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Roto-translation motion of the stars in close binary systems

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Abstract. This article has to show that the model of p-h which is used to determine the change of the semi major axis of the relative orbit stars is incorrect and leads to large errors in the determination of semi-major axis. The new model, suitable for the elliptical orbits of the stars. To determine relative motion of stars in a close binary system in this paper uses a numerical integration of the equations of motion with the reactive forces, including the rotational component of attraction between the stars and the stream flows into the substance. The calculations of elliptical orbits of close binary stars show that the effect of the reactive force on the evolution of the orbits of stars may be different. The results can be refined by introducing other disturbing factors and making new assumptions based on observations.

1. Introduction

In the mid-20th century the research of the motion of stars in close binary systems was started. This problem is reflected in the works of: A. Kruszewskij [1], D. Hadzhidimetriu [2] S. Piotrowskij [3] etc. That research led to discovery of the conservative mass exchange in a close binary system was obtained a simplified dependence of variation of the semi-major axis a relative stars orbit from the constant rate of increase of the mass of the host star $\dot{M}_2$ in the form

$$\dot{a} = 2a\dot{M}_2\left(\frac{1}{M_1} - \frac{1}{M_2}\right). \tag{1}$$

This dependence, which is called Paczynskij-Huangs formula, is used in all researches of close binary systems with conservative mass exchange, when $M_1 + M_2 = M = \text{const}$. This formula has been derived using assumptions of the equation of motion of the stars with variable masses permit an integral of the moment of the motion amount.

$$\mathbf{J} = M_1\mathbf{R}_1 \times \mathbf{V}_1 + M_2\mathbf{R}_2 \times \mathbf{V}_2 = \mathbf{const}, \tag{2}$$

where $M_i, \mathbf{R}_i, \mathbf{V}_i,\ (i = 1, 2)$ - the mass, radius-vector and the motion speed of the stars.

The assumption of the existence of the integral of the moment of the motion amount is wrong. To let us prove this suggestion please take a look at, based on the most common form of the equations for the Meshcherskijs motion task of two bodies with variable masses:

$$M_1\frac{d\mathbf{V}_1}{d\,t} = G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_1, \qquad M_2\frac{d\mathbf{V}_2}{d\,t} = -G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_2, \tag{3}$$

where $\mathbf{R} = \mathbf{R_2} - \mathbf{R_1}$, $\mathbf{Q}_1$ $\mathbf{Q}_2$ — the reaction forces acting on the star S_1 and S_2.

The existence of the integral (2) in the task of two bodies with variable masses is possible only in the Meshcherskij-Levi-Civitas problem, the equations of the motion are obtained from (3), if the reaction forces are equal:

$$\mathbf{Q}_1 = -\dot{M}_1\mathbf{V}_1, \qquad \mathbf{Q}_2 = -\dot{M}_2\mathbf{V}_2, \tag{4}$$

where $\dot{M}_1 = -\dot{M}_2 = \text{const}$, $\dot{M}_2 > 0$.

However, using the Meshcherskij-Levi-Civitas model to research the motion of the stars in a close binary system with a conservative exchange content is unreasonable because reaction forces (4) in close binary system does not exist.

Nowadays, the department of celestial mechanics SAI MSU (L.Lukyanov, S. Hasanov) developed a model, which determines the motion of close binary systems in the case of the elliptical orbit. In this work, we consider a further adjustment of the trajectories of the stars with the influence of the possible distribution of the flowing stream parameters.

2. Restricted elliptical three-body task

To determine the motion of the particles flowing masses we will use a flat elliptical restricted three-body task, the equations of motion, which are located in a rotating and pulsating Shapners barycentric coordinate system

$$\begin{aligned} \frac{d^2x}{dv^2} - 2\frac{dy}{dv} = \rho\frac{\partial U}{\partial x}, \\ \frac{d^2y}{dv^2} + 2\frac{dx}{dv} = \rho\frac{\partial U}{\partial y}, \end{aligned} \tag{5}$$

where x axis is always directed at the star-accretor S_2, v — true anomaly stars $\rho = 1/(1+e\cos v)$ — dimensionless radius stardom S_2 relatively S_1. U denotes the Jacobis function in restricted elliptic three-body task

