Quantification is Neither Necessary Nor Sufficient for Measurement

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Abstract. Being an infrastructural, widespread activity, measurement is laden with stereotypes. Some of these concern the role of measurement in the relation between quality and quantity. In particular, it is sometimes argued or assumed that quantification is necessary for measurement; it is also sometimes argued or assumed that quantification is sufficient for or synonymous with measurement. To assess the validity of these positions the concepts of measurement and quantitative evaluation should be independently defined and their relationship analyzed. We contend that the defining characteristic of measurement should be the structure of the process, not a feature of its results. Under this perspective, quantitative evaluation is neither sufficient nor necessary for measurement.

1. Introduction

Being an infrastructural, widespread activity, performed by human beings for millennia, measurement is laden with stereotypes, some of which concern the role of measurement in the relation between quality and quantity (or: between qualitative and quantitative properties). As an authoritative source, consider, for example: "The primary aim of a given theory of measurement is to show in a precise fashion how to pass from qualitative observations to the quantitative assertions needed for more elaborate theoretical stages of science." [1]. Contained in this statement are two basic suppositions. First, qualities such as beauty and wisdom are basic elements of human knowledge, but only on quantities such as length and weight can objective information be obtained, and only on quantity values can mathematical equations be computed; thus, the transition from qualities to quantities is a worthy endeavor. Second, measurement is the privileged tool to achieve the desired quantitative representations. The path leading to the recognition that a property is in fact a quantity, or can be represented as a quantity (that quantities are particular properties is a position accepted by the International Vocabulary of Metrology (VIM) [2]), is an evolutionary one, implying an advancement of knowledge related to the scale type on which the property can be evaluated [3] and therefore of the semantics of the property itself [4]. Against this background, and given the further acknowledgment that measurement is the tool that historically enabled physics to represent properties quantitatively, the conclusion is sometimes reached that there is a strict tie between quantities and measurement. Such a relation is differently instanced, as follows.

In terms of measurement as a process: each quantity evaluation is a measurement (i.e., the only way to assign a value to a quantity is by means of measurement: quantitative evaluation is sufficient

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for measurement); alternatively, each measurement is a quantity evaluation (i.e., only quantities can be measured: quantitative evaluation is necessary for measurement).

In terms of the ontology of the measurability of properties: each quantity is measurable (i.e., for a property to be a quantity is a sufficient condition for it to be measurable); alternatively, each measurable property is a quantity (i.e., if a property is not quantitative, then it cannot be measured: for a property to be a quantity is a necessary condition for it to be measurable).

These are the positions we intend to challenge here.

2. Framing measurement

Our claim is that measurement is not a natural entity, existing independently of human beings and discovered by them, but it is a designed-on-purpose process. Hence, some conventionality in the definition of measurement is unavoidable, and at least partially different concepts of measurement can be maintained in different contexts. On the other hand, one may safely assume that, for example, measurement results and subjective opinions are not the same. The relation between measurement and quantitative evaluation is a critical component of an appropriate characterization of measurement. It can be studied along three orthogonal dimensions (for the sake of simplicity, each of them will be considered as a Boolean feature). Is the evaluated property: 1. a quantity? 2. evaluated as a quantity? 3. evaluated by means of a process conveying good-quality information (in a sense to be specified)?

Issue 1 relates to claim on what the property *is*, and is thus *ontological*: is *to be* a quantity sufficient, and/or necessary, to be measurable? A positive answer would rule out, in particular, nominal properties as potentially measurable entities. Issue 2 acknowledges that a property that is a quantity might be nevertheless evaluated in a non-quantitative way. For example the length of rigid objects may be evaluated according to the condition "is it longer than 1 m and shorter than 2 m?", resulting in a pure, binary classification, even though at the ontological level length remains (interpreted as) a quantity. This issue is then *operative*, and adds a specification to the ontological one: is *to be evaluated* quantitatively sufficient, and/or necessary, for measurement? Issue 3 focuses on the information obtained in the evaluation, and questions whether measurement should be characterized in terms of desirably epistemic properties, e.g., in terms of objectivity and inter-subjectivity [5, 6]. This issue is *epistemic*: is *to be evaluated so to convey objective and inter-subjective information* (in a sense to be specified) sufficient, and/or necessary, for measurement?

