

Are fluctuating measurement results *instable*, *drifting* or *non-stationary*? – A survey on related terms in Metrology.

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Abstract. We describe properties of measurement quantities and measurement processes verbally by using accepted terms, some of which are sanctioned by international standards and guides. However, some of these terms are used inconsistently, as compared to definitions in base sciences like for example Mathematics, Signal and System Theory, Estimation and Optimisation Theory, Stochastics and Statistics. This is the case for different terms in Metrology, like *stability*, *drift* and *stationarity*. In this paper we show systematic relations between these and similar terms and discuss the adequacy of their application in practice. We have to admit that we incorrectly implement the terms *stable* / *instable* unconsciously, when we describe properties of measurement results, due to the use of common colloquial habits.

1. Introduction

Measurement instrumentation and measurement results fulfil defined requirements. We certify them by qualitative verbal descriptions and / or by quantitative logical and mathematical relationships, according to accepted rules.

For example, we say in colloquial language that certain situations and procedures are *stable* or *instable*. Sometimes data are reported to be *drifting*. And many instances appear *nonstationary*. Are such descriptions identical, synonymous, similar or related? Of course, they may serve as hints for the time being. But what shall we do, if we have to deliver quantitative information? We suppose that there must be different meanings behind these terms, which are all used regularly in practice.

Apparently, there must be a concept, which is able to deliver coherent tools and which is applicable to all fields of science and supports top-down considerations. Indeed, there are facilities, provided by Signal and System Theory and by Stochastics and Statistics [1; 2; 3].

In the following sections we will focus on the classification and definition of the terms of interest mentioned above. We will see that some of them refer to *signals* (models of real-world physical or other *quantities*) and some of them refer to *systems* (models of real-world physical or other *processes*) [9]. This means that all terms are well defined properties of signals or systems, which we may describe mathematically, if necessary.

Interestingly enough, no or just a few helpful definitions concerning our field of consideration can be found in the basic Guides of Metrology like the GUM [4] and the VIM [5]. Even the Standards of Statistics refrain from offering significant and precise support, when dealing with these terms [6, 8].

The following investigation will justify, which term has to be applied in which situation. In particular, it will become obvious that the terms *stable* and *unstable* are often used incorrectly for quantitative descriptions (models).



2. Signals and Systems

Terms like stability, drift and stationarity seem to describe properties of signals and of systems as well. However, properties of systems show up in their behaviour first and do so in their output signals afterwards. So, we have to distinguish carefully between definitions for signals and definitions for systems. This will be done in the following sections.

Properties of Signals and Systems

Quantitatively we describe *properties* of a signal by *structures* of equations on the one hand and by the vector of *parameters* of these equations on the other hand. The same is true in the first instance for systems: Quantitatively we describe *properties* of a process by *structures* of equations on the one hand and by the vector of *parameters* of these equations on the other hand. But, for systems the equations have a special meaning: They describe the relations between cause and effect, between input and output, relations, which we will call *behaviour* for defined circumstances.

Behaviour of Systems

Generally speaking, every signal comes from some source, which we call system. Signals depend on systems; they are manifestations of systems. May we then state that signals don't have their own behaviour and that we can only describe their momentary properties?

On the other side, this is not true for systems; these reveal behaviour indeed. The overall-behaviour links internal signals $\mathbf{x}(t)$ and output signals $\mathbf{y}(t)$ as responses to arbitrary excitations (stimulation, impact, action, effect, drive, load, question, disturbance) by the input signals $\mathbf{u}(t)$ (Figure 1).

