

Static potential and singlet free energy at non-zero temperature

A. Bazavov and P. Petreczky

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract. We study Wilson loops at non-zero temperature and extract the static quark potential from them. The extracted potentials are larger than the singlet free energies and do not show screening for $T < 190$ MeV. We also calculate the singlet free energy and find that only at high temperatures the potential approaches the singlet free energy

1. Introduction

Quarkonium suppression was proposed by Matsui and Satz as a signature of formation of deconfined medium in heavy ion collisions [1]. The basic idea behind this proposal is that color screening in the deconfined medium will modify the heavy quark potential, eventually leading to the dissolution of the heavy quarkonium states. The problem of dissolution of quarkonium states at high temperatures could be formulated in terms of spectral functions. Early attempts to calculate spectral functions on the lattice have been presented in Refs. [2]. However, extraction of the spectral functions from lattice results on Euclidean correlation functions is quite difficult [3] and one should also be careful with cutoff effects in the spectral functions extracted from the lattice [4]. Furthermore, the Euclidean time correlators may not be sensitive to the melting of the bound states at high temperatures due to the fact that the Euclidean time extent is limited to $< 1/(2T)$ (see e.g. discussions in Ref. [5]). The effective field theory framework for heavy quark bound states, namely pQNRCD could be a useful tool for calculating quarkonium spectral functions [6]. The effective field theory approach allows to rigorously define the concept of the static quark anti-quark potential both at zero and non-zero temperatures. One of the main outcomes of the effective field theory analysis is the finding that at non-zero temperature the potential has also an imaginary part, which has important consequences for the dissolution of the quarkonium states. While pNRQCD is formulated in the weak coupling framework it is possible to extend it to the non-perturbative regime. For example, if the binding energy is the smallest scale in the problem all the other scales, like the thermal scales, the inverse size of the bound state and Λ_{QCD} can be integrated out. In this case the potential should be determined non-perturbatively and is identical to the energy of a static $Q\bar{Q}$ pair. If one further neglects the dipole interactions one gets the generalization of the simple potential model to the case of high temperatures [7]. However, one still needs to specify the potential. In the past model considerations based on lattice calculations of the so-called singlet free energy have been used (see e.g. discussion in Ref. [7]). In Ref. [8] it has been suggested to extract the energy of a static $Q\bar{Q}$ pair using the spectral decomposition of the temporal Wilson loops at non-zero temperature. In this contribution we discuss the calculation of the static quark anti-quark energy in 2+1 flavor QCD based on this idea. We also compare the static energy with the singlet free energy.



2. Numerical results

In lattice QCD calculations the static $Q\bar{Q}$ energy is extracted from Wilson loops $W(r, \tau)$. At large Euclidean time separations the exponential decay of the Wilson loops is governed by the static energy or potential, $W(r, \tau) \sim \exp(-V(r)\tau)$. More generally one can write a spectral decomposition for the Wilson loops [8]

$$W(r, \tau) = \int_{-\infty}^{\infty} d\omega \sigma(r, \omega) e^{-\omega\tau}. \quad (1)$$

At zero temperature the spectral function is proportional to $\delta(\omega - V(r))$ plus a sum of delta functions corresponding to the excited states (hybrid potentials). At non-zero temperature the delta function becomes a Lorentzian with the width related to the imaginary part of the potential. In Ref. [8] the potential at non-zero temperature was extracted by inverting Eq. (1) via the maximum entropy method (MEM). This procedure overestimated both the real and the imaginary parts of the potential. The problem is similar to the problem of extracting meson spectral functions [2, 3], where the width of the bound state peaks is mostly an artifact of MEM. Refined methods to extract the potential have been presented in Ref. [9] but the extraction of the imaginary part of the potential remains very challenging.

The usual problem that appears in the calculation of the potential is that the Wilson loops become noisy at large spatial separations r . To deal with this problem smeared gauge fields are used in spatial links when constructing Wilson loops on the lattice. Alternatively, one can fix the Coulomb gauge and calculate the correlation functions of two temporal Wilson lines separated by distance r without connecting them by spatial links [10]. At zero temperature where one is only interested in the energy levels both choices are equally good, and merely correspond to different choices of static meson interpolating operators. The same should be true at non-zero temperature provided the imaginary part of the potential is not too large.

