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Magnetic field generation in galactic molecular clouds

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Abstract. We discuss magnetic field generation in weakly ionized turbulent gas. We consider the case of nonzero mean magnetic field. We assume that fluctuations are isotropic, and derive evolution equation for pair correlation function of magnetic fluctuations. Stationary solution of this equation is presented.

1. Introduction

A few years ago Pamela satellite has observed an excess of positrons in cosmic rays at 10-100 GeV energy range. Several theoretical explanations of this effect were presented, among them generation of positrons in pulsars or in dark matter annigalation process. We propose another source of positrons: cosmic rays acceleration in galactic giant molecular clouds and secondary particle production there. This mechanism has been studied earlier in [1],[3]. In these papers an exess of positrons in cosmic rays has been predicted. Appearance of appropriate observations requires a detailed investigation of this mechanism, with taking into account modern data.

Molecular clouds are clusters of molecular hydrogen with a complex inhomogeneous structure. According to observations, gas in them is strongly turbulent. The turbulence has a power-law Kolmogorov-like spectrum. In addition, gas is partially ionized, $N_i/N_n = 10^{-8} - 10^{-5}$. In such a system stochastic magnetic field arises, which can accelerate charged particles. Polarization observations [2], which were carried out for dozens molecular clouds, have shown that magnetic field directions in distant points of cloud may be similar. Probably, mean homogeneous magnetic field exists in clouds along with stochastic field.

In this study we investigate magnetic field generation in weakly ionized turbulent gas with nonzero mean magnetic field.

2. Magnetohydrodinamics eguations for weakly-ionized gas

In our problem for description of gas motion one can use double-fluid hydrodynamic equations. We denote $\mathbf{v}$ and $\mathbf{u}$ velocity of neutral and ionized gas components, ρ_i - dencsity of ionized component and μ_{in} - ion-neutral collision rate. We consider inertial range of scales $L_\nu < L < L_0$, where L_0 is determined by the size of the system, and L_ν corresponds to the viscous scale. Typical values for molecular clouds - $L_0 = 10^{19}$ cm, $Re = 10^8$, hence for the Kolmogorov turbulence $L_\nu = 10^{13}$ cm. In this range of scales viscosity can be neglected. We will also assume gas to be incompressible.

Since the concentration of ions is very small, neutral gas does not feel them. Ions motion, by contrast, is completely determined by the motion of the neutral gas. Thus we can treat the motion of the neutral component known - it coincides with the ordinary hydrodynamic turbulence. For ionized components only two forces are essential - the force of friction on the neutral gas and the Lorentz force. Therefore the equation of motion of the ionized component becomes

$$\frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho_i} - \mu_{in}(\mathbf{u} - \mathbf{v}) = \mathbf{0} \tag{1}$$

Well-known induction equation for magnetic field is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \tag{2}$$

Let us denote

$$a = \frac{1}{4\pi\rho_i\mu_{in}} \tag{3}$$

As far as we assume gas to be incompressible, $a = \mathrm{const}$. Expressing velocity of ionized component and substituting it in the induction equation (2), we obtain

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - a\nabla \times (\mathbf{B} \times (\nabla \times \mathbf{B}) \times \mathbf{B}) \tag{4}$$

This equation gives the dependence of the magnetic field on the velocity $\mathbf{v}$ of neutral gas. See [1] for details of derivation of (4).

3. Correlator's evolution equation

We will use a model, introduced by Kazantsev [4]. To describe the turbulent motion of the neutral gas, we assume its velocity $\mathbf{v}(t)$ to be a Gaussian stochastic process with zero mean $< \mathbf{v} >= 0$. All information about it is contained in the pair correlation function $< v_i(\rho, t)v_j(\rho', t') >$. The angle brackets here and below denote averaging over an ensemble of realizations. We will consider the simplest case of a delta-correlated in time stochastic process

$$< v_i(\rho, t)v_j(\rho', t') >= v_{ij}(\rho - \rho')\tau_c\delta(t - t') \tag{5}$$