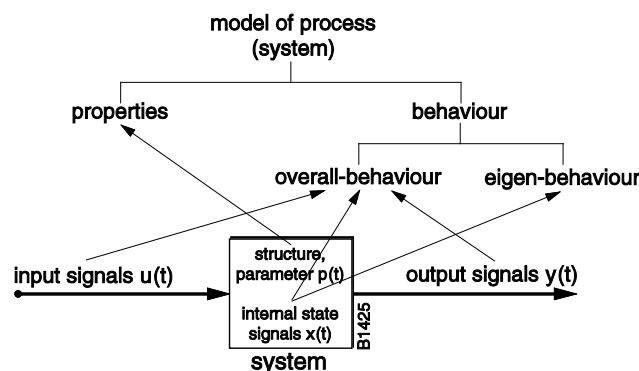


Figure 1. Signals, properties, parameters and behaviour concerning a system

Exactly, this is the immediate interpretation of the mathematical solution of differential equations, which describe systems. The *eigen-solutions*, according to the values of the initial conditions $\mathbf{x}(0)$, together with the *forced solutions*, according to the input excitations $\mathbf{u}(t)$ deliver the *total solution* in form of the output signals $\mathbf{y}(t)$. In this context *solution means behaviour*. Or: Behaviour describes, how a system reacts according to *external* (input) signals and to *internal* (initial) signals. This type of description is called *system transfer response description* [2]. We will use it in order to discuss the term *stability*.

Up to now we have assumed constant parameters $\mathbf{p}$ within the system. However, according to the cause and effect principle we additionally have to consider a second description of a system as soon as the parameters $\mathbf{p}$ vary over time and become independent variables. Then we describe the relations between process parameters $\mathbf{p}(t)$ and output quantities $\mathbf{y}(t)$. This type of description is called *system sensitivity description* [7]. We will use it in order to discuss the term *drift*.

3. The Term Stability

We frequently say that a procedure is unstable if a signal drifts away. Does this mean that *instability* and *drift* are synonymous terms? Not at all! The causes or the mechanisms of instability and drift in

systems are completely different (see section 5). There are *instable systems* and there are *drifting systems*. A system can be stable and drifts nevertheless. Actually, this is even the normal situation.

Output signals, emerging from instable systems, cannot be annotated as instable too. Instead, they are qualified as varying, fluctuating, oscillating, limit cycling, drifting, shifting, and as transient, non-constant, nonstationary, unsteady, unbounded, and so on. Again: Those types of signals may come from stable systems alike, due to dedicated input signals, which are able to generate these types of output signals just as well. Quantitatively, this is true for corresponding mathematical descriptions of output signals of course. There exists no definition or mathematical formalism for an "instable signal"!

Therefore we concentrate on *systems* and define *stability of a system* qualitatively as its ability (behaviour) to keep its output signals *bounded*, provided that all input signals are bounded too [2].

Definitions

Qualitative: Once a system is instable, at least one output signal will run away unbounded

- even if all *input signals* remain bounded
- and
- even if all *system parameters* remain bounded.

Quantitative: The definition of stability is simple for linear dynamic systems, but cumbersome for nonlinear systems. System Theory normally describes a linear time-independent dynamic system (LTI) by a set of ordinary, linear differential equations (ODE) with constant coefficients. When is it stable?

The stability of a linear system depends on both system properties, on *structures* and *parameters*:

1. System (model) *structure*

There are systems, which cannot get instable for structural reasons:

- linear nondynamic systems
- linear dynamic systems of less than third order

All other systems *may* become instable.

2. System (model) *parameters*

In addition, the stability of a linear dynamic system of higher than second order depends on the *magnitudes* of the parameters, which are elements of the so called characteristic equations of the system:

- In the *time domain*: A linear dynamic system is asymptotically stable, if all poles (eigenvalues) of the set of *characteristic equations* of the *differential equations* have negative real parts and are therefore located in the left half of the complex plane.
- In the *frequency domain*: A linear dynamic system is asymptotically stable if all poles (eigenvalues) of the *denominator polynomial* of the *spectral transfer response function* $g(s)$ have negative real parts and are therefore located in the left half of the complex plane.

There are more stability criteria seen from different points of views. The simplest are the two mentioned above. Mathematically they are equal and the statements are identical. Considering the magnitudes of the parameters, stability criteria deliver stability conditions, stability margins, stability bounds and stability areas, useful for system synthesis and system engineering.

Instability is a binary property (stable / instable) and is based on system properties and not on signal properties, even though it is noticed by apparent output trajectories. Therefore, we also have to avoid the frequently used term *long-term stability*, which concerns *drift* and not stability issues (see section 5).

Besides: In Metrology we seldom encounter instable systems, in particular there are no instable sensors. However, we sometimes have instable processes with implemented stable sensors. Very seldom observer, filter and adaptation structures may raise stability questions.

Note: An *instable* dynamic system does not behave *chaotic*, since its behaviour is completely deterministic and can be predicted, provided the model of the system is known. Chaotic systems follow different rules.

