

Stabilization of atoms in strong non-classical electromagnetic fields

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Abstract. The dynamics of an atom in a strong single mode non-classical electromagnetic cavity field is studied. The reconstruction of the energy spectrum of atom and the effect of the ionization suppression in the presence of non-classical field are discussed. It is found that Kramers – Henneberger mechanism of the ionization suppression is realized in strong nonclassical fields also. For squeezed vacuum states the threshold of the stabilization is approximately three times lower than for classical (coherent) state of the same intensity.

1. Introduction

Recent progress in laser physics makes it possible to obtain so-called squeezed states of electromagnetic wave field [1-4]. Such states with the large amount of energy are an example of macroscopic quantum effect, and hence they are of significant interest from both theoretical and practical points of view. Among different problems of interaction of non-classical electromagnetic fields with matter the case of high-intensity fields seems to be attractive and promising as high-order multiphoton processes should be taken into account [5]. The interest for such fields is based on the fact that the dispersion of the number of photons for strong squeezed states is much larger than the dispersion of photons for the classical electromagnetic field with the same intensity. Hence probabilities or cross sections for different multiphoton processes can be extremely large in such squeezed states of the field. This idea was confirmed for a number of processes, among them are the nonlinear polarization response and high-order nonlinear susceptibilities [6,7], strong-field atomic dynamics and multiphoton ionization [8,9], laser-induced free-free transitions for atomic and negative ion systems [10].

The effect of the ionization suppression (or stabilization) is one of the most intriguing nonlinear effects arising in the interaction of the strong electromagnetic fields with atoms or molecules. This effect was firstly predicted in theoretical papers [11-13] and later it was observed in experiments [14-16]. Two different mechanisms of the stabilization phenomenon can be distinguished. One of them is the interference stabilization proposed in [11] for Rydberg atomic states. Another one is the adiabatic or Kramers – Henneberger (KH) stabilization [12, 13]. This mechanism is predicted to exist for both ground and excited states of a quantum system. The detail description of the stabilization phenomenon can be found in [17, 18]. For both cases the stabilization is the result of the reconstruction of atomic spectra in the presence of the strong external electromagnetic field and formation of the atom



“dressed” by the field and resistant against ionization. In the strong field limit the energy spectrum structure of this “dressed” atom can be differ significantly from the spectrum of the field-free atom.

Till nowadays the problem of the strong-field atomic stabilization was studied only in the classical field limit. The only exception is the recent paper [19], where the dynamics of highly excited (Rydberg) states in the intense non-classical electromagnetic field was analyzed. In particular, it was found that such states are significantly more resistant against ionization if the field mode is in the squeezed vacuum state.

The interaction of the quantum system being initially in the ground state with the strong single mode non-classical electromagnetic field is studied in this paper for the case of the single photon transition from the ground state to the continuum. For the optical frequency band such situation is realized dominantly for negative atomic ions which have the only bound state in the discrete spectrum¹. The reconstruction of the atomic spectrum in the presence of the non-classical field and the formation of the “dressed” atom are analyzed. The stabilization phenomenon similar to the KH stabilization in the classical field is found to exist for the case of non-classical electromagnetic field states.

2. Quantized electromagnetic field and its interaction with an atomic quantum system

In this paper we will consider only the single mode of the electromagnetic field of frequency ω . The interaction of such a field with an atomic quantum system can be realized experimentally in a cavity or a resonator of micron size. Each radiative field mode can be considered as an oscillator with the Hamiltonian H_f . In the “pseudocoordinate” representation the expression for H_f can be written as

$$H_f = \frac{\hbar\omega}{2} \left(-\frac{\partial^2}{\partial \varepsilon^2} + \varepsilon^2 \right). \quad (1)$$

Here ε is the dimensionless electric field strength, $\varepsilon_0 = \sqrt{4\pi\hbar\omega/L^3}$ is the normalization constant and L is the spatial size of the cavity.

From the quantum-mechanical point of view the classical electromagnetic field is a coherent state of the field mode with a large average number of photons [20]. This state can be decomposed over the stationary (Fock) states

$$\phi_c(\varepsilon) = \sum_k \alpha_k |k\rangle, \quad (2)$$

with the amplitudes α_k given by a Poisson statistics.