$$U = \frac{x^2+y^2}{2} + p^3\left(\frac{1-m}{r_1} + \frac{m}{r_2}\right) - \frac{p^2}{2}(3 + m^2 - m), \tag{6}$$

r_1, r_2 — the distance from the particle stream flowing respectively to the centres of mass of the first and second stars

$$r_1 = \sqrt{(x+pm)^2 + y^2}, \quad r_2 = \sqrt{(x+pm-p)^2 + y^2}, \tag{7}$$

where $1 - m = M_1/M$ and $m = M_2/M$ — relative masses of the stars, $p = a(1 - e^2)$ — focal parameter of the stars orbit, a and e — axis and eccentricity, respectively.

Well assume that the host star has a spherical

$$(x + pm - p)^2 + y^2 = \frac{P^2}{\rho^2}, \tag{8}$$

shape radius of P which in the process of mass transfer changes according to the dependence.

$$P = P_0\sqrt[3]{\frac{m}{m_0}}. \tag{9}$$

Outflow of the matter from the donor star starts when the energy level of the particles is larger than the energy level of the singular point of Euler L_1. Speed of expiration V_0 of the mass particles from the star S_1 is always directed toward the star S_2, forming the axis Ox an angle $\pm\alpha$ and has a value

$$V_0(v) = V_{00} V_1(v)\,\beta, \quad V_1(v) = m\sqrt{\frac{GM}{p}}\sqrt{1 + 2e\cos v + e^2}, \tag{10}$$

In the initial conditions, the angle α between the jet and the horizontal axis changes within the limits of $-\pi/2 < \alpha < \pi/2$, and the parameter β – between $0.5 < \beta < 2$.

Numerical integration is performed on the interval $v_0 \le v \le v_0 + \tau$, where the value of the true anomaly $v_0 + \tau$ determined being deposited on the surface of the particles of the second star at $x_2 = x(v_0 + \tau), y_2 = y(v_0 + \tau)$. Without doubts these results can be corrected by introducing other disturbing factors and making new assumptions based on observations.

As the result of numerical integration the components of the speed of expiration $\mathbf{W}_1$ of the star S_1 and of the speed mass flow $\mathbf{W}_2$ of the star S_2 are obtained

$$W_{1x} = V_c,\ W_{1y} = 0,\ W_{2x} = a_x + b_x(\omega + \Omega),\ W_{2y} = a_y + b_y(\omega + \Omega), \tag{11}$$

where

$$a_x = x'(v_0 + \tau)\dot{v},\ b_x = y\,(v_0 + \tau),\ a_y = y'(v_0 + \tau)\dot{v},\ b_y = -x\,(v_0 + \tau) - pm + p$$

reaction forces, influencing the stars S_1 and S_2, are considered to be attached to their center of mass and are given by

$$\mathbf{Q}_1 = \dot{m}M\{-V_c, 0\}, \quad \mathbf{Q}_2 = \dot{m}M\{x'(v_0 + \tau)\dot{v}, y'(v_0 + \tau)\dot{v}\}. \tag{12}$$

A numerical integration is determined as the mass of the steady stream S_3 and coordinates x_3, y_3 its center of mass

$$M_3 = \frac{\tau}{\dot{v}_c}\dot{m}M, \quad x_3 = \frac{1}{\tau}\int\limits_{v_0}^{v_0+\tau} x(v)dv, \quad y_3 = \frac{1}{\tau}\int\limits_{v_0}^{v_0+\tau} y(v)dv, \tag{13}$$

where

$$\dot{v}_c = \frac{1}{2\pi}\int\limits_0^{2\pi} \dot{v}\,dv = \frac{1}{2\pi}\int\limits_0^{2\pi}\sqrt{\frac{GM}{p^3}}\,(1 + e\cos v)^2\,dv = \sqrt{\frac{GM}{p^3}}\left(1 + \frac{e^2}{2}\right)$$

— the mean angular velocity of the orbital motion.