4. The Term Stationarity

The term *stationarity* refers to signals (time series data) and parameters, but not to systems. There are several classes of stationarity. We only mention *weak stationarity*, which shows the main characteristics of interest already. Signals are (weakly) stationary concerning time and space, if characteristic *values* (like mean values, standard deviations values, covariance values and so on) and characteristic *functions* (like probability density functions, correlation functions, spectral power density functions and so on) are independent concerning time and space, or frequency and wavelength.

The simplest example refers to a stationary signal or parameter $x(t)$, for which the following equation (description) holds [10]:

$$\mathcal{AVG}\{x(t)\} = \mu_x = \lim_{t_{\text{obs}} \rightarrow \infty} \frac{1}{\Delta t_{\text{obs}}} \int_{t=-t_{\text{obs}}}^{+t_{\text{obs}}} x(t) dt = \mathcal{AVG}\{x(t) + \Delta\tau\}$$

with

$\mathcal{AVG}\{\dots\}$ averaging (expectation) operator

μ_x arithmetic mean value

t_{obs} observation time value

$\Delta\tau$ arbitrary time shift intervall

This means that for a stationary signal or parameter the arithmetic mean value is independent of time t . However, this seemingly simple definition asks for an infinite range of the independent variable, here of time t , which is never available. So, we normally define *short-time stationarity* within finite observation (averaging) intervals (windows) Δt_{obs} (Figure 2).

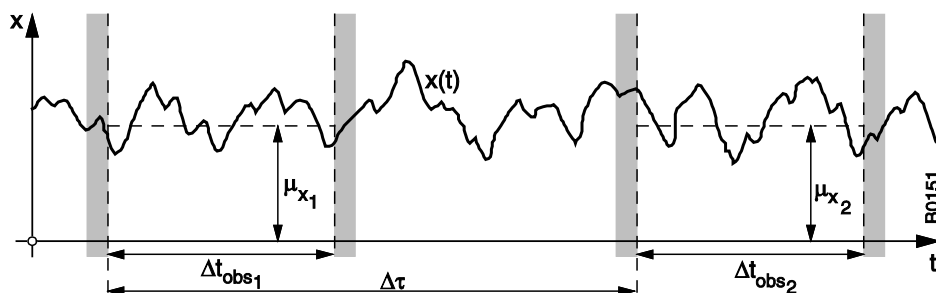


Figure 2. Definition of short-time stationarity

Note: Normally, stationarity will be superposed by other properties of the signal, drift for example. Sometimes, a separation of different signal content can be achieved by dedicated filtering.

Remark: The following source from the National Institute of Standards and Technology (NIST) serves as an example how delicate the use of common language terms can be, if it is not based on scientifically agreed conventions: "A manufacturing process cannot be released to production until it has been proven to be stable. Also, we cannot begin to talk about process capability until we have demonstrated stability in our process. A process is said to be stable when all of the response parameters that we use to measure the process have both constant means and constant variances over time, and also have a constant distribution." [8]. Here stationarity is meant and not stability.

5. The Term Drift

A rough characterization in technical and natural sciences concerning *drift* means that an observed signal changes on average in *time* versus a certain direction. When measuring or collecting data from a process, we may occasionally say: "Our data drift away". Besides, there is the related term *shift* too. It refers to a signal, which changes on average in *space* into a certain direction.

Features

1. Within a dynamic system in steady state condition ($\mathbf{u}(t) = \text{constant}$ or stationary) drift appears as a temporal deviation of at least one system parameter from the nominal value in a particular direction.
2. Drift is perceived as a slow movement of behaviour compared to all other temporal patterns in the dynamic system.
3. Regular input-output results of a system and its drift features superpose. Several drift features may interfere with each other.

Definitions

1. Drift, perceived in signals (data), is the special result of *temporal parameter variations* $\mathbf{p}(t)$ of a system and excludes drift as a result of varying input quantities $\mathbf{u}(t)$ of this system.
2. Drifting deviations of system properties or behaviour from explicitly stated nominal properties or behaviour are erroneous properties or behaviour. They give rise to *systematic drift errors* $\mathbf{e}_{\text{drift}}(t)$, accompanied by *drift uncertainties* $\mathbf{u}_{\text{drift}}(t)$ (Figure 3).