With the explicit position that measurement is primarily an epistemic process, we argue that the relation between measurement and quantitative evaluation is loose: *not all quantitative evaluations are measurements* and *not all measurements are quantitative evaluations*; furthermore, *being a quantity is neither sufficient nor necessary for being measurable*. Our argument develops along two complementary lines. First quantities are, historically and conceptually, tied with *measures*, not measurement, and 'measure' and 'measurement' are different concepts: the results that hold for measures should not uncritically applied also to measurement. Second, the structure of measurement is generally independent of the possible quantitative structure of the property under consideration: that only quantities are considered to be measurable is just a matter of convention. In this paper we explore the first topic, thus mainly developing the *pars destruens* of the work.

3. A historical overview

In Euclid's *Elements*, sometimes thought of "the earliest contribution to the philosophy of measurement available in the historical record" [7], the concept 'measure' (as a noun) is often exploited, whereas 'measurement' is never used, because "in the geometrical constructions employed in the Elements [...] empirical proofs by means of measurement are strictly forbidden" (from the introductory notes of [8]). This highlights that 'measurement' and 'measure' should not dealt with as synonyms: even though they are terms so entangled with daily activities and speaking that their meaning and relations are often spurious, in the scientific context the former refers specifically to the *process* of measuring, not to the structure of its input or output entities – properties and property values respectively. For example, the VIM defines 'measurement' as "the process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity". Interestingly, the VIM avoids using "measure" as noun (but in the technical, idiomatic term "material measure") to reduce ambiguity, by preferring "measurement result" to denote the *outcome* of the process. Indeed, while "measurement" appears to have a relatively well-defined meaning even in everyday use – "the action of measuring", according to the *Oxford English Dictionary* [9] – "measure" is remarkably

polysemic, with several distinct meanings. One of them is particularly important here: a measure is "a quantity contained in another an exact number of times; a divisor". This is indeed the definition that can be traced back to the Elements. The oft-quoted first two definitions of Book 5 are [8] "A magnitude is a part of a(nother) magnitude, the lesser of the greater, when it measures the greater" and "the greater (magnitude is) a multiple of the lesser when it is measured by the lesser". Given that the Greek verb for 'to measure' contains the root "metr-" one might conclude that there is a conceptual continuity (or even identity) between the Euclidean 'to measure' and what nowadays is the object of metrology, i.e., (physical) measurement. But let us consider another definition by Euclid, now from Book 7: "A number is part of a(nother) number, the lesser of the greater, when it measures the greater". Definitions 3.1 and 7.3 have the same structure: "An x is a part of a(nother) x, the lesser of the greater, when it measures the greater", and in both cases 'to measure' is used. But while in the first case x = magnitude, in the second case x = number ("numbers *measure* one another"), thus showing that here *'to measure' does not necessarily have an empirical connotation*. These definitions are followed by several others in which divisibility between numbers is presented in terms of their "measurability": *the Euclidean 'to measure' is actually a shortcut for 'to be (integer) part of'*.

The generalization of measures as additive functions is immediate, via the hypothesis that such parts are pairwise disjoint. Indeed, the mathematical literature proposes definitions where "the study of measures and their application to integration is known as *measure theory*." [10]. This shows the conceptual continuity (even though surely not an identity) between the traditional, Euclidean concept of measure and the contemporary measure theory, which – now the emphasis should be obvious – is *not* a measurement theory. This highlights that sentences such as "Euclid's concept [...] explained the place of numbers in measurement [...] and what it is that is being estimated in measurement" [7], and then, as apparently a direct consequence, "to understand measurement theory, it is necessary to revisit the theory of integration and, particular, Lebesgue measure theory" [11], are plainly false.

With a path including the contributions by, e.g., Newton and Maxwell, and more recently Campbell, what in the classical world was a feature of a measure – numerical ratios – has become a definitional characteristic of measurement. Even more critically, the emphasis on measurement as representation has generated a shift from the original ontological claim to much weaker representability conditions. For example, according to [1] an appropriate representation theorem "makes the theory of finite weak orderings a theory of measurement, *because of its numerical representation*" (emphasis added): even though weakly ordered entities surely do not satisfy the Euclidean conditions on a measure, they are considered measurable because they can be represented by means of quantity values. From this point of view, Michell's call to stronger conditions is appropriate: unfortunately it just misses the point.

4. Measurement and quantities

The structure of the position that identifies measurement and quantitative evaluation is transparent:

(a) physical laws are written as mathematical equations whose variables represent quantities;
(b) properties can be mutually related via physical laws only if they are quantities;
(c) only quantities are measurable.

While (a) and (b) are patent facts, they do not entail (c), which is explained in the light of the equation:

(d) quantity = (Euclidean) magnitude

together with the stipulation that:

(e) objects of measurement are (Euclidean) magnitudes.