We have calculated the correlation functions of temporal Wilson lines in 2+1 flavor QCD using the Highly Improved Staggered Quark (HISQ) action [11] and improved gauge action with the physical value of the strange quark mass and light quark masses corresponding to pion mass of 160 MeV in the continuum limit. The calculations have been performed in Coulomb gauge on $48^3 \times 16$ and $48^3 \times 12$ lattices for bare gauge coupling $\beta = 10/g^2 = 7.5$ as well as on $24^3 \times 6$ lattices at various gauge couplings that have been used in the study of the chiral and deconfinement transition by HotQCD [12]. As in Ref. [12] the lattice spacing was set by the r_1 scale extracted from the static potential and using the value $r_1 = 0.3106$ fm. We also used the renormalization constants for the Wilson line correlators determined in Ref. [12]. On $24^3 \times 6$ lattices we also calculated on-axis smeared and unsmeared Wilson loops. We used iterative APE and HYP smearing with 5, 10 and 20 smearing steps for APE smearing and 1, 5 and 10 smearing steps for HYP smearing. It should be noted that for $\tau T = 1$ the correlator gives the so-called singlet free energy [13], $F_1(r, T) = -T \ln W(r, \tau = 1/T)$.

The correlators of the Wilson lines as well as smeared Wilson loops show exponential τ -dependence, except close to $\tau T \simeq 1$, where the correlators show a slight increase. This increase is due to the contribution of a backward propagating state $\sim \exp(-E_{back}(1/T - \tau))$, that arises from the fact that static quarks propagate in gauge field background that is periodic in τ . Similar behavior has been observed in the Wilson loop calculations at non-zero temperature in pure gauge theory [8] as well as in full QCD calculations of bottomonium spectral functions within the non-relativistic formulation [16]. For the largest two values of N_τ considered here the backward propagating state does not cause any problem and we get stable results for the potential by performing single exponential fits in the τ -interval around the mid-point $\tau T = 1/2$. However, the results are quite noisy for $rT > 1$ in this case. Thus to explore the potential at larger distances we use $24^3 \times 6$ lattices for which statistical errors are small. Here the results are sensitive to how the fits are done. We performed three type of fits. First we used only $\tau T = 1/3$ and $\tau T = 1/2$ to

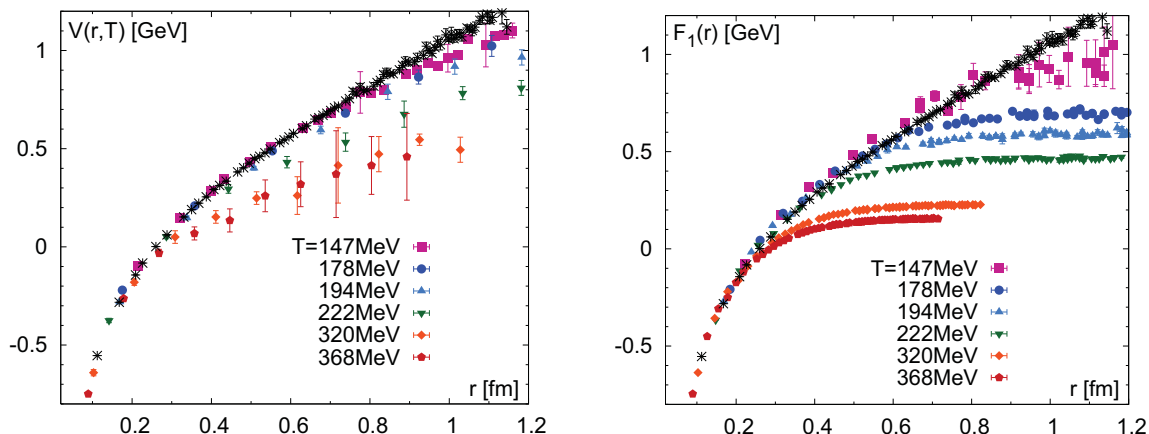


Figure 1. The static quark anti-quark potential (left) and the singlet free energy in Coulomb gauge (right) calculated on $24^3 \times 6$ lattice. The zero temperature potential is shown as black crosses.

extract the potential. Then we determined the backward propagating contribution by performing fits with $\tau T = 1$ and $\tau T = 2/3$. Subtracting the backward propagating contribution from the correlator we extracted the potential using $\tau T = 1/2$ and $\tau T = 2/3$ points. This gives our central value of the extracted potential. Finally, we performed single exponential fits for $\tau T = 1/2$ and $\tau T = 2/3$ which gave us the lower value of the potential. While the Coulomb gauge results and smeared Wilson loops give similar results excited state contributions are visible in τ -dependence of the unsmeared Wilson loops.

