

Problems in implementing some of the new measurement unit definitions of the SI

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Abstract. The paper illustrates and discusses some problems that should be taken into account, should the proposed use of fundamental constants in the definition of measurement units of the SI be implemented: (a) more base units being multi-dimensional, instead of fixing the present problems in this respect; (b) the multidimensionality in the definitions; (c) the use of CODATA adjusted values of the constants for this specific purpose; (d) formal issues in stipulating algebraic expressions of the definitions, and in respect to the rounding or truncation of the numerical values in their transformation from uncertain to exact values; (e) formal issues with the use of the integer number N_A ; (f) limitations that can arise from the stipulation of the values of several constants for the CODATA Task Group to continue performing in future meaningful least squares adjustments of the fundamental constants taking into account future data.

1. Introduction

The use of some ‘fundamental constants’ (c_0, h, e, k_B, N_A) with stipulated numerical values has been proposed for the re-definition of some of the present *base* units of the International System of Units, the SI [1]. This proposal was submitted at its 2011 meeting to the CGPM, which decided in its Resolution 1 to “*take note of the intention of the CIPM*” for “*the possible future revision of the International System of Units*” [2].

Problems can be found in the literature over the past five years concerning the proposal on the floor related to four main areas: (a) how meaningful a progress can be achieved by using fundamental constants in the definition of the measurement units; (b) which should the constants used and how the definitions should use them; (c) how the definitions should precisely be formulated; (d) how to deal with future data.

In this paper, some of the issues arising from the above problems, which should be taken into account in the implementation of the proposal, are discussed:

- (1) having more base units multi-dimensional instead of fixing the present problems in this respect;
- (2) how to take into account the multi-dimensionality in the definitions;
- (3) using or not using CODATA ‘adjusted values’ of the constants for this specific purpose, being the main purpose of the adjustments rather to obtain the best measure of the *consistency* of a much wider set of constants;
- (4) formal issues in stipulating algebraic expressions in the definitions, and in respect to the rounding or truncation of the numerical values in their transformation from uncertain to exact values;
- (5) formal issues when using the integer number N_A ;



(6) handling limitations that can arise from the stipulation of the values of several constants, namely for the CODATA Task Group to continue performing in future meaningful least squares adjustments of the fundamental constants, and taking into account future data.

It is *not* the aim of this paper to discuss the authority of the CODATA Task Group in this respect, nor the general validity of the Least Squares Adjustment (LSA) method [3].

2. Problems of principle

The aim of the International System of Units (SI) is to provide a system of units, one for each of a specified (and limited) list of quantities necessary to express measured properties in nature.

The above items (1) to (3) fall under this heading.

2.1. Meaning of ‘base unit’ of the SI

A long-lasting debate led to seven quantities that are called fundamental and whose units are called “base” [4]. Each of these units was originally aimed at being defined without having to resort to the unit of any other quantity. They are assumed to be dimensionally *independent* [5]. All the remaining are conversely called “derived units”, since one has to express each of them as an algebraic expression of the seven base units.

There are some properties of unit systems that are desirable, and there are some that are essential. A property that is not essential but is desirable is that the system of units is *coherency*. For detail on this see [6].

A property that is mandatory for any system of units, and consequently often assumed achieved also by the SI, is internal *consistency*. For base units defined independently from each other this is necessarily the case.

However, this principle is *presently not* implemented for several base units of the SI (see section 2.2). In this situation, consistency cannot be assumed *a priori*. For non-independent definitions, formal or logical consistency, in the sense of a syntactic property relating unit definitions, can be quite problematic to prove. There is another, more specifically metrological, meaning of the term consistency that is often used. It is defined in [7] as a kind of “metrological compatibility”, a property of a set of experimental determinations affected by uncertainty. For details see [14].

