

# Consideration of the Measurement Uncertainty in the Assembling of Components Characterized by High Dimensional Variation

**C Hernández, R Tutsch**

Institut für Produktionsmesstechnik, Technische Universität Braunschweig,  
Schleinitzstrasse 20, D-38106 Braunschweig, Germany.

E-mail: carlos.hernandez@tu-bs.de

**Abstract.** The Statistical Dynamic Specifications Method (SDSM) relies on in-line measurements to manage dimensional specifications, target and tolerance. SDSM is the cornerstone of an innovative assembling technique based on the Statistical Feed-Forward Control Model (SFFCM) for processes in which the stacked dimensional variation of the assembled components reaches the order of the total allowed tolerance. Since the magnitude of such variation might jeopardize the process capability, it is a matter of interest to study the inclusion of the measurement uncertainty when applying SDSM on a target subprocess. By means of simulating the production of assemblies made of parts having high dimensional variation, a set of experiments were designed to compare the impact of different levels of measurement uncertainty on the capability of a target subprocess. Simulation results showed that depending on the magnitude of the uncertainty the capability index  $c_p$  of the target subprocess increases between 2.5% and 34.5% from 1.27 to 1.82 as a direct consequence of adjusting the respective tolerance. Thus, the inclusion of the measurement uncertainty in the proposed SDSM has a significant impact in its practical realization since a decrement in  $c_p$  implies an increment of the scrap percentage of the target subprocess.

## 1. Introduction

The Statistical Feed-Forward Control Model (SFFCM) concentrates the efforts on managing the specifications and tolerances of the inner components of a given assembly by means of applying iteratively the Statistical Dynamic Specifications Method (SDSM) so that the variation of the resulting assemblies' dimension can be kept under control [1].

Assuming the presence of a detectable long-term component or drift in the variation of the dimension of interest, SFFCM divides the component lots in subsets of items produced consecutively in a short-time interval from which some items are inspected to estimate the sample mean and the sample standard deviation of each subset so that necessary adjustments can be determined and triggered.

To carry out the experiments the manufacture of one thousand assemblies made of two components having high dimensional was simulated. The measurement uncertainty was increased stepwise to quantify its influence on the estimation of the adjusted specifications and on the capability of the target subprocess.



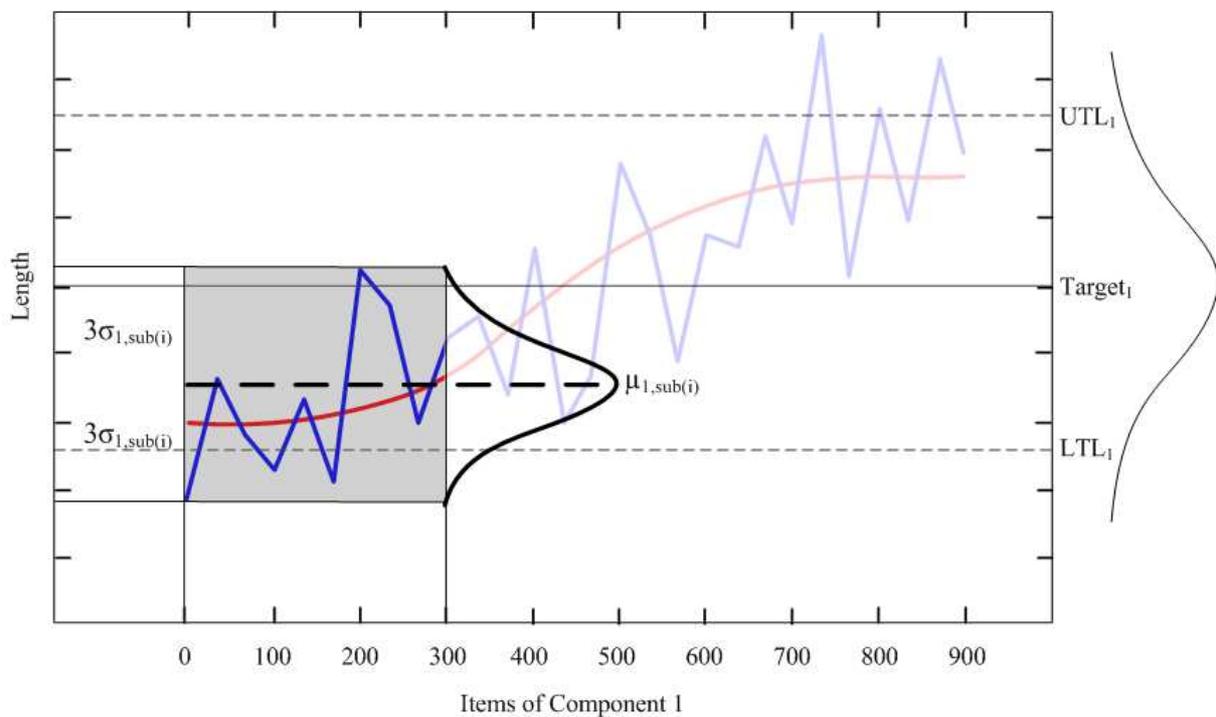
**2. Statistical Dynamic Specifications Method (SDSM)**