3. Differential equations stars

We explain stars' motion in the inertial coordinate system on a basis of the model which uses the following equations

$$\begin{aligned} M_1\frac{d\mathbf{V}_1}{dt} &= G\frac{M_1 M_2}{R^3}\mathbf{R} + \mathbf{Q}_1 + G\frac{M_1 M_3}{r_{13}^3}\mathbf{r}_{13}, \\ M_2\frac{d\mathbf{V}_2}{dt} &= -G\frac{M_1 M_2}{R^3}\mathbf{R} + \mathbf{Q}_2 + G\frac{M_2 M_3}{r_{23}^3}\mathbf{r}_{23}, \end{aligned} \tag{14}$$

where $r_{13} = \sqrt{(x_3 + pm)^2 + y_3^2}$, $r_{23} = \sqrt{(x_3 + pm - p)^2 + y_3^2}$ — the distances between the centres of mass of the stars and the jet.

Differential equations in osculating elements for semi-major axis of the relative orbit a, the eccentricity e and the angular rotational speed ω of the star S_2 are presented as

$$\begin{aligned} \frac{da}{dt} &= \frac{2a^2}{\sqrt{GMa(1-e^2)}}[e\sin v \cdot S + (1+e\cos v)\cdot T], \\ \frac{de}{dt} &= \sqrt{\frac{a(1-e^2)}{GM}}\left[\sin v \cdot S + \left(\cos v + \frac{\cos v + e}{1+e\cos v}\right)\cdot T\right], \\ \frac{d\omega}{dt} &= \frac{5}{2}\frac{\dot m}{m}\sin\gamma\frac{W_{otn}}{P} = -\frac{5}{2}\frac{\dot m}{P^2 m}\left[A(\omega+\Omega)+B\right] \end{aligned} \tag{15}$$

Perturbing acceleration of S and T is becoming known after the integration of the equations (5):

$$\begin{aligned} S &= \frac{\dot m}{m}\left(a_x + b_x(\omega+\Omega) + \frac{mV_c}{1-m}\right) + GM\tau\frac{\dot m}{\dot v_c}\left(\frac{x_3+pm-p}{r_{23}^3} - \frac{x_3+pm}{r_{13}^3}\right), \\ T &= \frac{\dot m}{m}\left[a_y + b_y(\omega+\Omega)\right] + GM\tau\frac{\dot m}{\dot v_c}\, y_3 r_3, \quad r_3 = \frac{1}{r_{23}^3} - \frac{1}{r_{31}^3} \end{aligned} \tag{16}$$

By choosing the true stars anomaly v and performing averaging using the independent variable, we obtain

$$\begin{aligned} \frac{d\tilde a}{d\tilde v} &= \frac{1}{2\pi}\int\limits_0^{2\pi}\frac{2a^2}{\dot v\sqrt{GMa(1-e^2)}}[e\sin v\cdot S + (1+e\cos v)\cdot T]dv, \\ \frac{d\tilde e}{d\tilde v} &= \frac{1}{2\pi}\int\limits_0^{2\pi}\frac{1}{\dot v}\sqrt{\frac{a(1-e^2)}{GM}}\left[\sin v\cdot S + \left(\cos v + \frac{\cos v + e}{1+e\cos v}\right)\cdot T\right]dv \\ \frac{d\tilde\omega}{d\tilde v} &= \frac{5}{2\dot v}\frac{\dot m}{m}\sin\gamma\frac{W_{otn}}{P} = -\frac{5}{2}\frac{\dot m}{\dot v P^2 m}\left[A(\omega+\Omega)+B\right] \end{aligned} \tag{17}$$

if, in addition to the independent variable m we find according to the formula $\frac{d}{d\tilde v} = \frac{\dot m}{\dot v_c}\frac{d}{dm}$, the equations of motion for the average of elements $\tilde a$, $\tilde e$ $\tilde\omega$ with regard to the expression $\dot v$ will take the form