Among different non-classical states of the field mode the most interesting is the squeezed vacuum state. This state is given by the function

$$\phi_{sq}(\varepsilon) = \frac{1}{\sqrt{\beta}\sqrt{\pi}} \exp(-\varepsilon^2/(2\beta^2)), \quad (3)$$

where β is the parameter of squeezing. Though average electric field strength in a squeezed vacuum state is equal to zero for any instant of time, the average field energy for the case $\beta \ll 1$ (or $\beta \gg 1$) is high and given by the expression:

$$\langle E \rangle = \hbar\omega(\langle k \rangle + 1/2). \quad (4)$$

Here $\langle k \rangle = (\beta - 1/\beta)^2 / 4$ is the average number of photons in a squeezed vacuum state. The decomposition of the squeezed vacuum state over the Fock states

$$\psi_{sq}(\varepsilon) = \sum_k \alpha_{2k} |2k\rangle, \quad (5)$$

¹ One of the examples of such a system is a negative Hydrogen ion. Its ionization potential is equal to 0.75 eV, and the spatial size is of several atomic units.

gives the non-Poisson statistics of photon with

$$|\alpha_{2k}|^2 = \frac{1}{\sqrt{1+\langle k \rangle}} \cdot \frac{(2k)!}{2^{2k} (k!)^2} \left(\frac{\langle k \rangle}{1+\langle k \rangle} \right)^k, \quad (6)$$

and all odd coefficients equal to zero. The comparative analysis of the photon statistics for coherent and squeezed vacuum state is presented in [8, 9]. As the dispersion of the number of photons for coherent and squeezed vacuum states are $D^{coh} = \langle k \rangle$ и $D^{sq} = 2(\langle k \rangle^2 + \langle k \rangle)$, one obtains that for $\langle k \rangle \gg 1$ the probability to find the number of photons significantly different from the average value $\langle k \rangle$ is extremely high in the squeezed state.

The interaction of atomic system with a single-mode quantized electromagnetic field is considered in the dipole approximation. The total Hamiltonian of the system “atom + electromagnetic field” is given by

$$H(\vec{r}, \epsilon) = H_0(\vec{r}) + H_f - \epsilon_0 (\vec{d} \vec{e}_\lambda) \epsilon, \quad (7)$$

where H_0 is the atomic Hamiltonian with the only stationary state in discrete spectrum $|g\rangle$ and the energy E_g , and the continuum states $|E\rangle$; $\vec{r}$ is electronic coordinate; $\vec{d}$ is the dipole moment and $\vec{e}_\lambda$ is the polarization of the field. Further, we will suppose that $\hbar\omega > I$ ($I = -E_g$ is the ionization potential).

The total time-dependent wave-function of the system $|\Psi\rangle$ we will expand in terms of interaction-free eigenstates of the atomic and light field Hamiltonians:

$$\Psi(\vec{r}, \epsilon, t) = \sum_k C_{g,k}(t) |g\rangle |k\rangle \exp\left(-\frac{i}{\hbar}(E_g + E_k)t\right) + \sum_k \int dE \cdot C_{E,k}(t) |E\rangle |k\rangle \exp\left(-\frac{i}{\hbar}(E + E_k)t\right), \quad (8)$$

Here $E_k = \hbar\omega(k + 1/2)$ is the energy of the field subsystem, and coefficients $C_{g,k}$ ($C_{E,k}$) are the amplitudes to find the atom in a state $|g\rangle$ ($|E\rangle$), and the field in a state $|k\rangle$. Substituting (8) to the nonstationary Schroedinger equation, one obtains:

$$\begin{aligned} i\hbar \frac{dC_{E,k}}{dt} &= -\frac{\epsilon_0}{\sqrt{2}} d_{Eg} \exp\left(\frac{i}{\hbar}(E - E_g)t\right) \left\{ C_{g,k+1} \sqrt{k+1} \exp(-i\omega t) + C_{g,k-1} \sqrt{k} \exp(i\omega t) \right\} \\ &\quad - \frac{\epsilon_0}{\sqrt{2}} \int dE' d_{EE'} \exp\left(\frac{i}{\hbar}(E - E')t\right) \left\{ C_{E',k+1} \sqrt{k+1} \exp(-i\omega t) + C_{E',k-1} \sqrt{k} \exp(i\omega t) \right\} \\ i\hbar \frac{dC_{g,k}}{dt} &= -\frac{\epsilon_0}{\sqrt{2}} \int dE' d_{gE'} \exp\left(\frac{i}{\hbar}(E_g - E')t\right) \left\{ C_{E',k+1} \sqrt{k+1} \exp(-i\omega t) + C_{E',k-1} \sqrt{k} \exp(i\omega t) \right\} \end{aligned} \quad (9)$$

Here d_{gE} , $d_{EE'}$ are the atomic dipole matrix elements.

3. AC Stark shift in the non-classical electromagnetic field

Let us start from the study of the reconstruction of the energy spectrum and the atomic potential by the non-classical light field. We suppose that for initial instant of time the atomic state is $\phi_g(\vec{r}) = |g\rangle$, and the state of the field mode is $\phi^{field}(\epsilon) = \sum_k \alpha_k |k\rangle$ with definite values of α_k . In the second order of perturbation theory for low enough time of consideration one derives from the set of equations (9)

$$C_{gk}^{(2)} = -\frac{i}{\hbar} \frac{\epsilon_0^2}{2} \alpha_k \int \frac{|d_{Eg}|^2}{\hbar} \left\{ \frac{2\omega_{Eg}}{\omega_{Eg}^2 - \omega^2} k + \frac{1}{\omega_{Eg} + \omega} \right\} dE \cdot t,$$

and hence the function of the system “atom + electromagnetic field” can be written in a form:

$$\Psi(\vec{r}, \epsilon, t) \approx \sum_k \alpha_k \left(1 - \frac{i}{\hbar} \frac{\epsilon_0^2}{2} \chi_k t \right) |g\rangle |k\rangle \exp\left(-\frac{i}{\hbar} (E_g + E_k)t\right) + \text{continuum}, \quad (10)$$

where $\chi_k = \int \frac{|d_{Eg}|^2}{\hbar} \left\{ \frac{2\omega_{Eg}}{\omega_{Eg}^2 - \omega^2} k + \frac{1}{\omega_{Eg} + \omega} \right\} dE$, and «continuum» implies the superposition of states

to find electron in continuum. From (10) one obtains the following expression for the atomic wave function of the bound state

$$\psi(\vec{r}, t) \approx \sum_k |\alpha_k|^2 \left(1 - \frac{i}{\hbar} \frac{\epsilon_0^2}{2} \chi_k t \right) \phi_g(\vec{r}) \exp\left(-\frac{i}{\hbar} E_g t\right) \approx \phi_g(\vec{r}) \exp\left(-\frac{i}{\hbar} (E_g + \delta E_g) t\right),$$

where

$$\delta E_g = -\frac{\epsilon_0^2}{2} \int \frac{|d_{Eg}|^2}{\hbar} \left\{ \frac{2\omega_{Eg}}{\omega_{Eg}^2 - \omega^2} \langle k \rangle + \frac{1}{\omega_{Eg} + \omega} \right\} dE$$

is the shift of the energy of the bound state $\phi_g(\vec{r})$. It is important to notice, that the shift obtained is defined only by the average number of quanta $\langle k \rangle$ and do not depend on the photon statistics. The shift appears to exist also for the case of $\langle k \rangle = 0$. It is well known Lamb shift, that appears from interaction of the atomic system with the electromagnetic vacuum. For intense fields ($\langle k \rangle \gg 1$) this effect can be neglected. Then, taking into account the expression $\langle \epsilon^2 \rangle = 2\langle k \rangle \epsilon_0^2$ [8, 9], we derive:

$$\delta E_g = -\chi \langle \epsilon^2 \rangle / 4, \quad \chi = \int \frac{|d_{Eg}|^2}{\hbar} \frac{2\omega_{Eg}}{\omega_{Eg}^2 - \omega^2} dE, \quad (11)$$

where χ is the atomic susceptibility. The expression (11) corresponds to the semiclassical limit of the problem [17], when the atomic subsystem is considered as a quantum one, while the field is supposed to be classical. Indeed, if the light mode is in the coherent state with $\langle k \rangle \gg 1$, then $\langle \epsilon^2 \rangle \equiv \langle \epsilon \rangle^2$ and expression (11) can be written in a typical form

$$\delta E_g = -\chi \langle \epsilon \rangle^2 / 4, \quad (12)$$

where $\langle \epsilon \rangle$ is the averaged over quantum state amplitude value of the electric field strength.

4. Ionization suppression in a strong non-classical field

In the classical field limit the stabilization of atoms is a result of the reconstruction of the atomic spectrum by a strong electromagnetic field and the formation of the “dressed” atom resistant against ionization. For the case of single photon transition to the continuum this “dressed” atom exists in a form of KH atom [17,18]. The emergence of the KH potential is typically associated with the nearly free oscillations of the atomic electron in a strong laser field with the amplitude $a_e = e\epsilon/m\omega^2$. It was found [21], that in relatively weak fields corresponding to the quiver motion amplitude $a_e \leq a_0$ (a_0 is the typical size of the quantum system) the structure of the energy spectrum of the KH atom corresponds to the field-free atomic spectrum with AC Stark shift taken into account. It was demonstrated in section 3, that the AC Stark shift in the quantized field does not feels the photon statistics and depends only on the average number of photons. It means that at least in such intensity limit the reconstruction of the atomic energy spectrum in a quantized field can be interpreted in terms of appearance of the KH atom. As the intense non-classical field can be found in the state with zero value of the average electric field strength, we conclude that the emergence of the KH potential do not result from the quiver motion of the atomic electron.

It was found in [21, 22] that for the classical electromagnetic field limit the KH stabilization can be interpreted in terms of coherent repopulation of the atomic continuum states by free-free transitions and the interference of the different multiphoton order transitions from the initial bound state $|g\rangle$ to the final continuum state $|E\rangle$. Similar approach can be developed for non-classical field states also. In this paper we restrict ourselves to the consideration of the interference of the direct single-photon transition to the continuum and the three-photon transition via the intermediate continuum state (figure 1). In this case the total amplitude of the single- and three-photon transition to the continuum state $|E\rangle$ and the number of quanta in the field mode equal to k can be written in a form:

$$C_{E,k} = C_{E,k}^{(1)} + C_{E,k}^{(3)} = -\frac{i}{\hbar} \frac{\epsilon_0}{\sqrt{2}} \alpha_{k+1} \sqrt{k+1} \cdot \left\{ d_{E,i} + \frac{\epsilon_0^2}{2} k \int dE' dE'' \frac{d_{EE''} d_{E'',E'} d_{E',i}}{\hbar^2} \frac{1}{(\omega_{E',i} - \omega)(\omega_{E'',i} - 2\omega)} \right\} \int \exp(i(\omega_{E,i} - \omega)t) dt. \quad (13)$$

The probability of ionization can be derived for (13) as a result of summation over photon number states and integration over energy spectrum of photoelectrons $W_i = \sum_k \int |C_{E,k}|^2 dE$.

Taking into account that $\sum_k |\alpha_k|^2 k^2 = \langle k^2 \rangle$ and applying the pole approximation [17] for calculation of the integral in (13) we obtain the expression for the ionization rate of quantum system

$$\dot{W} = \frac{2\pi}{\hbar} |d_{E,i}|^2 \frac{\epsilon_0^2}{2} \left[\langle k \rangle - \pi^2 \epsilon_0^2 \langle E | d | E + \hbar\omega \rangle^2 (\langle k^2 \rangle - \langle k \rangle) \right]. \quad (14)$$

Here $E = \hbar\omega - I$ and I is the ionization potential.