SDSM consists of a collection of steps that help managing dimensional specifications and tolerances of inner components of an assembly [1].

Let  $L_{assy}$  and  $t_{assy}$  be the target and tolerance of an assembly made of two components whose specifications have been set to  $L_j$  and  $t_j$  and let the variation of the length of the items of Component 1 be the superposition of a random component and a potentially controllable long-term drift.

$$L_{assy} = L_1 + L_2 \tag{1}$$

$$t_{assy} = \sqrt{t_1^2 + t_2^2} \tag{2}$$



**Figure 1.** Mean and standard deviation of the subset  $i$ .

If a small subset  $i$  of items produced consecutively in a short-time interval were taken from the lot of Component 1, it would be found that 99.73% of the items fall in the band  $\mu_{1,sub(i)} \pm 3\sigma_{1,sub(i)}$  (Figure 1). Since the influence of the long-term drift is only partial here, the standard variation of this subset is expected to be smaller than the one of the whole lot [2]. Hence, it is reasonable to think that at least for the subset  $i$  the nominal tolerance  $t_1$  had not been fully used and that part of it could have been spared to complement the nominal tolerance  $t_2$  of a matching subset  $i$  of Component 2. In fact, it would have been possible to define another tolerance  $t_{2,adj,sub(i)}$  as follows:

$$t_{1,sub(i)} = 3\sigma_{1,sub(i)} \tag{3}$$

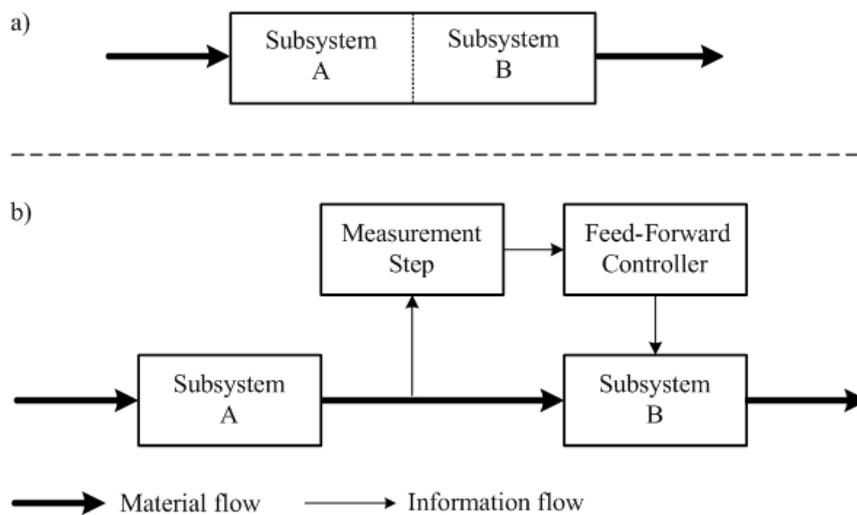
$$t_{2,adj,sub(i)} = \sqrt{t_{assy}^2 - t_{1,sub(i)}^2} \tag{4}$$