$$\begin{aligned} \frac{d\tilde a}{dm} &= \frac{2+\tilde e^2}{(1-\tilde e^2)^2}\frac{\widetilde W_{2y}}{m} + 2\tilde a^3\sqrt{1-\tilde e^2}\,\tau y_3 r_3 + \frac{\tilde a^2(2+\tilde e^2)}{\sqrt{GM\tilde a}(1-\tilde e^2)}\frac{\tilde b(\tilde\omega+\Omega)}{m}, \\ \frac{d\tilde e}{dm} &= \frac{2+\tilde e^2}{2\tilde a(1-\tilde e^2)}\frac{\tilde e}{1+\sqrt{1-\tilde e^2}}\frac{\widetilde W_{2y}}{m} - \frac{3}{2}\tilde e\,\tilde a^2\sqrt{1-\tilde e^2}\,\tau y_3 r_3 - \\ &\quad - \frac{3\tilde a\tilde e(2+\tilde e^2)}{4\sqrt{GM\tilde a}(1-\tilde e^2)}\frac{\tilde b(\tilde\omega+\Omega)}{m}, \\ \frac{d\tilde\omega}{dm} &= -\frac{5}{4}\frac{2+\tilde e^2}{(1-\tilde e^2)^{3/2}P^2 m}\left[\widetilde W_{2x}\tilde y + \widetilde W_{2y}\tilde b + (\tilde b^2+\tilde y^2)(\omega+\Omega)\right] \end{aligned} \tag{18}$$

where

$$\tilde x = x\,(v_0+\tau), \quad \tilde y = y\,(v_0+\tau), \quad \widetilde W_{2x} = x'(v_0+\tau), \quad \widetilde W_{2y} = y'(v_0+\tau),$$
$$\tilde b = p - pm - \tilde x = \tilde a(1-\tilde e^2)(1-m) - \tilde x.$$

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4. Numerical results

Account of the interaction of gravity of stars, star attraction jet and reactive power with the translational and rotational motion

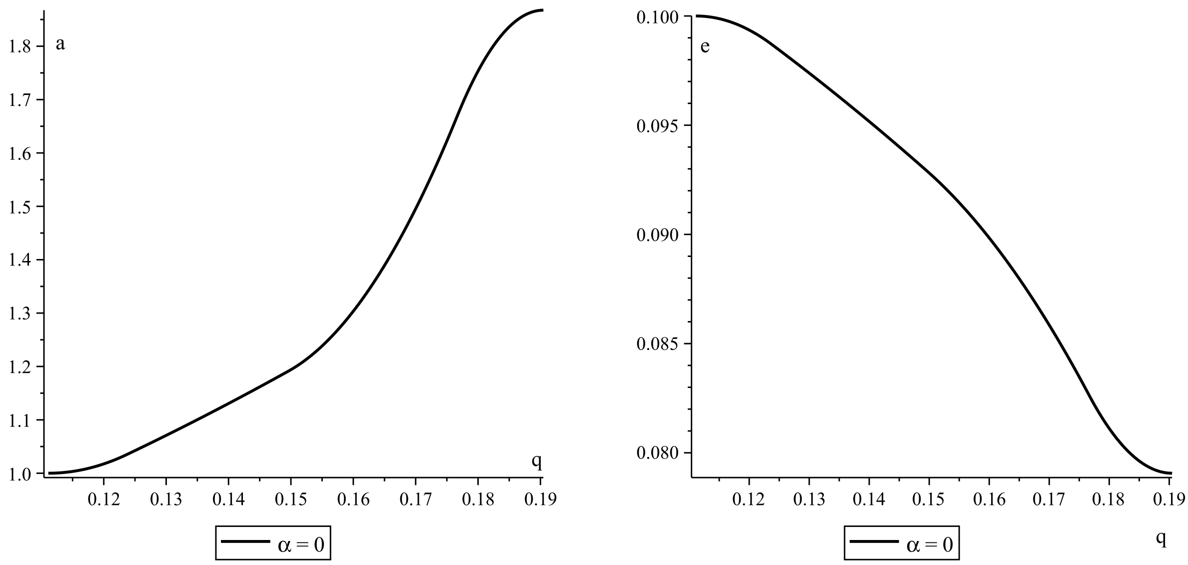


Figure 1. On the left picture is dependence of the semi-major axis of the mass ratio, on the right dependence on the mass ratio of eccentricity