Neutral gas in our problem is a homogeneous isotropic medium. We assume it to be incompressible, hence $\operatorname{div}\mathbf{v} = \mathbf{0}$, consequently $\partial_i v_{ij} = \partial_j v_{ij} = 0$, which leads us to the general form of the correlation tensor

$$v_{ij}(r) = 2V(r)\delta_{ij} + rV'(r)(\delta_{ij} - \frac{r_i r_j}{r^2}). \tag{6}$$

The magnetic field we also consider to be a Gaussian stochastic process, but with nonzero mean. Denote its mean and fluctuating components by $\mathbf{H}$ and $\mathbf{b}$ respectively:

$$\mathbf{B} = \mathbf{H} + \mathbf{b}, \qquad < \mathbf{B} >= \mathbf{H} \tag{7}$$

We believe mean component to be constant in space and time $\mathbf{H} = \mathrm{const}(\mathbf{r}, t)$. In the presence of the mean magnetic field appears a preferred space direction, so the correlator of the magnetic field can be anisotropic. But for simplicity we will assume it to be isotropic. This can be done because, as we will see below, in the case of small mean field H the amplitude of the fluctuations $|b| \simeq H^{1/2}$ is greater than H.

Maxwell equation $\operatorname{div}\mathbf{b} = 0$ is similar to the incompressibility equation, so tensor structure of the magnetic field correlator is

$$< b_i(\mathbf{x}, t)b_j(\mathbf{x} + \mathbf{r}, t) >= 2Q(r)\delta_{ij} + rQ'(r)(\delta_{ij} - \frac{r_i r_j}{r^2}) \tag{8}$$

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Using equation (4), one can derive an evolution equation for the magnetic field correlator, that is to compute $\partial Q/\partial t$. To do this, we have to split the arising correlators like $< vb^2 >$ and $< b^4 >$. Since all our random processes are Gaussian, to split the first correlator we use Furutsu-Novikov formula, and to split the second we present it as a product of pair correlators. To obtain an isotropic tensor, we have to replace

$$h_i h_j \Longrightarrow \frac{1}{3}\delta_{ij}$$

where $h_i = H_i/H$ - unit vector in the direction of the mean field. After some tensor algebra we get an equation

$$\frac{1}{2\tau_c}\frac{\partial Q(r)}{\partial t} = \left(V(0) - V(r) + \lambda + \frac{2a}{\tau_c}\frac{H^2}{3}\right)(Q'' + \frac{4Q'}{r}) - V'Q' - \frac{1}{r}(4V' + rV'')(Q + \frac{1}{6}H^2) \quad (9)$$

where we denote an unknown parameter

$$\lambda = \frac{4a}{\tau_c}Q(0) \quad (10)$$

4. Stationary correlation function

Let us reduce our equation to dimensionless one. For distance unit we take the size of the molecular cloud L_0, for unit of magnetic field - the value $\sqrt{\frac{\tau_c}{2a}V(0)}$, a for unit of time the value

$$\frac{L_0^2}{2\tau_c V(0)} = \frac{\tau_{max}^2}{\tau_c}$$

where τ_{max} - eddy turnover time at scale L_0.

Thus, we reduce equation (9) to the form

$$\frac{\partial Q(r)}{\partial t} = \left(V(0) - V(r) + \lambda + \frac{1}{3}H^2\right)(Q'' + \frac{4Q'}{r}) - V'Q' - \frac{1}{r}(4V' + rV'')(Q + \frac{1}{6}H^2) \quad (11)$$

where $\lambda = 2Q(0)$, and $V(0) = 1$.

For typical parameters of molecular clouds: concentration of neutrals $N_n = 10^3\,\mathrm{cm}^{-3}$, and ions $N_i = 10^{-2}\,\mathrm{cm}^{-3}$, $L_0 = 10^{19}$ cm, $L_\nu = 10^{13}$ cm, the unit of measurement of the magnetic field is equal to 5μG.

We choose a function $V(r)$, which describes the motion of the neutral gas, in the form

$$V(r) = \begin{cases} 1 - r^\alpha & \text{when } r < 1, \\ 0 & \text{when } r > 1 \end{cases} \quad (12)$$

Below we consider only the case of $\alpha = 2/3$, which corresponds to Kolmogorov turbulence.