In the same way, if the mean  $\mu_{1,sub(i)}$  of the subset  $i$  had been known *a priori* then it would have been possible to define an adjusted target  $L_{2,adj,sub(i)}$  for a matching subset  $i$  of Component 2 to help meeting the desired  $L_{assy}$  as follows:

$$L_{2,adj,sub(i)} = L_{assy} - \mu_{1,sub(i)} \tag{5}$$

### 3. Statistical Feed-Forward Control Model (SFFCM)

SFFCM separates the system in two, a feeding and a controlled subsystem, to place an intermediate measurement step between. Thus, in the presence of a detectable long-term drift in the output of the feeding Subsystem A, corrective adjustments could be made on the parameters of the controlled Subsystem B by means of applying iteratively SDSM on small subsets of items [3]. The size of the subsets will determine the final number of adjustments needed to assemble the whole lot.



**Figure 2.** Statistical Feed-Forward Control Model.

### 4. Measurement Uncertainty

The target of the matching subset  $i$  of Component 2 can be computed directly with the following estimator:

$$\hat{L}_{2,adj,sub(i)} = L_{assy} - \bar{x}_{1,sub(i)} \tag{6}$$

However, the computation of the corresponding tolerances is not trivial because the measurement uncertainty has to be considered this time.

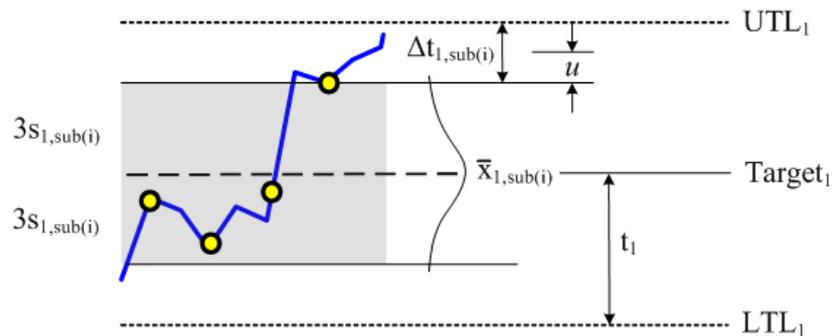
$$\hat{t}_{1,sub(i)} = 3s_{1,sub(i)} \tag{7}$$

$$\hat{t}_{1,sub(i),unc} = \hat{t}_{1,sub(i)} + u \tag{8}$$

If  $\Delta t_{1,sub(i)}$  defines the difference between the nominal tolerance  $t_1$  and half of the band of  $6s_{1,sub(i)}$  where 99.73% of the items' lengths are believed to fall and  $u$  is the measurement uncertainty (Figure 3), the relation between  $\Delta t_{1,sub(i)}$  and  $u$  can be expressed by means of the following ratio (Equation 10):

$$\Delta t_{1,sub(i)} = t_1 - 3s_{1,sub(i)} \tag{9}$$

$$X \% = \frac{u}{\Delta t_{1,sub(i)}} \tag{10}$$



**Figure 3.** Representation of the measurement uncertainty.

To avoid proceeding with an analysis in terms of a given value  $u$  of the measurement uncertainty, Equation 8 was rewritten in terms of the variable ratio  $u/\Delta t_{1,sub(i)}$ .

$$\hat{t}_{1,sub(i),unc} = \hat{t}_{1,sub(i)} + (X\%) \Delta t_{1,sub(i)} \tag{11}$$

$$\hat{t}_{1,sub(i),unc} = 3s_{1,sub(i)} + (X\%)(t_1 - 3s_{1,sub(i)}) \tag{12}$$

Thus, the adjusted tolerance of a matching subset  $i$  of Component 2 can be computed as follows:

$$\hat{t}_{2,adj,sub(i),unc} = \sqrt{t_{assy}^2 - \hat{t}_{1,sub(i),unc}^2} \tag{13}$$

