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Turn-on delay of QD and QW laser diodes: What is the difference?

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Abstract. Turn-on delay of laser diodes with quantum-sized active media is investigated both theoretically and experimentally. In this research we show the striking difference in turn-on delay of quantum dot and quantum well laser diodes: With quantum-well lasers turn on delay tends to zero in the limit of high pumping, while with quantum dot lasers turn-on delay has the non-vanishing component which is independent of pumping.

1. Introduction
Laser diodes (LDs) with quantum-sized active media such as quantum wells (QWs) and quantum dots (QDs) are widely used and intensively studied due to their efficiency, compactness and high flexibility of properties allowing for implementation of many extremely different application-oriented constructions. In recent years, much attention is paid to the pulsed operation of LDs allowing for orders of magnitude higher pumping densities [1,2]. However, the research of dynamical properties of laser diodes, especially those based on QDs, concentrate mostly on relaxation oscillations and related phenomena such as damping of relaxation oscillations [3–5]. Except the seminal work [6], little attention is paid to study of the turn-on behavior of LDs where the pump is quickly changed from below to above threshold that allows to deeply explore the dynamical response under various lasing conditions. In this work we study the difference between the turn-on of QW and QD LDs with special attention to the nonlinear and non-instantaneous capturing of the carriers into a quantum dot which is known to strongly affect the recovery of QD material [7], but remains unresolved except for recent publication [8,9].

2. Turn-on delay of QW LD
We first consider the turn on delay in QW LD. Theory is based on the simple rate equation system:

$$\begin{cases} \dfrac{dN}{dt} = \dfrac{j}{ed} - A(N - N_{tr})S - \dfrac{N}{\tau} \\ \dfrac{dS}{dt} = \Gamma A(N - N_{tr})S - \dfrac{S}{\tau_p}, \end{cases} \tag{1}$$

where N is the carrier concentration, S is the photon density in cavity, j is the current density, e is the electron charge, d is the active layer width, A is the linear gain coefficient, N_{tr} is the transparency concentration, τ is the carrier lifetime, τ_p is the photon lifetime in cavity, Γ is the optical confinement coefficient. It is convenient to normalize the rate equations (1) before solution. It is well known that for threshold concentration N_{th} one can write:

$$\Gamma A(N - N_{tr}) = \Gamma A(N - N_{th}) + \tau_P^{-1} \tag{2}$$

Substituting (2) into the system (1) we get:

$$\begin{cases} \dfrac{dN}{dt} = \dfrac{j - j_{th}}{ed} - \dfrac{N - N_{th}}{\tau} - A(N - N_{th})S - \dfrac{S}{\Gamma\tau_p} \\ \dfrac{dS}{dt} = \Gamma A(N - N_{th})S \end{cases} \tag{3}$$

where $j_{th} = edN_{th}/\tau$ is the threshold current density. If we denote $N - N_{th} = \delta N$, then:

$$\begin{cases} \dfrac{d\delta N}{dt} = \dfrac{j - j_{th}}{ed} - \dfrac{\delta N}{\tau} - AnS - \dfrac{S}{\Gamma\tau_p} \\ \dfrac{dS}{dt} = \Gamma AS\delta N \end{cases} \tag{4}$$

Now introduce new normalized variables $D = 1 + \Gamma A\tau_p \delta N$, $J = 1 + \tau\tau_p A\Gamma \dfrac{j - j_{th}}{ed}$, $\eta = \dfrac{\tau_p}{\tau} << 1$, $I = \tau AS$, $t_1 = \dfrac{t}{\tau_p}$ and get new normalized rate equations for intensity I and carrier quantity D [10]:

$$\begin{cases} \dfrac{dI}{dt_1} = I(-1 + D), \\ \dfrac{dD}{dt_1} = \eta(J_+ - D(1 + I)), \end{cases} \tag{5}$$