With the new proposed definitions, the above property would become greatly complicated to prove. In fact, the constants c_0, h, e, k_B, N_A (and $\nu(^{133}\text{Cs})$, not a “fundamental constant”) are *not* each the direct expression of a base SI unit: one has to express the relevant units as algebraic expressions of the constants (where the use of the stipulated values for the constants,¹ indicated with an asterisk, is mandatory—for comments on their numerical values see later):

$$\begin{aligned} [c_0/\nu(^{133}\text{Cs})] &= [\text{metre}], c_0^*/\nu(^{133}\text{Cs})^* = 3.261\,225\,561\,106\,45\dots \cdot 10^{-2} \text{ m} \\ [h\nu(^{133}\text{Cs})/c_0^2] &= [\text{kilogram}], h^*\nu(^{133}\text{Cs})^*/c_0^{*2} = 6.777\,265\,181\,006\,83\dots \cdot 10^{-41} \text{ kg} \\ [h\nu(^{133}\text{Cs})/k_B] &= [\text{kelvin}], h^*\nu(^{133}\text{Cs})^*/k_B^* = 4.411\,764\,153\,914\,89\dots \cdot 10^{-1} \text{ K} \\ [e\nu(^{133}\text{Cs})] &= [\text{ampere}], e^*\nu(^{133}\text{Cs})^* = 1.472\,821\,956\,237\,59\dots \cdot 10^{-9} \text{ A} \\ [1/N_A] &= [\text{mole}], 1/N_A^* = 1.660\,539\,000\,996\,49\dots \cdot 10^{-24} \text{ mol} \end{aligned} \quad (1)$$

2.2. Definition of a base unit making use of other base or derived units

As indicated before, already at present some base units are not dimensionally independent: the unit of length involves the second; the unit of amount of substance involves the kilogram; the unit of electric current involves the newton and the metre; the unit of luminous intensity involves the hertz, the watt and the steradian; the unit of amount of substance involves the kilogram. As shown in the expression (1) this fact is now aggravated for the proposed new units. In principle, the text of each definition should also make explicit reference to all other units used. For an alternate possibility for fixing this issue see [6].

¹ The present numerical values originate from the set of measurement units that are presently defined—and realised.

2.3. Use of CODATA adjusted values of fundamental constants in the definition of a unit

Since 1969 a CODATA Task Group [8] performs a valuable check of consistency of a large set of ‘fundamental constants’ by means of the so-called Least Squares Adjustment (LSA) algorithm. At regular intervals, these evaluations produce a set of “adjusted values” and of the corresponding associated uncertainties (see in [8] for the full list of references). The aim of the CODATA Task Group, as expressed in their reports and publications is twofold: (i) “*a least squares adjustment (LSA) is one of the few ways in which the over-all consistency of physical theory can be systematically investigated. Moreover, it provides a consistent set of constants at a particular epoch which can be used by all workers requiring them*” [10]; (ii) to get “*particular numerical values obtained for a set of ‘best’ or ‘recommended’ set of constants ...*” [9].

For aim (ii), the sentence reported above from [9] ends by saying that this feature “... *is only of secondary importance*”, because the main importance is the “*information gained during the course of the critical review which necessarily accompanies [precedes] the adjustments*” [9]. Actually, extensive warnings in respect to the use of the LSA method form the whole Section C in [10] (see [8]). Another critical feature requiring attention is that pointed out in a “Warning!” in [9]: “*Because of the intimate relationship which exist among least-squares adjusted values of the fundamental constants, a significant shift in the numerical value of one will generally cause significant shifts in others. Consequently, for any critical application of these numbers, the user is urged to refer to the original article[s] as well as to the current literature ...*”. Their use in the definition of measurement units is certainly a very critical application.

It is a fact that consistency is a most important requirement for a set of constants. However, its check is performed with tools suitable for that purpose, like the LSA, which are *different* from the tools used to obtain the numerical values of the chosen summary statistics directly evaluated from the—critically-selected—experimental data available for each constant. Thus, confounding the results of the LSA with the several choices possible for the latter (e.g., mean, weighted mean, median, ...) should be considered incorrect because:

- (a) The value of (at least) one member of the set—at arbitrary choice—is to be set fixed in the LSA, so that all the adjusted numerical values are relative to this choice (*relatively-adjusted values*)—while any pair difference is invariant irrespective of the choice of the fixed member;
- (b) The original values of the adjustable member(s) of the set depend on the chosen *inter*-subjective criteria, and are *altered* according to the LSA optimisation algorithm. The latter operation sets an “intimate relationship” [9] between *all* members of the set, which may have no physical meaning for some of them in the specific case of the fundamental constants. The issue here is not a possible resulting ‘discrepancy’ between data in the set, but the fact that the obtained values only optimise the *consistency*, i.e. primarily the value of the statistical parameter used for evaluating the *uncertainty* of the set. For this there is a cost for the values: divorcing by a certain amount from the original physics world. This cost is generally irrelevant in other applications of the CODATA analysis. Also, it is not a characteristic of the LSA only, but of all methods having the same purpose, e.g. the methods with fixed effects. However, in the specific case of the measurement units, the CIPM may want (or need) to take advantage of the best accuracy allowed by the current experimental data, and this is a critical condition making incorrect the use of the CODATA relatively-adjusted values, and that may even result in missing the aimed goal [15]: see next section.
- (c) The above difficulties are generally also involving the *adjusted uncertainties* typically lower than the experimental ones, associated with the relatively-adjusted values: it may happen that the uncertainty aimed level is satisfied by some adjusted constants, but not yet achieved in actual experimental determinations.

