

Parton-hadron matter in- and out-of equilibrium

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Abstract. We study the shear and bulk viscosities of partonic and hadronic matter - as well as the electric conductivity - as functions of temperature T within the Parton-Hadron-String Dynamics (PHSD) off-shell transport approach. Dynamical hadronic and partonic systems in equilibrium are studied by the PHSD simulations in a finite box with periodic boundary conditions. The ratio of the shear viscosity to entropy density $\eta(T)/s(T)$ from PHSD shows a minimum (with a value of about 0.1) close to the critical temperature T_c , while it approaches the perturbative QCD (pQCD) limit at higher temperatures in line with lattice QCD results. For $T < T_c$, i.e. in the hadronic phase, the ratio η/s rises fast with decreasing temperature due to a lower interaction rate of the hadronic system and a significantly smaller number of degrees-of-freedom. The bulk viscosity $\zeta(T)$ - evaluated in the relaxation time approach - is found to strongly depend on the effects of mean fields (or potentials) in the partonic phase. We find a significant rise of the ratio $\zeta(T)/s(T)$ in the vicinity of the critical temperature T_c , when consistently including the scalar mean-field from PHSD, which is also in agreement with that from lQCD calculations. Furthermore, we present the results for the ratio $(\eta + 3\zeta/4)/s$, which is found to depend non-trivially on temperature and to generally agree with the lQCD calculations as well. Within the PHSD calculations, the strong maximum of $\zeta(T)/\eta(T)$ close to T_c has to be attributed to mean-fields (or potential) effects that in PHSD are encoded in the temperature dependence of the quasiparticle masses, which is related to the infrared enhancement of the resummed (effective) coupling $g(T)$. We also find that the dimensionless ratio of the electric conductivity over temperature σ_0/T rises above T_c approximately linearly with T up to $T = 2.5T_c$, but approaches a constant above $5T_c$, as expected qualitatively from perturbative QCD (pQCD).

1. Introduction

High energy heavy-ion reactions are studied experimentally and theoretically to obtain information about the properties of nuclear matter under the extreme conditions of high density and/or temperature. Ultra-relativistic heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) at CERN have produced a new state of matter, the quark gluon plasma (QGP). The produced quark gluon plasma behaves as a strongly-interacting fluid unlike a weakly-interacting gas [1, 2]. Large values of the azimuthal asymmetry of charged particles in momentum space, in particular the elliptic flow v_2 , observed in these experiments [3, 4, 5, 6, 7] could quantitatively be well described by means of ideal hydrodynamics up to transverse momenta on the order of 1.5 GeV/c [8, 9, 10, 11, 12]. An ideal fluid has been defined as having a zero shear viscosity η ; yet semiclassical arguments have been



given suggesting that the shear viscosity cannot be arbitrarily small [13]. Indeed, the lower bound for the shear viscosity to entropy density ratio $\eta/s \geq 1/4\pi$ was obtained by Kovtun-Son-Starinets (KSS) [14] for infinitely coupled supersymmetric Yang-Mills gauge theory based on the AdS/CFT duality conjecture. Recent relativistic viscous hydrodynamic calculations using the Israel-Stewart framework require a very small η/s of $0.08 - 0.24$ in order to reproduce the RHIC elliptic flow v_2 data [15, 16, 17, 18]. The main uncertainty in these estimates results from the equation of state and the initial conditions employed.

There is strong evidence from atomic and molecular systems that η/s should have a minimum in the vicinity of the phase transition (or rapid crossover) between the hadronic matter and the quark-gluon plasma [19], and that the ratio of bulk viscosity to entropy density ζ/s should be maximum or even diverge at a second-order phase transition [20, 21, 22, 23, 24].

The shear and bulk viscosities of strongly-interacting systems have been calculated within different approaches. Calculations have been performed at extremely high temperatures, where perturbation theory can be applied [25, 26], as well as at extremely low temperatures [26, 27, 28]. First results for shear and bulk viscosities obtained within lattice QCD simulations just above the critical temperature of pure gluon/gluonball matter have been published in [29, 30, 31, 32]. In the literature there are several methods for the calculation of the shear and bulk viscosities, such as the Relaxation Time Approximation (RTA) [34], the Chapman-Enskog (CE) method [35] and the Green-Kubo method [36, 37].