Let us find stationary solutions of (11). We have linear second-order equation for the function $Q(r)$. It has two independent solutions. When $r \to 0$ ($r_\nu \ll r \ll \lambda^{1/\alpha}$) one can find the asymptotic behavior of solutions, and when $r > 1$ one can obtain an explicit solution

$$r \to 0 \;:\quad Q^{(1)} \sim 1; \quad Q^{(2)} \sim r^{-3} \quad (13)$$

$$r > 1 \;:\quad Q^{(1)} = 1; \quad Q^{(2)} = r^{-3} \quad (14)$$

We are interested in a solution, bounded at zero and decreasing at infinity. One can show that such a solution does exist, and it fix the value of the parameter λ (for each value of the

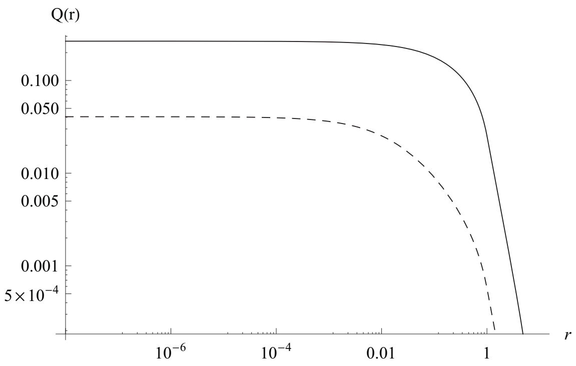

Figure 1. Correlation function $Q(r)$ for $H = 1$ - solid line, and for $H = 0.1$ - dotted line

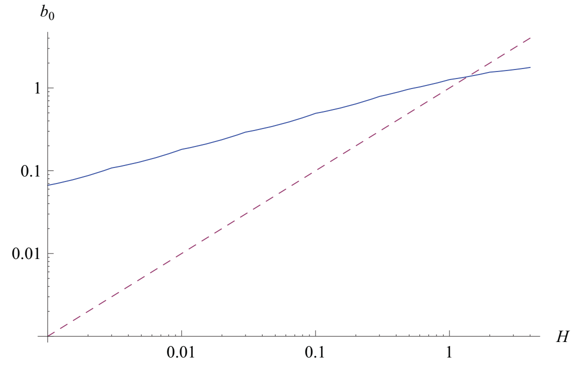

Figure 2. Fluctuations amplitude b_0 against the value of mean field H. For comparison, line $b_0 = H$ is shown dashed

average magnetic field H). Graph of this solution for the values of $H = 1$, $H = 0.1$ is shown in Figure 1. Since

$$< b_i(\mathbf{r}) b_i(\mathbf{r}) >= b_0^2 = 3\lambda \tag{15}$$

where b_0 - the amplitude of the fluctuations of the magnetic field, we obtain the dependence of b_0 on H, shown in Figure 2. In the case $H < 1$ one can see that $b_0 \sim \sqrt{H}$. Therefore, when $H \ll 1$, we obtain $b_0 \gg H$ - random field exceeds the mean field.

It should be noted that when $H \to 0$ the value of the random field b_0 decreases to zero, and in the limit $H = 0$ our solution becomes identically zero. The fact is that if $H = 0$, there are no (non-zero) solutions, bounded at $r \to 0$ and decreasing at infinity.

5. Discussion

In the papers [5], [6] similar problem of a turbulent dynamo in a conducting fluid is studied for the case $\nu \ll \eta$, where ν, η - kinematic and magnetic viscosity, respectively. The role of our unknown parameter λ plays η, which is considered to be known.

In [5] authors try to take into account the presence of the mean magnetic field, but they derive evolution equation for correlators only for zero mean field. Therefore, in paper mentioned, our general equation (11) was solved only in special case $H = 0$. In this case, for the Kolmogorov turbulence $V(r) = 1 - r^\alpha$ with $\alpha = 2/3$ there are no solutions, growing with time, see [5] (and there are no limited solutions of the stationary equation). To avoid this problem, Rogachevskii et al and Malyshkin et al use rather artificial method - they suppose the correlation time τ_c to be a non-constant function of scale. In this approach, Kolmogorov turbulence corresponds to $\alpha = 4/3$ and growing solutions appear (and for stationary equation - limited solution appears). In this work we have demonstrated that when mean feild is taken into account, limited solution can exist even at $\alpha = 2/3$.

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