### 5. Simulation

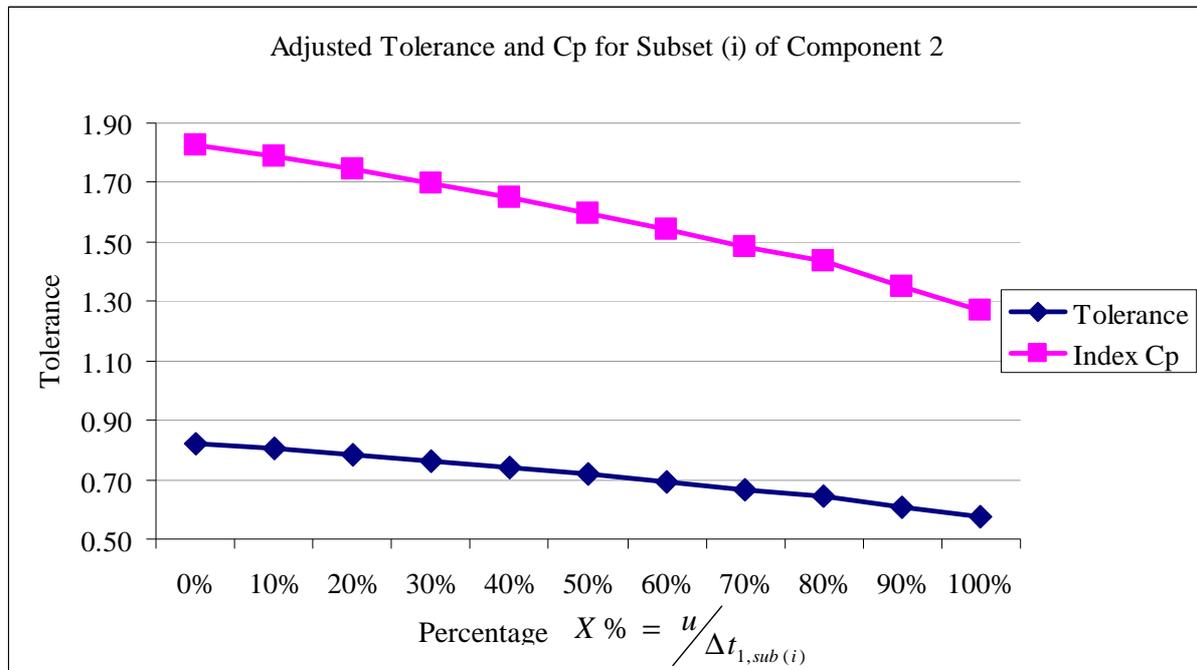
For the purposes of this work the production of a lot of 1,000 assemblies made of two components having high dimensional variation was simulated (Table 2). Each of the experiments mentioned above was replicated 500 times employing new populations generated by a Monte Carlo simulation.

**Table 1.** Components' nominal specifications [mm] and processes' characteristics

	Target	Tolerance	Mean	Std. Deviation	$c_p$
Assembly	30.00	1.00	29.55	0.29	1.15
Component 1	20.00	0.82	19.60	0.25	1.09
Component 2	10.00	0.58	9.95	0.15	1.29