In this contribution we extract the shear and bulk viscosities from the ‘infinite’ parton-hadron matter employing different methods within the Parton-Hadron-String Dynamics (PHSD) transport approach [38, 39], which is based on generalized transport equations on the basis of the off-shell Kadanoff-Baym equations [40, 41] for Green’s functions in phase-space representation (in first order gradient expansion beyond the quasiparticle approximation). The approach consistently describes the full evolution of a relativistic heavy-ion collision from the initial hard scatterings and string formation through the dynamical deconfinement phase transition to the strongly-interacting quark-gluon plasma (sQGP) as well as hadronization and the subsequent interactions in the expanding hadronic phase. In the hadronic sector PHSD is equivalent to the Hadron-String-Dynamics (HSD) transport approach [42, 43] that has been used for the description of pA and AA collisions from SIS to RHIC energies in the past and the partonic dynamics is based on the Dynamical QuasiParticle Model (DQPM) [44, 45], which describes QCD properties in terms of single-particle Green’s functions (in the sense of a two-particle irreducible (2 PI) approach) and reproduces lattice QCD results – including the partonic equation of state – in thermodynamic equilibrium. The PHSD approach [38, 39, 46] has been applied to pp , $p + A$ and $A + A$ collisions from $\sqrt{s_{NN}} = 5$ GeV up to 2.76 TeV providing a consistent description of hadronic single-particle spectra, collective flow coefficients of hadrons as well as dilepton radiation from the partonic and hadronic phase. Since transport coefficients are no inherent parameters of PHSD we may ask the question: what are the transport coefficients from PHSD in equilibrium - in particular as a function of temperature - that are compatible with the experimental observations? Here we summarize our findings from Refs. [47, 48, 49].

2. Shear viscosity coefficient: the Kubo formalism

In this Section we concentrate on the extraction of the shear viscosity from the ‘infinite’ parton-hadron matter employing the Kubo formalism. We simulate the ‘infinite’ matter within a cubic box with periodic boundary conditions at various values for the quark density (or chemical potential) and energy density. The size of the box is fixed to 9^3 fm^3 . The initialization is done by populating the box with light (u, d) and strange (s) quarks, antiquarks and gluons. The system is initialized out of equilibrium and approaches kinetic and chemical equilibrium during its evolution by PHSD. If the energy density in the system is below the critical energy density ($\varepsilon_c \approx 0.5 \text{ GeV/fm}^3$), the evolution proceeds through the dynamical phase transition (as

described in Ref. [47]) and ends up in an ensemble of interacting hadrons. For more details we refer the reader to Ref. [47].

The Kubo formalism relates linear transport coefficients such as heat conductivity, shear and bulk viscosities to non-equilibrium correlations of the corresponding dissipative fluxes, and treats dissipative fluxes as perturbations to local thermal equilibrium [36, 37]. The Green-Kubo formula for the shear viscosity η is given by [50]:

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \langle \pi^{xy}(\mathbf{0}, 0) \pi^{xy}(\mathbf{r}, t) \rangle, \quad (1)$$

where T is the temperature of the system, t refers to the time after the system equilibrates, which is set at $t = 0$, while $\langle \dots \rangle$ denotes the ensemble average in thermal equilibrium and π^{xy} is the shear component (traceless part) of the energy momentum tensor $\pi^{\mu\nu}$:

$$\pi^{xy}(\mathbf{x}, t) \equiv T^{xy}(\mathbf{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{p^x p^y}{E} f(\mathbf{x}, \mathbf{p}; t), \quad (2)$$

where the mean-field U_s enters in the energy $E = \sqrt{\mathbf{p}^2 + U_s}$.