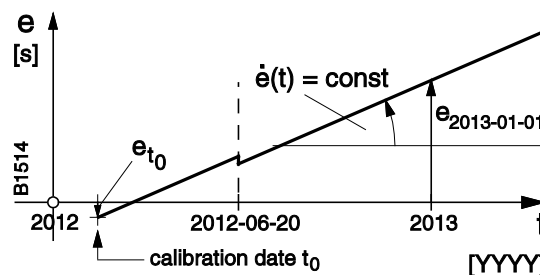


Figure 3. Example of the systematic drift error e of an output signal of a drifting watch.

Since we assume constant or stationary input signals $\mathbf{u}$ for the consideration of drift, we concentrate on the *system sensitivity description* $\mathbf{p}(t) \rightarrow \mathbf{y}(t)$, the second type of system description mentioned above [7].

At least one parameter, the drift parameter $\mathbf{p}_{\text{drift}}(t)$, changes in one direction. The parameter deviation (error) $\Delta \mathbf{p}_{\text{drift}}(t)$ makes the dynamic system a *time variant dynamic system*. We usually assume that the parameter drift proceeds linearly with time t : $\Delta \mathbf{p}_{\text{drift}}(t) = \mathbf{v}_{\text{p}_{\text{drift}}} t$ [$\{\mathbf{p}\}$]. The constant (average) drift velocity (drift rate) of the parameter $\mathbf{p}$ is $\mathbf{v}_{\text{p}_{\text{drift}}} = \dot{\mathbf{p}}_{\text{drift}}$ [$\{\mathbf{p}\} \text{s}^{-1}$]. It may change with time t too. This is a description of system properties.

Thus output signals will show effects due to these drift properties. The simplest case is the *constant output signal* $\mathbf{y}$, which drifts in time t . Thus, the signal is not constant any more. The nominal temporal trajectory $\mathbf{y}_{\text{nom}}$ is superposed by a temporal component $\Delta \mathbf{y}_{\text{drift}}(t)$, which systematically tends in one direction.

$$\mathbf{y}(t) = \mathbf{y}_{\text{nom}} + \Delta \mathbf{y}_{\text{drift}}(t) \quad [\{\mathbf{y}\}]$$

For a typical *stationary random output signal* $\mathbf{y}(t)$ the nominal arithmetic mean value drifts [10]. Thus the random signal is *not stationary* any more.

$$\mu_{\mathbf{y}}(t) = \mu_{\mathbf{y}_{\text{nom}}} + \Delta \mu_{\text{drift}}(t) \quad [\{\mathbf{y}\}]$$

Note that the drift signal $\Delta \mathbf{y}_{\text{drift}}(t)$ can be a random signal with a drifting mean value too.

So we always distinguish between the nominal trajectory (signal) $\mathbf{y}_{\text{nom}}(t)$ and the deviating trajectory (signal) drift $\Delta \mathbf{y}_{\text{drift}}(t)$.

These remarks are true for nondynamic and dynamic systems. Questions about the effects of system parameter errors $e_p(t)$ on the output signals $y(t)$ or on the output signal errors $e_y(t)$ have to be answered by system sensitivity analysis.

Drifting systems are regularly stated as *instable*. This is not correct in the strict sense of Signal and System Theory. The International Vocabulary of Metrology (VIM) is misleading too [5] with its definition 4.19: "stability of a measuring instrument: property of a measuring instrument, whereby its metrological properties remain constant in time" with a further quantification of this term: "In terms of the change of a property over a stated time interval." This would be the appropriate definition of a *time invariant system* and not of a *stable system* on the one hand and the suitable specification of a *drifting output signal* on the other hand according to system parameter drift.

6. Conclusion

- The term *instable* characterises a system and not a signal (data). Signals of instable systems move *boundlessly*. Instable systems are rare; they are not useful considering practical applications. On the other side, they can be stabilised by control tools.
- Signals are constant and stationary on the one side and drifting, shifting, varying, fluctuating, oscillating, limit cycling, transient as well as nonconstant, nonstationary, unsteady, unbounded, and so on the other side.
- Constant parameters make a system *time invariant*; varying parameters make it *time variant*.
- The term drift characterises a special parameter varying system, whereby drifting signals arise.

The terms discussed above are standard terms of Signal and System Theory and Stochastics and Statistics. However, their treatment in everyday use and the statements concerned in International Guides and Standards of other fields are more or less inconsistent.

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