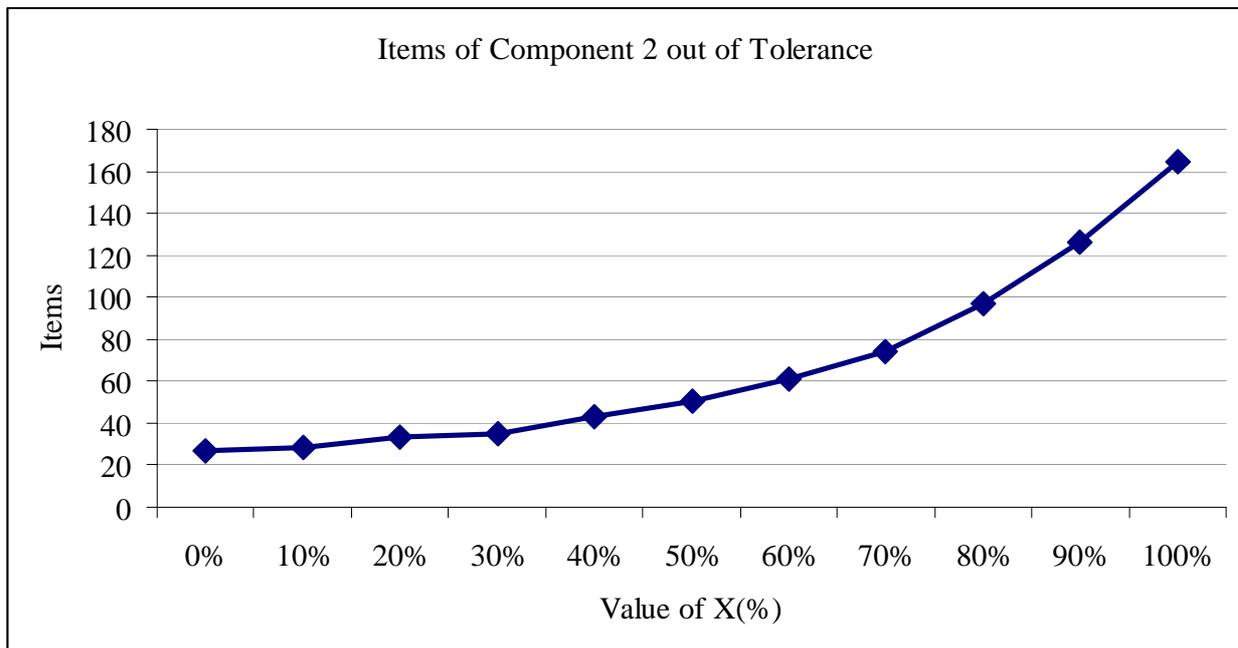
According to the definition of SFFCM, Component 1 was meant to represent the feeding Subsystem A and Component 2 the controlled Subsystem B.

Simulation results showed that depending on the magnitude of the measurement uncertainty, the adjusted tolerance of the subsets of Component 2 decreased in average from 0.83 to 0.57 (Figure 4). Consequently, the capability of the subprocess corresponding to Component 2 decreased in average from 1.84 to 1.27.



**Figure 4.** Adjusted tolerance  $t_{2,adj,sub(i),unc}$  and  $c_{p,2,adj,sub(i)}$  for different ratios  $u/\Delta t_{1,sub(i)}$ .

In practice, the measurement uncertainty that is added to  $t_{1,sub(i)}$  to determine  $t_{1,sub(i),unc}$ , and thus, to compute the adjusted  $t_{2,adj,sub(i),unc}$ , will impact directly in the number of items of Component 2 that fall out of their respective tolerance range and that, in consequence, can be considered as defective units or scrap (Figure 5).



**Figure 5.** Average number of items of Component 2 out of tolerance.

## 6. Conclusion

When dealing with assembling processes characterized by high dimensional variation whose cumulative variation reaches the order of the allowed tolerance for the whole assembly, the magnitude of the measurement uncertainty considered to estimate the adjusted specifications using SDSM has a significant impact on its practical realization because any loss of capability (fall of the index  $c_p$ ) might produce an increment of the scrap percentage of the target subprocess.

## 7. References

- [1] Hernández C and Tutsch R 2012 *Statistical Dynamic Specification Method for Allocating Tolerances: 12th CIRP Conference on Computer Aided Tolerancing (Huddersfield, UK, 18-19 April 2012)*
- [2] Burr I 1979 *Elementary Statistical Quality Control* Ch.8 p.192. (Marcel Dekker Inc.).
- [3] Hernández C and Tutsch R 2012 *Sampling Strategies and Long Term Variation Modelling for a Statistical Feed-Forward Controller: XX IMEKO World Congress Metrology for Green Growth ( Busan, Republic of Korea, 9-14 September 2012)*

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