In our numerical simulation the volume averaged shear component of the energy momentum tensor can be written as

$$\pi^{xy}(t) = \frac{1}{V} \sum_{i=1}^N \frac{p_i^x p_i^y}{E_i}, \quad (3)$$

where V is the volume of the system and the sum is over all particles in the box at time t . Note that the scalar mean-field contribution U_s only enters via the energy E . Taking into account that point particles are uniformly distributed in our box (implying $\pi^{xy}(\mathbf{r}, t) = \pi^{xy}(t)$), we can simplify the Kubo formula for the shear viscosity to

$$\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0) \pi^{xy}(t) \rangle. \quad (4)$$

The correlation functions $\langle \pi^{xy}(0) \pi^{xy}(t) \rangle$ are empirically found to decay exponentially in time,

$$\langle \pi^{xy}(0) \pi^{xy}(t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle e^{-t/\tau}, \quad (5)$$

as shown in Fig. 1 (lhs), where τ is the relaxation time. Finally, we end up with the Green-Kubo formula for the shear viscosity,

$$\eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau, \quad (6)$$

which we use to extract the shear viscosity from the PHSD simulations in the box.

3. The relaxation time approximation

3.1. Calculation of the shear and bulk viscosities

The starting hypothesis of the relaxation time approximation (RTA) is that the collision integral can be approximated by

$$C[f] = -\frac{f - f^{eq}}{\tau}, \quad (7)$$

where τ is the relaxation time. In this approach it has been shown that the shear and bulk viscosities (without mean-field or potential effects) can be written as [51, 52, 53]:

$$\eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|^4}{E_a^2} \tau_a(E_a) f_a^{eq}(E_a/T), \quad (8)$$

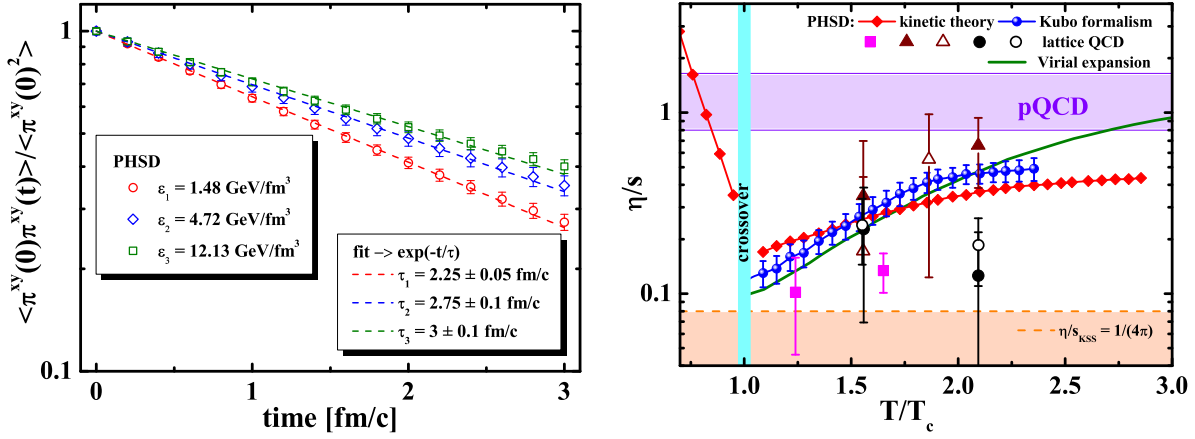


Figure 1. (lhs) The correlation functions $\langle \pi^{xy}(0)\pi^{xy}(t) \rangle$, which are normalized by $\langle \pi^{xy}(0)^2 \rangle$, as a function of time obtained by the PHSD simulations in the box (open symbols) for systems at different energy densities and corresponding exponential fits (dashed lines) with extracted relaxation times.

(rhs) The shear viscosity to entropy density ratio η/s as a function of temperature of the system obtained by the PHSD simulations using different methods: the relaxation time approximation (red line+diamonds) and the Kubo formalism (blue line+dots). The others symbols denote lattice QCD data for pure $SU_c(3)$ gauge theory from [29] (magenta squares), from [31] (wine open and full triangles), and from [32] (black open and full circles). The orange dashed line demonstrates the Kovtun-Son-Starinets bound [14] $(\eta/s)_{KSS} = 1/(4\pi)$. For comparison, the virial expansion approach (green line) [55] is shown as a function of temperature, too.