3. Formal problems

The items (4) to (5) in the Introduction fall under this heading.

3.1. Stipulation of numerical values, namely using algebraic expressions of them

The current proposal requires stipulating the numerical values of c_0 , h , e , k_B and N_A (presently only c_0 is). In the general case of stipulation, in principle one could decide to use any arbitrary numerical value. However, this would lead to metrological undesirable consequences. Should a discontinuity in the size of the new units be avoided, as is considered to be preferable in metrology, the stipulated value cannot be chosen anymore arbitrarily. It should coincide with the ‘best’ value in a statistical sense of the set of experimental data available at the time of the stipulation of that constant, not being the CODATA one.

Another critical issue arising from the suppression of the explicit information on uncertainty in stipulated numerical values is the rounding or truncating of the stipulated numbers. For an uncertain number, this operation follows precise rules: rounding is performed according to the uncertainty level and limited to two uncertain digits (e.g., in the CODATA values), sometimes to one. However, since after stipulation the uncertainty information is lost in the definition, the fact is questionable that the first uncertain digit could be retained in stipulation, a number exact by definition. It seems possible to consistently use only the digits unaffected by uncertainty.

Another issue deserving attention in stipulation is the (frequent) case where a constant is expressed (indirectly) as an algebraic combination of other constants. The algebraic expressions of stipulated values, for example $R = k_B \cdot N_A$ or $F = N_A e$ or $K_J = 2e/h$ or $R_K = h/e^2$, or the expressions (1) above, are often assumed to have in turn stipulated exact numerical values. This opinion is not a direct consequence of the concept of stipulation. In the expressions (1), the numerical values are shown limited to the 15 digits allowed by the use of MS Excel for computation. It obviously happens that the resulting number is sometimes rational, in general irrational, with no obvious rounding or truncation rules applicable. That is a general consequence of algebraic combinations.

The stipulation of the constants forming the algebraic expression does not automatically generate a stipulated numerical value in the definition of the units: a *distinct decision* should be taken in the latter respect, and a *separate specific* stipulation (number rounding or truncation). In particular, the use of the inverse of a constant in the new definitions places the same problem. For more details see [11].

3.2. Representation, use and stipulation of incompletely-known integer numbers

An example of an intrinsic integer number in measurement is the result of a count. An example of a count is the Avogadro number, whose known value is usually expressed as $6.022\,141\,29 \cdot 10^{23}$.

Is it correct? This representation does not convey at all the information that the number is an integer, of which, contrarily to usual cases, only some of the *most significant digits are known*. For more details see [6, 11].

4. Problems in perspective for science and metrology

The item (6) falls under this heading.

4.1. Future possibility of considering new determinations of the fundamental constants, and of the use of the Least Squares Analysis by CODATA

A question arises: is it useful (or correct, depending on the viewpoints) that CODATA analysis and outcomes always incorporate constraints arising from a basically regulatory field like metrology, for decisions concerning the measurement units? It is true that the numerical values of the constants completely depend on the size of the units on which the measurements are based. However, may general science, which is basically interested on consistency of the physical theories, be tied to LSA evaluations of consistency that are actually *biased* by metrological issues?

If the CGPM will eventually adopt the CIPM proposal, more constants will have their numerical values stipulated. Whence, many more will also get fixed numerical values because of the inter-relationship between constants placed by the LSA method, considerably reducing the number of the freely-adjustable ones according to the present philosophy. The number of the remaining constants might be so small that the whole CODATA task could become irrelevant or even terminated.

This situation would also place delicate question marks on the handling of future experimental determinations of the constants, namely of the stipulated ones. The traceability checks in the case of constant-based inter-related unit definitions are likely, at least, to be quite more indirect and complex.