with boundary conditions

$$\begin{cases} D(0^+) = J_- \\ I(0^+) = I_0 << 1 \end{cases}. \tag{6}$$

With these normalized variables the threshold current $J_{\mathrm{th}}=1$ and threshold carrier density $D_{\mathrm{th}}=1$. On the front edge of the pump pulse current values are: $J(0^-)=J_-<1$ and $J(0^+)=J_+>1$. In what follows we will use t instead of t_1 in all formulae for simplicity. Steady state solution of (5) at fixed current J is:

$$\begin{cases} I = 0 \\ D = J \end{cases} \quad if \;\; J < 1, \quad D = I = J - 1 \;\; \text{If } J \geq 1 \tag{7}$$

Eq. (7) shows that LD turns on if the carrier density reaches threshold value. The turn on delay may be found from the second equation of the system (5) if we assume that intensity below threshold is zero $I=0$:

$$\frac{dD}{dt} = \eta(J_+ - D), \quad D(0) = J_- \tag{8}$$

Solution of eq. (8) is:

$$D(t) = J_+ - (J_+ - J_-)\exp(-\eta t), \tag{9}$$

and from condition $D(\Delta t) = D_{th} = 1$ we find turn-on delay:

$$\Delta t = \frac{1}{\eta} \ln\left(\frac{J_+ - J_-}{J_+ - 1}\right). \tag{10}$$

Which for non-normalized rate equations transforms to the well known formula:

$$\Delta t = \tau \ln\left(\frac{j_+ - j_-}{j_+ - j_{th}}\right). \tag{11}$$

At strong pump $j >> j_{\mathrm{th}}$ we get a hyperbolic function for turn-on delay:

$$\Delta t \approx \tau \frac{j_{th}}{j_+}. \tag{12}$$

Formulae (10-12) were obtained in assumption that LD turns on when the carrier concentration D reaches the threshold value D_{th}. Another type of boundary condition is that laser turns on when the intensity reaches a reference value I_{ref}. This approach is more convenient for experimental research. Solving the system (5) with new boundary conditions for turn-on by substituting I=0 into the first equation we can transform the second to:

$$I = I_0 \exp\left(\frac{1}{\eta} F(s)\right), \tag{13}$$

where $s=\eta t$ is the normalized time and:

$$F(s) \equiv (J_+ - 1)s - (J_- - J_+)(\exp(-s) - 1). \tag{14}$$

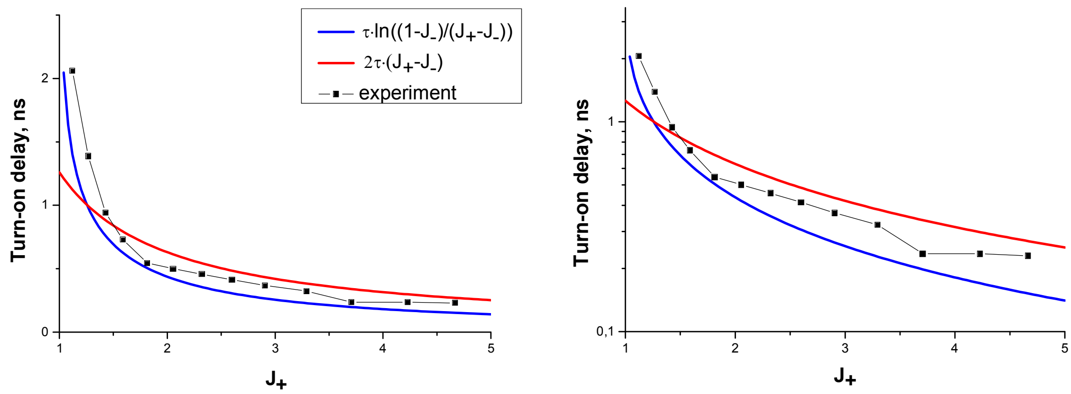

Figure 1. Turn-on delay of QW LD vs normalized current in linear (a) and log-scale (b): experiment (black rectangles), fit with eq. (10) (blue line), fit with eq. (19) (red line). Fitting values are: J_-= 0 and τ = 0.63 ns.