$$\zeta = \frac{1}{9T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} [(1 - 3v_s^2)E_a^2 - m_a^2]^2 f_a^{eq}(E_a/T), \quad (9)$$

where the sum is over particles of different type a (in our case, $a = q, \bar{q}, g$). In the PHSD transport approach the relaxation time can be expressed in the following way:

$$\tau_a = \Gamma_a^{-1}, \quad (10)$$

where Γ_a is the width of particles of type $a = q, \bar{q}, g$, which is defined within the DQPM. In our numerical simulation the volume averaged shear and bulk viscosities assume the following expressions:

$$\eta = \frac{1}{15TV} \sum_{i=1}^N \frac{|\mathbf{p}_i|^4}{E_i^2} \Gamma_i^{-1}, \quad (11)$$

$$\zeta = \frac{1}{9TV} \sum_{i=1}^N \frac{\Gamma_i^{-1}}{E_i^2} [(1 - 3v_s^2)E_i^2 - m_i^2]^2, \quad (12)$$

where the speed of sound $v_s = v_s(T)$ is taken from [54] or the DQPM, alternatively. Note that $v_s(T)$ from both approaches is practically identical since it is governed by the DQPM, which reproduces the lattice QCD results.

In Fig. 1 (rhs) we present the shear viscosity to entropy density ratio as a function of temperature of the system extracted from the PHSD simulations in the box employing different methods: the relaxation time approximation (red line+diamonds) and the Kubo formalism (blue line+dots). For comparison, the virial expansion approach (green line) [55] and lattice QCD data for pure $SU_c(3)$ gauge theory are shown as a function of temperature, too.

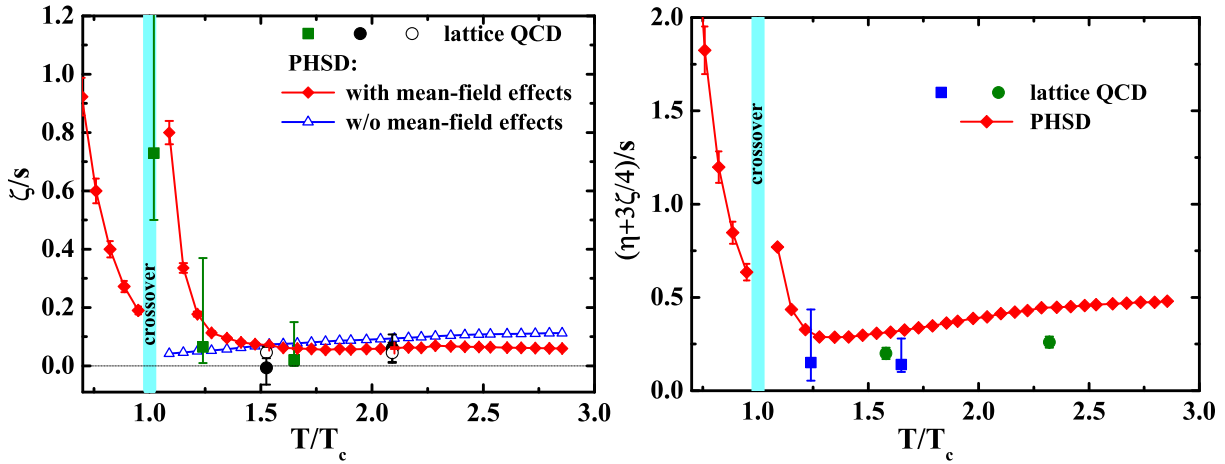


Figure 2. (lhs) The bulk viscosity to entropy density ratio as a function of temperature of the system extracted from the PHSD simulations in the box using the relaxation time approximation including mean-field effects (red line+diamonds) and without potential effects (blue line+open triangles). The available lattice QCD data from [30] (green squares) and from [32] (black open and full circles) are shown, too.

(rhs) The specific sound channel $(\eta+3\zeta/4)/s$ as a function of temperature of the system obtained by the PHSD simulations in the box using the relaxation time approximation including mean-field effects (red line+diamonds). It is compared with lattice QCD data from [33] (green circles) and from combining results of [30] and [32] (blue squares).