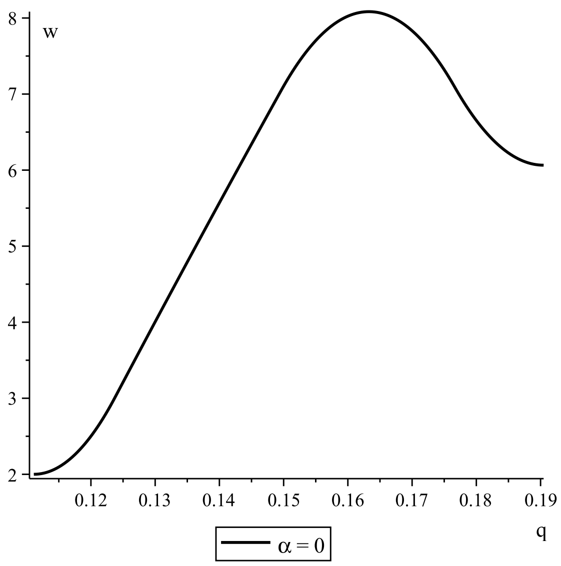
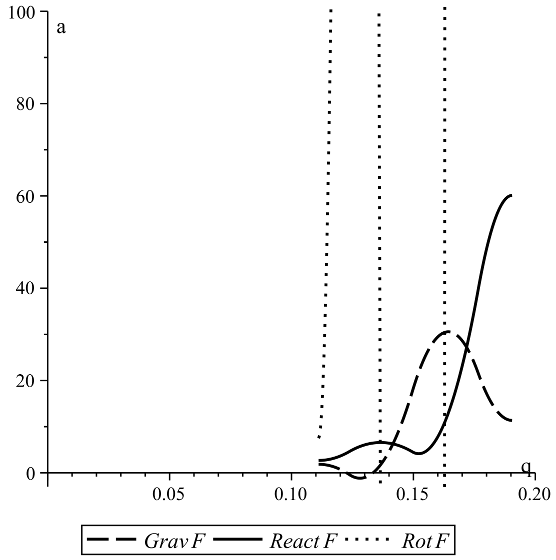


Figure 2. Dependence of the angular velocity of the mass ratio

Here shows the power modules for dependencies.



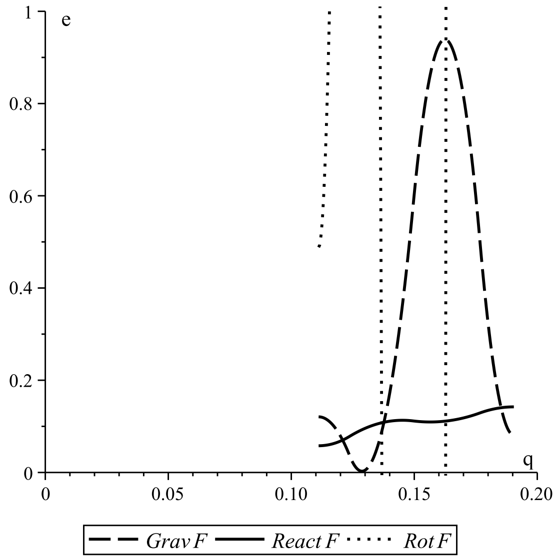


Figure 3. On the left Dependence of the semi-major axis of the mass ratio, on the right dependence on the mass ratio of eccentricity

5. Conclusions

For half a century the incorrect model of Paczynskij-Huang was used to determine the relative orbit of close binary stars. But to this day in the work associated with the close binary systems, the model still has been used. Because we consider this model is wrong, the purpose of this paper was to show fallacy of this model using other methods. To determine the relative motion of stars in a close binary system in this paper we use a numerical integration of the equations of motion with the reactive forces of attraction and stars flowing stream. The calculations of elliptical orbits of close binary stars show that the effect of the reactive force on the evolution of the orbits of stars may be different. There is no doubt that the results can be corrected by introducing other disturbing factors and making new assumptions based on observations.

6. References

[1] Kruszevski A. 1966. *Adv. Astron. and Astrophys.* **4**.P.233.
[2] Hadjidemetriou J. 1969. *Astrophys. and Space Sci.* **3**.P.330.
[3] Piotrovskij S.L. 1967. *Astron. Jorn.* **44**.P.241.