We can find s_{on} from the condition $I(s_{on})=I_{ref}$:

$$F(s_{on}) = \eta \ln(I_{ref} / I_0) \tag{15}$$

with η << 1 and assuming logarithm is a slow function we can approximate:

$$F(s_{on}) \approx 0. \tag{16}$$

and in condition of strong pump s << 1 and J_+ >> 1, we simplify (14):

$$F(s) = -(1 - J_-)s + \frac{J_+}{2} s^2, \tag{17}$$

so that:

$$s_{on} \simeq 2(1 - J_-)J_+^{-1}. \tag{18}$$

And now we can write expression for the turn-on delay:

$$\Delta t \simeq 2\tau \frac{1-J_-}{J_+} \tag{19}$$

From here, it is easy to see that $\Delta t \to 0$ with $J_+ \to \infty$.

Experimental measurements of the turn-on delay were made with edge-emitting ridge stripe QW LD with generation wavelength 1060 nm. In our experiments we use pulses of 1 ns rise-time (measured at 10%–90% level) obtained from a high power (up to 1.5A current) pulse source. The laser output was detected using a high-speed pin detector with a cut-off frequency of 30 GHz and a 50 GHz digital oscilloscope. The turn-on delay was derived as the time difference between the rise-up of the pump current pulse measured at the laser diode and that of the signal from the photodetector. The optical and electrical path lengths were carefully estimated, and the difference between the two lengths was taken into account. Further details of the experimental technique can be found in Ref. [11]. Fig.1 shows experimental dependence of the turn-on delay time Δt on the normalized current J_+. It is clearly seen that experiment has a good agreement with (10) for low current, while for of high currents experimental dependence tends to curve corresponding to (19). In other words, the first approach for characterization of the turn-on delay is appropriate near the threshold while the second one is applicable at high pumping.

3. Turn-on delay of QD LD

In this part of the paper we will consider the turn-on delay of QD LD. The main difference between QW and QD LDs in this consideration is the nonlinear and non-instantaneous capturing of the carriers into a dot. It is known to strongly affect the recovery of QD material [7] and thus can have an impact on the laser turn-on delay [9]. In QD LDs, carriers are injected at first in to the wetting layer, and only after that they can be captured in QDs. Rate equations for this two-stage process are represented by three: the first is for the photon density in the cavity S, the second is for the QD filling probability ρ and the third is for the carrier density in the wetting layer n scaled to the two-dimensional QD density per layer [5,12-14]:

$$\begin{cases} \dfrac{dS}{dt} = -\dfrac{S}{\tau_p} + v_g g_0 (2\rho - 1) S \\ \dfrac{d\rho}{dt} = \dfrac{\rho}{\tau_p} + \dfrac{2n(1-\rho)}{\tau_{cap}} - v_g g_0 (2\rho - 1) S \\ \dfrac{dn}{dt} = \dfrac{n}{\tau} + \dfrac{j}{e} - \dfrac{2n(1-\rho)}{\tau_{cap}} \end{cases}, \tag{20}$$

where τ_d is the carrier lifetime in dot, v_g is the group velocity, σ is the interaction cross-section of carriers with photons in a dot, g_0 is the differential gain, τ_{cap} is the carrier capture time in dot. Rate for the carrier capture in a dot is described by $2n(1-\rho)/\tau_{cap}$. It is proportional to the carrier density in the wetting layer n and to a probability for carrier to find a vacant dot. Factor 2 accounts for the spin degeneracy in quantum dot, while factor $1-\rho$ satisfies Pauli principle and describes nonlinear interaction between quantum dots and the wetting layer and constitutes the most important difference between QD and QW rate equations.