3.2. Mean-field or potential effects

In the absence of a chemical potential for quarks there should be no sizeable vector or tensor fields, only scalar fields. This affects the bulk viscosity, but not the shear viscosity. The expression for the bulk viscosity with potential effects is given by [53]

$$\zeta = \frac{1}{T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} f_a^{eq}(E_a/T) \times \left[\left(\frac{1}{3} - v_s^2 \right) |\mathbf{p}|^2 - v_s^2 \left(m_a^2 - T^2 \frac{dm_a^2}{dT^2} \right) \right]^2. \quad (13)$$

In the numerical simulation the volume averaged bulk viscosity with mean-field effects is used

$$\zeta = \frac{1}{TV} \sum_{i=1}^N \frac{\Gamma_i^{-1}}{E_i^2} \left[\left(\frac{1}{3} - v_s^2 \right) |\mathbf{p}|^2 - v_s^2 \left(m_i^2 - T^2 \frac{dm_i^2}{dT^2} \right) \right]^2. \quad (14)$$

Using the DQPM expressions for masses of quarks and gluons (for $\mu_q = 0$)

$$m_q^2(T) = \frac{1}{3} g^2(T) T^2, \quad m_g^2(T) = \frac{3}{4} g^2(T) T^2$$

we can calculate the derivative dm^2/dT^2 as well as v_s^2 .

In Fig. 2 (lhs) we show the bulk viscosity to entropy density ratio as a function of temperature of the system obtained by the PHSD simulations in the box employing the relaxation time approximation including mean-field (or potential) effects (red line+diamonds) and without potential effects (blue line+open triangles) as well as the available lattice QCD data from [30, 32].

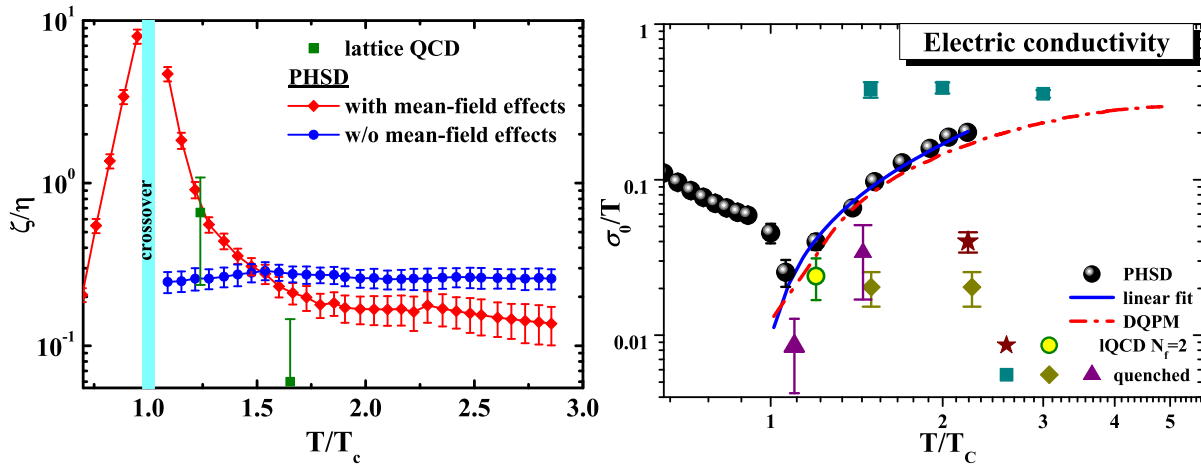


Figure 3. (lhs) The bulk to shear viscosity ratio ζ/η as a function of temperature of the system obtained by the PHSD simulations in the box employing the relaxation time approximation including mean-field effects (red line+diamonds) and without potential effects (blue line+circles).

(rhs) The ratio of the electric conductivity over temperature σ_0/T as a function of the scaled temperature T/T_c ($T_c = 158$ MeV). The full round symbols show the PHSD results, the solid blue line is a linear fit to the PHSD results (above T_c), while the dash-dotted red line gives the corresponding ratio in the relaxation-time approach (employing the DQPM parameters). The scattered symbols with error bars represent the results from lattice QCD calculations: triangles – Refs. [56], diamonds – Ref. [57], squares – Ref. [58], star – Ref. [59], open circle – Ref. [60].

