do it. Now imagine pulling the D8-brane all the way from one pole of the $\mathbb{C}P^3$, where it has a vanishing size, to the other pole, where it again has a vanishing size. This deforms the original $\mathcal{N} = 6$ solution with $F_0 = 0$, to a new $\mathcal{N} = 0$ solution with $F_0 = 1$. We can repeat the process and produce backgrounds with higher values of F_0. Furthermore, as long we keep to a small number of D8-branes relative to N and k, we can use the probe approximation, and the background metric remains the same. Namely, the $SO(6)$ symmetry is preserved, and the solution we obtain is the $\mathcal{N} = 0$ $AdS_4 \times \mathbb{C}P^3$ solution.

Now let us turn to Type IIB string theory, and describe the T-dual brane configuration. The brane configuration used in the ABJM model consists of an NS5-brane, a $(1, k)$5-brane (a bound state of an NS5-brane and k D5-branes), and a number of D3-branes arranged as follows [26]:

	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•		•		•		•			
$(1,k)5$	•	•	•	$\cos\theta$	$\cos\theta$	$\cos\theta$		$\sin\theta$	$\sin\theta$	$\sin\theta$
D3	•	•	•				•			

The angle θ is the relative orientation of the two 5-branes in the 3-7, 4-8 and 5-9 planes, and is related to k as $\tan\theta = k$ (for $g_s = 1$ and $C_0 = 0$). The coordinate x^6 is compact, and the D3-branes can either wind around it, or be suspended between the two 5-branes. This describes a three-dimensional $\mathcal{N} = 3$ Yang-Mills-Chern-Simons theory with a gauge group $U(N + l)_k \times U(N)_{-k}$, and two bi-fundamental hyper-multiplets. The two ranks $N + l, N$ are

given by the number of D3-branes on either side of the circle separated by the two 5-branes. For $l \leq k$ this theory flows in the IR to the $\mathcal{N} = 6$ superconformal CSM theory with the same gauge group.

The object dual to the D8-brane described above is a D7-brane, which is oriented as follows:

$$
\begin{array}{ccccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\text{D7} & \bullet & \bullet & \bullet & \bullet & \bullet & & & \bullet & \bullet & \bullet
\end{array}
$$

The new configuration breaks supersymmetry, and the D7-brane is repelled from the D3-branes in the x^5 direction. The position of the D7-brane in x^5 corresponds to the position of the D8-brane in the $\mathbb{C}P^3$, and the instability of the D3-D7 configuration corresponds to a "slipping" mode of the D8-brane. The D8-brane deformation described above corresponds to moving the D7-brane from $x^5 \to -\infty$ to $x^5 \to +\infty$ across the D3-branes (Fig. 1b). The key property of the D7-brane is that it sources a monodromy for the RR scalar potential in the 5-6 plane $C_0 \to C_0 + 2\pi$, which one can regard as occuring across a branch cut emanating from the D7-brane [27]. As the D7-brane is taken to $x^5 \to \infty$ we are left with a piecewise constant C_0 background that jumps by 2π across the cut. This leads to an additional CS term on the D3-brane that the cut intersects, that comes from the 4d RR coupling:

$$
\int_{R^{1,2}} \int_{x^6} C_0 \mathrm{Tr} \left(F \wedge F \right) = 2\pi S_{CS} . \tag{16}
$$

Depending on the position of the D7-brane on the circle, the theory will be either $U(N + l)_k \times U(N)_{-k+1}$ or $U(N + l)_{k+1} \times U(N)_{-k}$. More generally, for q D7-branes the resulting theory is $U(N + l)_{k_1} \times U(N)_{k_2}$, with $k_1 + k_2 = q$. There are $q + 1$ distinct theories, corresponding to the different ways of distributing the D7-branes between the two halves of the circle. Alternatively, we can describe the distinct theories in terms of configurations with all the D7-branes on the same side, but with additional D5-branes attached. Start with the configuration describing $U(N + l)_k \times U(N)_{-k+1}$ (Fig. 1b), and move the D7-brane across the NS5-brane. This leads to the creation of a D5-brane [28], and therefore changes the part of the NS5-brane below the D7-brane to a $(1,1)$5-brane (Fig. 1c). The resulting theory is $U(N + l)_{(k-1)+1} \times U(N)_{-(k-1)}$, and is identical to the original theory. We can therefore put all the D7-branes on one side, and describe the different configurations by the number of D5-branes. In the Type IIA picture these different configurations correspond to D8-branes with additional D6-branes ending on them. We will fix our convention by identifying the D8-brane embedding without attached D6-branes with the D7-brane to the *left* of the NS5-brane without attached D5-branes. This then tells us that the background with $F_0 = q$ and $F_2 = k$ is dual to the unique CFT with $U(N + l)_k \times U(N)_{-k+q}$.
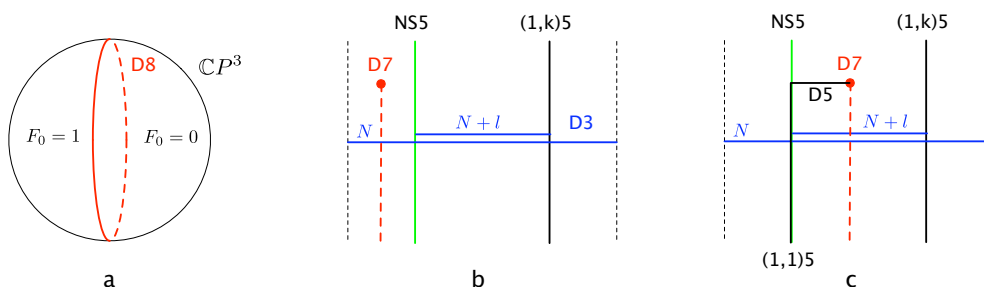

Figure 1. The D8-brane deformation and the dual Type IIB brane configuration.

Acknowledgments

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