We normalize these rate equations similarly to the earlier QW case. New normalized variables are: $t_1 = t/\tau_p$, $I = v_g \sigma S \tau_d$, $\eta_d = \tau_p / \tau_d$, $\eta = \tau_p / \tau$ $g = \tau_p v_g g_0$, $B = \tau_d / \tau_{cap}$, $J = j\tau / q$ (pump current per dot). Substituting these variables in (20) we obtain the normalized rate equations:

$$\begin{cases} \dfrac{dI}{dt_1} = [-1 + g(2\rho - 1)]I \\ \dfrac{d\rho}{dt_1} = \eta_d[-\rho + Bn(1-\rho) - (2\rho - 1)I] \\ \dfrac{dn}{dt_1} = \eta[-n + J - 2Bn(1-\rho)] \end{cases} \tag{21}$$

In what follows we will use t instead of t_1 in all formulae for simplicity and assume that $\tau_d \approx \tau$ (hence $\eta_d \approx \eta$). The QD LD gain $g(2\rho\text{-}1)$ is defined by the dot density and the normalized differential gain g. For typical value τ=1 ns and τ_{cap}=10 ps we take factor $B\approx100$. The normalized threshold current of QD LD is J_{th}=2 (for QW LD J_{th}=1). This follows from the spin degeneracy in QD energy levels. With J_-<2, laser operates in stable OFF steady state, so that:

$$\begin{cases} I = 0 \\ \rho = \rho_0 = J_- / 2 + O(B^{-1}) \\ n = n_0 = B^{-1}J / (2 - J_-) + O(B^{-2}) \end{cases} \tag{22}$$

The boundary conditions are:

$$\begin{cases} \rho(0^-) = \rho_0 \\ n(0^-) = n_0 \\ I(0^-) = I_0 << 1 \end{cases} \tag{23}$$

Turn-on delay Δt in our consideration is the lag between the momentum of time $t=0^+$, when current simultaneously becomes J_+>2, to the momentum of time when $I=I_{ref}$. Assuming the intensity before laser turn-on is very small we substitute $J=J_+$ and I=0 in the second and the third equations (21) and obtain differential equation of the second order for ρ:

$$\frac{d\rho}{ds} = B\,[(z_0 - J_+)\exp(-s) + J_+ - 2\rho](1-\rho) - \rho, \tag{24}$$

where $s = \eta t$ and $z_0 \equiv 2\rho_0 + n_0 = J_- + O(B^{-1})$. Solution of the first equation in (21) takes the form:

$$I = I_0 \exp\left(\frac{1}{\eta}G(s)\right), \tag{25}$$

where $G(s)$ is defined by:

$$G(s) \equiv -(1+g)s + 2g\int_0^s \rho(u)du. \tag{26}$$

Now we can determine numerically Δt by substituting solution for eq. (24) in eq. (26) and then substituting result in eq. (25) and finding s for $I=I_{ref}$. Analytically we will investigate turn-on delay in some limit case similar to QW lasers. Namely, we will consider the limit of large $B\rightarrow\infty$, and the limit of large $J_+\rightarrow\infty$. In the limit of $B\rightarrow\infty$ and $1-\rho >> B^{-1}$ (24) simplifies to Bernoulli equation for $1-\rho$:

$$\frac{d\rho}{ds} \simeq B[(J_- - J_+)\exp(-s) + J_+ - 2\rho](1-\rho). \tag{27}$$

which has analytical solution for ρ:

$$\rho = 1 - \frac{\exp(-BF(s))}{(1-\rho_0)^{-1} + 2B\int_0^s \exp(-BF(u))du}, \tag{28}$$

with $F(s)$ defined by:

$$F(s) \equiv (J_+ - J_-)(\exp(-s) - 1) + (J_+ - 2)s. \tag{29}$$

In the limit of $J_+\to\infty$ the approximate expression for $F(s)$ will be very useful. This approximation can be clearly seen from simplification of (29) for large J_+:

$$F(s) \simeq J_+[(\exp(-s)-1)+s] \tag{30}$$

Consideration of (28) shows that it has initial level and saturates at level $\rho=\rho_s\equiv1+O(B^{-1})$. For $\rho\approx1$ expression (26) simplifies to:

$$G=(g-1)s \tag{31}$$

Substituting (31) in (25) we obtain:

$$I \simeq I_0 \exp\left(\frac{1}{\eta}(g-1)s\right). \tag{32}$$

Then the minimum turn-on delay for QD LD is:

$$\Delta t = \frac{\tau_p}{\tau_p v_g g_0 - 1}\ln\left(\frac{I_{ref}}{I_0}\right). \tag{33}$$

This formula shows the striking difference in turn-on delay of QD and QW LDs. With quantum-well lasers turn-on delay is zero in the limit of $J_+\to\infty$. With QD lasers the situation is very different as the turn-on delay has the independent of pumping non-vanishing component given by (33).

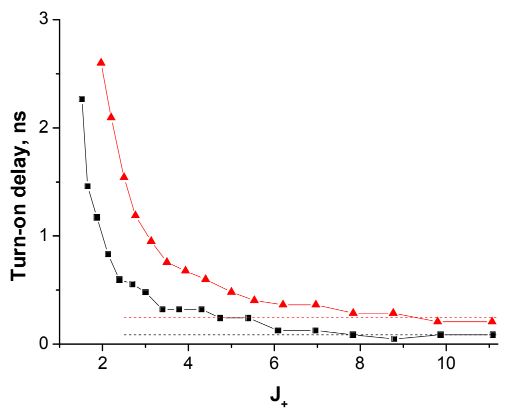

Figure 2. Experimental results for turn-on of QD LDs with 1 QD layer (red triangles) and 5 QD layers (black rectangles).

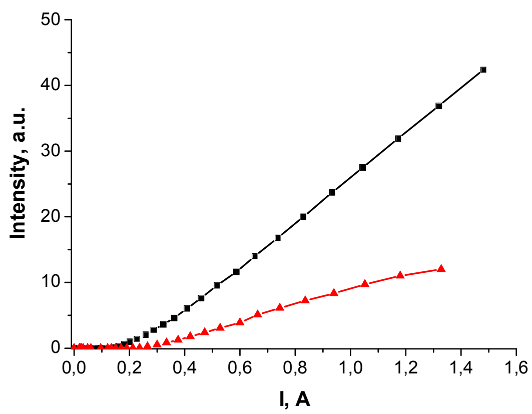

Figure 3. Light-current curves of lasers with 1 QD layer (red triangles) and 5 QD layers (black rectangles).

In our experiments we investigate lasers with 1 and 5 QD layers. Growth and basic properties of similar QD laser structures have been described elsewhere [15]. Length of all lasers was 4.5 mm and the generation line was at 1290 nm (corresponding to the ground state lasing). Experimental technique was the same as with QW LDs and results of measurements are shown in Fig. 2. *L-I* curves for these lasers are shown in Fig. 3. It is seen from fig. 2 that despite of some scattering, measurements with QD LDs clearly indicate the quasi-stable pump-independent value for higher currents. The difference of the values of the minimum turn-on delay for QD LDs with 1 and 5 QD layers is clear if the differential gain of these lasers is taken into account (see expression (33)). Difference of gain constants g_0 for these lasers is seen from almost two-fold difference of their thresholds in Fig. 3. Taking into account the input of the transparency current to the laser threshold, the three-fold difference of the minimum turn-on delay values is in a good agreement with the theoretical expression (33) for the minimum turn-on delay of QD LD.

4. Summary

Theoretical and experimental investigation of the turn-on delay of quantum well and quantum dot laser diodes is presented. We show that in contrast to the quantum wells, the nonlinear and non-instantaneous capturing of the carriers into a dot leads to the minimum turn-on delay of QD LDs being non-zero at any pumping current. The value of this non-vanishing turn-on delay depends strongly on the gain factor.

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