A sound wave propagation in the z -direction with wavelength $\lambda = 2\pi/k$ is damped according to

$$T_{03}(t, k) \propto \exp \left[-\frac{(\frac{4}{3}\eta + \zeta)k^2 t}{2(\varepsilon + p)} \right], \quad (15)$$

where T_{03} is the momentum density in the z -direction, ε is the energy density and p is the pressure. Thus both the shear η and bulk ζ viscosities contribute to the damping of sound waves in the medium. In Fig. 2 (rhs) we present the specific sound channel $(\eta + 3\zeta/4)/s$ as a function of temperature of the system obtained by the PHSD simulations in the box using the relaxation time approximation including mean-field effects (red line+diamonds). It is compared with lQCD data from [33] (green circles) and from combining results of [30] and [32] (blue squares).

Furthermore, in Fig. 3 (lhs), we show the bulk to shear viscosity ratio ζ/η as a function of temperature of the system extracted from the PHSD simulations in the box using the relaxation time approximation including mean-field (or potential) effects (red line+diamonds) and without potential effects (blue line+circles). This ratio shows a pronounced maximum close to T_c when including the expected mean-field effects.

3.3. Electric conductivity

A further quantity of interest is the electric conductivity in the partonic/hadronic matter which controls the electromagnetic emissivity of the system in equilibrium. Here we briefly present the results from Ref. [49] which are displayed in Fig. 3 (rhs) for the dimensionless ratio of the electric conductivity over temperature σ_0/T which shows a pronounced minimum close to T_c and approaches a constant at higher temperature.

4. Summary and conclusions

We have employed the off-shell Parton-Hadron-String Dynamics (PHSD) approach in a finite box with periodic boundary conditions for the study of the shear and bulk viscosities - as well as the electric conductivity - as a function of temperature (or energy density) for dynamical infinite partonic and hadronic systems in equilibrium. The PHSD transport model is based on a lQCD equation of state [69] and well describes the entropy density $s(T)$, the energy density $\varepsilon(T)$ as well as the pressure $p(T)$ in thermodynamic equilibrium in comparison to the lQCD results. We have employed the Kubo formalism as well as the relaxationtime approximation to calculate the shear viscosity $\eta(T)$. We find that both methods provide very similar results for the ratio η/s with a minimum close to the critical temperature T_c while approaching the perturbative QCD (pQCD) limit at higher temperatures. For $T < T_c$, i.e. in the hadronic phase, the ratio η/s rises fast with decreasing temperature due to a lower interaction rate of the hadronic system and a significantly smaller number of degrees-of-freedom (or entropy density). Our results are, furthermore, also in almost quantitative agreement with the ratio $\eta(T)/s(T)$ from the virial expansion approach in Ref. [55] as well as with lQCD data for the pure gauge sector.

We have, furthermore, evaluated the bulk viscosity $\zeta(T)$ in the relaxation time approach and focused on the effects of mean fields (or potentials) in the partonic phase. Here we find a significant rise of the ratio $\zeta(T)/s(T)$ in the vicinity of the critical temperature T_c due to the scalar mean-fields from PHSD. The result for this ratio is in line with that from lQCD calculations. Additionally, the specific sound channel $(\eta + 3\zeta/4)/s(T)$ has been calculated and presents a non-trivial temperature dependence; the absolute value for this combination of the shear and bulk viscosities is in an approximate agreement with the lattice gauge theory. Furthermore, the ratio $\zeta(T)/\eta(T)$ within the PHSD calculations shows a strong maximum close to T_c , which has to be attributed to mean-field (or potential) effects that in PHSD are encoded in the infrared enhancement of the resummed coupling $g(T)$ (from the DQPM).

We also find that the dimensionless ratio of the electric conductivity over temperature σ_0/T rises above T_c approximately linearly with T up to $T = 2.5T_c$, but approaches a constant above $5T_c$, as expected from pQCD. This finding is naturally explained within the relaxation-time approach using the DQPM spectral functions. Below T_c the ratio σ_0/T rises with decreasing temperature because the system merges to a moderately interacting gas of pions with a larger charge (squared) to mass ratio than in the partonic phase and a longer relaxation time.

Since the PHSD calculations have proven to describe single-particle as well as collective observables and also dilepton data from relativistic nucleus-nucleus collisions from lower SPS to top RHIC energies [61, 62], the extracted transport coefficients $\eta(T)$, $\zeta(T)$ and $\sigma_0(T)$ are compatible with experimental observations in a wide energy density (temperature) range. Furthermore, the qualitative and partly quantitative agreement with lQCD results is striking.

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