Further, how will new determinations be taken into account in future? Possibly, it will not be in the interest of metrology to adjourn the stipulated value of a specific constant. On the other hand, it would certainly be advantageous for general science to have this new information taken in due account, even by the CODATA Task Group, according to the normal scientific method.

5. Possible ways out

Summarising, in this paper issue (1) (section 2.1) concerned point (a) of the Introduction; point (b) was not treated in this paper; the issues (2) to (5) (sections 2.2, 2.3, 3.1 and 3.2) concerned point (c); point (d) was treated in issue (6) (section 4.1).

Concerning point (a), there is an intrinsic contradiction between the SI principle of using ‘base’ quantities as distinct from ‘derived’ quantities and using fundamental constants in the base unit definitions, except possibly for the mole—apart from formal problems arising from the use of the inverse of N_A . In fact, there is no possible remedy when relaxing the basic principle of (dimensional) independence of the SI base units, like it would happen by using *multi-dimensional* constants in the definitions, unless the set of base quantities is changed—e.g., by replacing length with velocity. A fact already for some of the base units in the present set will be aggravated by the new proposal.

Furthermore, the new definitions will make necessary to have *practical* realisations of these units, called “mise en pratique”: the traceability chain will therefore start from the relationship between the unit definition and its *mise en pratique*. In all instances, the new proposed units will cause the metrological need—note, not simply the scientific will—for future measurements of those constants, in order to provide facts supporting evidence to the traceability chain. National traceability might only demand, within each NMI, (i) to realise an apparatus for the measurement of a constant, (ii) to assign the outcome of the measurements the stipulated value, and, (iii) to establish a relationship between these results and the *mise en pratique* of the relevant unit(s) of the NMI. In the case of constant-based multi-dimensional unit definitions, these checks of the resulting degree of internal traceability are likely to be quite more indirect and complex—and costly—than the present ones. For International *inter-pares* traceability, one *additionally* needs to resort, as usual, to between-laboratories information, the *inter-comparisons*, where the actual differences between realisations will show up [12, 13].

As to point (c), the *values* of the constants, the CODATA *relatively-adjusted* values, computed to test the consistency of the data set, *should not be conceptually confused* with the values obtained, directly or through computations, from the experimental determination(s) of the constants’ values, nor with the statistically-evaluated representative values of them—like the mean, weighted mean, As to the *uncertainty* associated with the value of a constant, the LSA method reduces, sometimes dramatically, the standard uncertainties associated with the adjusted value. This is a direct benefit of the method arising from statistical features when a larger set of values is involved, even without new direct determinations and beyond current experimental capability. The *uncertainty* levels associated with the adjusted values of the constants *should not be conceptually confused* with the uncertainties actually achieved experimentally.

The way out is to use the values and related uncertainties determined, at the time of the stipulation and for each individual constant, only by selection criteria of the ‘best’ value and uncertainty, in a strict statistical sense, of the set of available experimental data. Most statistical methods look appropriate—except fixed-effect methods, for the same reason why the LSA is not.

Concerning the expression of the above numerical values in the definitions of measurement units, some issues need attention in the step bringing from the initial uncertain to the stipulated values. The latter being exact by definition, they, with proper rounding, should not involve uncertain digits of the initial numbers, possibly except when the uncertainty affects the last digit by only $\pm \approx 1$ like in the case of c_0 . Algebraic expressions of stipulated values should be avoided in the definitions, including the inverse of a constant. In fact, a definition should be in itself comprehensive and self-consistent, and is

not supposed to allow arbitrary interpretations by the users. The allowed number of digits of the result of the numerical operations should appear within the definition: it can be set, before stipulation, only from the original uncertain number, information not available to all users. This is to say that only the result of the operations should appear in the definition as a numerical value stipulated, and with the correct number of digits.

Concerning the effects of the new definitions on the future CODATA task and on handling future experimental determinations of the stipulated constants, point (d), a serious concern is expressed for a possible divorcing of the metrological regulatory procedures from the outcomes of interest—and needs—for ‘general science’. In order to ensure the future possibility to take into account the fact that new experimental determinations might possibly lead to different ‘best’ values according to normal scientific practice, a possibility would be that the CODATA Task Group modifies its procedure. In this perspective, the analysed set of constants should concern *exclusively* the check for their best *consistency*, limiting to one, μ_0 , the non-adjusted constant. The adjustments should be recalculated back from 1983 with c_0 and ε_0 adjustable.

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