While (d) can be assumed as a definition of 'quantity', or just the acknowledgment of a synonymy, as we have just noted (e) cannot be justified by the Euclidean tradition, which does not deal with any (empirical) "action of measuring". It seems that (e) is simply a matter of convention or tradition, and thus the question arises: why should measurement be only related to quantities?

From the Joint Committee for Guides in Metrology comes a hint that the current conception of measurement is a moving target, instead of being stably bound to the Euclidean tradition. The already mentioned current version of the VIM maintains that only quantities are measurable (hypothesis (c) above), but at the same time introduces the concept of (measurable) 'ordinal quantity', "quantity, defined by a conventional measurement procedure, for which a total ordering relation can be established, according to magnitude, with other quantities of the same kind, but for which no algebraic operations among those quantities exist", thus breaking hypothesis (d): in the Euclidean perspective, the very term "ordinal quantity" would be an oxymoron.

The position (d) + (e) hints at a possible direction towards establishing the measurability of non-physical properties, and in fact it may partially explain the activities aimed at guaranteeing (i.e., discovering or constructing, depending on one's philosophical orientation) a quantitative structure to non-physical properties, such as those typically derived from (simultaneous) "conjoint measurement" [12]. In the lack of a well-established theoretical and metrological structure [13, 14], to study the structure of a property (and, possibly, to discover that it is quantitative) is a worthwhile endeavor, but in principle this should be acknowledged as a distinct task from measuring the property itself. As an example of the critical relation between quantification and measurement, consider the sentence "a demonstration that intelligence, say, satisfies (or fails to satisfy) the axioms of conjoint *quantification* would complete the scientific task of quantification for intelligence", taken from [15] and where the original term "conjoint measurement" has been replaced by "conjoint quantification". Hence [7] correctly notes that "conjoint measurement" refers "not so much [to] a method of measurement [but to] a context within which indirect evidence for quantitative structure could be collected". Were instead the mentioned replacement accepted, the sentence would be more or less tautological. The original sentence is meaningful only if 'measurement' and 'quantification' are assumed as (i) in principle distinct concepts and (ii) actually coincident. Nevertheless, we have shown that *this position is wrong*.

5. The concept of quantity and philosophies of measurement

Up to this point we have attempted to formulate both the positions that quantification is necessary and/or sufficient for measurement, and our own comments, in as philosophically agnostic a manner as possible. However, given that these claims are often stated in terms of or influenced by particular philosophical stances, and given also that our own position on measurement inevitably involves certain philosophical commitments, it is worth exploring the implications of two major philosophical schools of thinking about measurement. Our treatment here is necessarily brief (but see [16]).

Empiricist philosophical stances generally emphasize a commitment to direct observation as the basis for knowledge. Representational measurement theory, operationalism, and the writings of S.S. Stevens may all be considered empiricist approaches to measurement, in that they characterize measurement in terms of the manner in which numerical assignments are derived from observable relations. The idea that quantification is sufficient for measurement can be formulated as a direct corollary to operationalism, insofar as operationalism is commonly interpreted as holding that measurement is nothing more than the the results of applying a particular procedure ('measurement by fiat'); on one reading, Stevens' [17] declaration that "measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules" is consistent with this position as well, insofar as anything other than random assignment can be considered a rule. Others have argued, and we agree, that this trivializes the concept of measurement; certainly, if one accepts that measurement is an epistemic activity, it can easily be seen that there are instances of rule-based number-assignments that have nothing to do with the acquisition of knowledge, and thus cannot count as instances of measurement. For example, if a group of persons were each to flip a coin a given number of times, they could be assigned numbers based on how many times their coin landed heads-up – and further, these numbers or transformations thereof could be shown on some criteria to possess quantitative structure [18] – but one would be hard-pressed to argue that such numerical assignments should be considered to be values of a property of persons.

By contrast, *realist* philosophical stances on scientific inquiry emphasize the commitments that (a) there is a (single) natural world, which exists regardless of what any conscious being thinks or perceives; (b) scientific claims about the world are to be taken at face value, as possessing truth-values, and (c) so interpreted, true scientific claims constitute knowledge of the world. Michell's writings are consistent with (but do not explicitly require) *metaphysical realism,* which entails the *correspondence theory of truth*: statements are true if they directly correspond to facts in the world. In terms of measurement, the implication is that whether or not a property is a quantity is entirely an empirical matter – that is, a mind-independent fact about the way the world really is.