Our numerical results for the static quark anti-quark potential at different temperatures extracted from Coulomb gauge Wilson line correlators are shown in Fig. 1. The errors for the temperature dependent potential shown in the figure are mostly systematics and are estimated as described above. We also calculated the singlet free energy in Coulomb gauge and the results are shown in Fig. 1. Qualitatively the temperature dependence of the singlet free energy obtained in our calculations is similar to the temperature dependence of the singlet free energy obtained earlier with the p4 action [14, 15] but there quantitative differences. For the lowest temperature both the potential and the singlet free energy agree with the zero temperature result. For $T = 178\text{MeV}$ the singlet free energy is very different from the zero temperature potential, while the difference is small for the finite temperature potential. Furthermore, the in-medium potential does not show screening at distances smaller or equal to 1 fm. We fitted the potential with screened Coulomb plus linear form which gave us estimates for the screening length of about 2 fm. Screening effects become apparent in the potential at $T = 194\text{MeV}$ and happen at distances of about 1fm. At the same temperature the screening effects in the singlet free energy set in at distance of about 0.6fm. At higher temperatures screening effects in the potential set in at smaller and smaller distances and the difference between the potential and the singlet free energy becomes smaller. This is expected as at very high temperatures the singlet free energy should be equal to the potential [6]. At all temperatures the potential is larger than the singlet free energy, i.e. it seems to approach the singlet free energy from above. At the highest two temperatures our results are in qualitative agreement with the findings of the recent quenched study [9].

To study to what extent our conclusions depend on the choice of the correlators we compare the singlet free energy obtained from unsmeared and smeared Wilson loops as well as from Coulomb gauge correlators. The results are shown in Fig. 2. At low temperatures the results