The result obtained is the generalization of the expression from [22] for the arbitrary state of the field. If the state of the field is the coherent one, the expression for the ionization rate takes the form

$$\dot{W} = \frac{2\pi}{\hbar} |d_{E,i}|^2 \frac{\epsilon_0^2}{2} \langle k \rangle \left[1 - \pi^2 \epsilon_0^2 \langle E | d | E + \hbar\omega \rangle^2 \langle k \rangle \right], \quad (15)$$

which can be easily obtained in the frames of the semiclassical theory [22]. The appearance of the stabilization phenomenon follows from (15) and the threshold of the stabilization can be estimated as

$$\langle k \rangle^{(threshold)} \approx \frac{L^3}{4\pi^3 \hbar\omega \langle E | d | E + \hbar\omega \rangle^2}. \quad (16)$$

Estimating the value of the free-free matrix element transition as $\langle E | d | E + \hbar\omega \rangle \sim e a_0 / \hbar\omega$, where a_0 is the typical size of the atomic system, we find that

$$\langle k \rangle^{(threshold)} \approx \frac{1}{4\pi^3} \frac{\hbar\omega}{Ry} (L/a_0)^3, \quad (17)$$

where $Ry \approx 13.6$ eV. For example, for the cavity

of the micron size and $\hbar\omega = 2$ eV we have $\langle k \rangle^{(threshold)} \sim 10^{10}$, that is approximately corresponds to the intensity $\sim 10^{14}$ W/cm².

Different situation arises for the non-classical field. For example, for the squeezed vacuum state we obtain from (15)

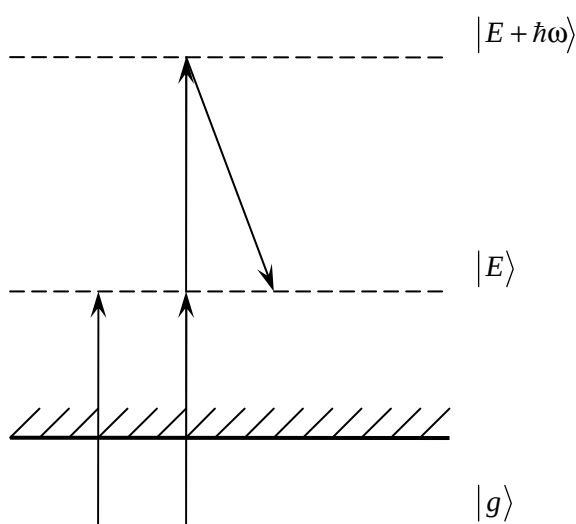


Figure 1. A scheme of bound-continuum and continuum-continuum transitions taken into account in the model.

$$\dot{W} = \frac{2\pi}{\hbar} |d_{E,i}|^2 \frac{\epsilon_0^2}{2} \langle k \rangle \left[1 - \pi^2 \epsilon_0^2 |\langle E | d | E + \hbar\omega \rangle|^2 (3\langle k \rangle + 1) \right]. \quad (18)$$

It means that for the squeezed vacuum state the stabilization threshold is approximately three times lower in comparison with the coherent state of the same intensity.

5. Conclusions

The phenomenon of the field reconstruction and formation of the “dressed” atom (KH atom) in the single mode cavity field is studied in the paper. It is found that for single-photon bound of the discrete spectrum and continuum this reconstruction depends on only the average number of quanta but not the photon statistic of the field mode. It is demonstrated that the KH stabilization appears to exist in the non-classical light also as a result of nonlinearity of the matrix element transition to the continuum. For squeezed field states the threshold of the stabilization can be several times lower than for classical field.

It should be emphasized also that decreasing of microcavity size leads to the decreasing of the number of photons necessary for stabilization. In principle, it is possible to suppose the situation when the stabilization will result only from few-photon interaction with an atom. It follows from (17) that in such a case the size of the cavity should be close to the size of the quantum system. For visible or IR frequency band it means that the size of the quantum system should be significantly greater than the atomic size. Probably, such situation can be realized in the modern nanostructure objects (quantum dots), where the Wannier-Mott exciton can serve as an object stable to ionization [24,25].

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