A number of challenges have been raised to metaphysical realism. In our view, the most compelling of these invoke the observation that there are simply too many ways in which beliefs and symbols can be mapped onto the world for it to be plausible that there is a single complete and true description of the way the world really is [19]. A commitment to scientific realism need not entail the belief that the world consists of a fixed totality of mind-independent objects and their properties;

rather, it is possible to maintain a realist view of measurement while acknowledging that *knowledge* is constructed by humans, and can be constructed in multiple ways depending in part on pragmatically determined frames of reference.

For example, we may wish to acquire knowledge about the extent to which students have mastered a set of concepts related to statistical reasoning. We may design an assessment of this property of students, based on our best available theories of learning and cognition, and we may employ statistical models such as the Rasch model [20], which treat the measured property of students as a quantity, to test hypotheses regarding both our cognitive theories and our assessment (see [21] for an example of how this might be done), and to represent differences amongst students in their degrees of mastery of statistical concepts. Doing so may (a) advance our collective understanding of how learning works, (b) communicate information about students to educators and other stakeholders in an efficient manner, and (c) suggest further avenues of exploration both for educational practice and educational psychology. Thus, in this example, information has been acquired and represented using the logic of measurement, to scientifically and practically productive ends, without it ever being necessary to assume that the measured property *is* a quantity in a mind-independent sense.

6. What, then, is measurement all about?

In this paper we have argued that the defining characteristic of measurement should be the structure of the process, rather than a particular feature of the results of the process (such as whether information can be represented on a ratio scale) or a feature of the input of the process (such as whether the property being evaluated is a quantity). Alternatively stated, measurement is an epistemic activity, and may be applied to a variety of properties of parts of the natural world. In our view, when one says that scientists are engaging in 'measurement activities', one is saying they are attempting to develop methods of obtaining objective and inter-subjective information about instances of a property. It may turn out that the property under investigation is truly quantitative (or can be treated as quantitative within a specified frame of reference and according to the available knowledge of it), in which case the particular method of obtaining objective and inter-subjective information about the property may indeed be to discover ratios of magnitudes of quantity relative to a standard unit. However, this may not be known from the outset (for example, it was only discovered after centuries of study that temperature was a quantitative property [15]; are we then to say that all pre-1760 work on discovering differences in temperatures do not count, *a priori* and independently of how they were performed, as measurement activities?); further, it may turn out that the property is not quantitative at all, and in our view it seems arbitrary and unnecessary to then disallow the use of the term 'measurement' when referring to the study of that property.

We certainly agree with Michell [20] that the *a priori* assumption that a given property is quantitative is problematic (one might even say 'pathological'). Part of the point of measurement activities, in our view, is to *test* hypotheses about properties, and in so doing learn more about how facts about them can be (accurately *and* usefully) represented. There are two points on which we depart from Michell: (a) we do not think that *quantitivity* is necessarily a mind-independent feature of properties, and (b) we do not think that *quantity* – mind-independent or not – should be considered a necessary condition for measurement.

As argued previously, empiricism is motivated by the intuition that the preferred method of acquiring knowledge is through observation and experience, while realism is motivated by the intuition that that scientific inquiry seeks to gain knowledge about a natural world. Neither of these intuitions contradicts the other. An example of a philosophic framework that is consistent with both is found in Putnam's recent (e.g., [19]) writings on *pragmatic realism*, which acknowledge that conceptual relativity is not at odds with realism, but rather, it is the interface between the world and the rich fabric of our concepts and linguistic schemes that jointly determines what we see. On this account, the existence of natural reality is not denied, but neither is it seen as directly presented to our senses; instead, our various substantive and methodological models and theories and pragmatic concerns cause us to organize, prioritize, and make sense of experience in a particular way. Further, the connection between natural reality and the outcomes of a measurement procedure is not in itself compromised by the fact that we choose to privilege certain contrast classes, levels of explanation, methods of summarization, and modes of description.

Measurement is a complex and challenging endeavor. In our view, the claims that quantification is sufficient or necessary for measurement both trivialize the concept of measurement: the former

position in effect denies that measurement is an epistemic activity, and the latter arbitrarily ties an empirical activity (measurement) to a specific mathematical concept (Euclidean magnitude). We have argued, instead, that locating the defining characteristics of measurement in the structure of the activity – rather than a specific feature of its subject matter or output – pays respect to the fact that measurement is an infrastructural, dynamic, designed-on-purpose epistemic activity.

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