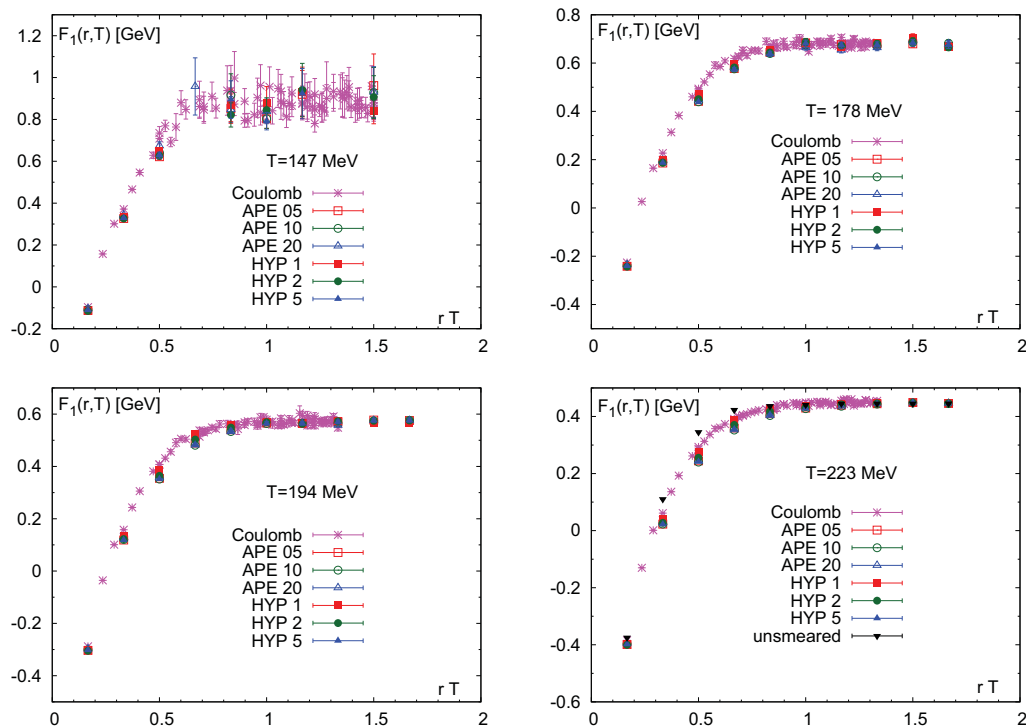


Figure 2. The singlet free energy at $T = 147$ MeV, 178 MeV, 194 MeV and 220 MeV calculated for different choices of the correlators.

obtained using unsmeared Wilson loops were too noisy and therefore we do not show them here. Smeared Wilson loops give results which are slightly smaller than Coulomb gauge results at intermediate distances, except for one iteration of HYP smearing. There is no dependence on the smearing if more than one smearing steps are used. The difference between the Coulomb gauge result and the results obtained using smeared Wilson loops is about the same or smaller than the errors of the extracted potential (c.f. Fig. 1). For $T = 222$ MeV we also show F_1 obtained from unsmeared Wilson loops, which turns out to be systematically larger than F_1 obtained in Coulomb gauge at intermediate distances. This could be due to the intersection divergences of cyclic Wilson loops [17]. At large distances the singlet free energy obtained from various correlators converges to the same value as expected.

3. Conclusion

We have calculated the correlation functions of Wilson lines at non-zero temperature for different τ and extracted the temperature dependent potential. The temperature dependent potential is always larger than the singlet free energy, approaching it from above. We do not see screening effects present in the potential for $T < 190$ MeV. The value of the extracted temperature potential at large distance is very close to the value of the phenomenological potential used in Ref. [7]. Our results are not very sensitive to the specific choice of the correlation function as long as Coulomb gauge Wilson line correlators and smeared Wilson loops are considered.

Acknowledgments

This work was supported by U.S. Department of Energy under Contract No. DE-AC02-98CH10886. The numerical simulations have been performed at NERSC and on BlueGene/L

computers at the New York Center for Computational Sciences (NYCCS) at Brookhaven National Laboratory.

- [1] T. Matsui and H. Satz, Phys. Lett. B **178** (1986) 416.
- [2] T. Umeda et al., Eur. Phys. J. C **39S1** (2005) 9; M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004); S. Datta et al., Phys. Rev. D **69**, 094507 (2004)
- [3] I. Wetzorke et al., Nucl. Phys. Proc. Suppl. **106**, 510 (2002).
- [4] F. Karsch, et al., Phys. Rev. D **68**, 014504 (2003)
- [5] P. Petreczky, J. Phys. G **39**, 093002 (2012)
- [6] N. Brambilla, et al, Phys. Rev. D **78**, 014017 (2008); Phys. Rev. D **82**, 074019 (2010)
- [7] P. Petreczky, C. Miao and A. Mocsy, Nucl. Phys. A **855**, 125 (2011)
- [8] A. Rothkopf et al, PoS LAT **2009**, 162 (2009); Phys. Rev. Lett. **108**, 162001 (2012);
- [9] Y. Burnier and A. Rothkopf, Phys. Rev. D **86** (2012) 051503; arXiv:1304.4154 [hep-ph]
- [10] C. Aubin et al., Phys. Rev. D **70**, 094505 (2004)
- [11] E. Follana et al., Phys. Rev. D **75**, 054502 (2007)
- [12] A. Bazavov et al., Phys. Rev. D **85**, 054503 (2012); J. Phys. Conf. Ser. **230**, 012014 (2010)
- [13] O. Kaczmarek et al., Phys. Lett. B **543**, 41 (2002)
- [14] P. Petreczky, J. Phys. G **37**, 094009 (2010)
- [15] O. Kaczmarek, PoS CPOD **07**, 043 (2007) [arXiv:0710.0498 [hep-lat]].
- [16] G. Aarts et al., Phys. Rev. Lett. **106**, 061602 (2011)
- [17] M. Berwein, N. Brambilla, J. Ghiglieri and A. Vairo, JHEP **1303** (2013) 069 [arXiv:1212.4413 [hep-th]].