

Rapidity Dependence of Transverse Momentum Correlations from Fluctuating Hydrodynamics

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Abstract. Interest in the development of the theory of fluctuating hydrodynamics is growing [1]. Early efforts suggested that viscous diffusion broadens the rapidity dependence of two-particle transverse momentum correlations [2]. That work stimulated an experimental analysis by STAR [3]. In this work we present results of correlation observables computed using the second order Israel-Stewart hydrodynamics with stochastic noise and latest theoretical equations of state and transport coefficients. We also compute these observables using the first Navier-Stokes theory. We find that the second order theory better describes the data and also provides some novel features forced by causal constraints of the second order theory.

1. Introduction

Experiments carried out at Relativistic Heavy Ion Collider (RHIC) provided some clear signatures of formation of a deconfined state of quark and gluons, or quark-gluon plasma (QGP) [4, 5]. It was concluded that QGP behaves like a nearly perfect liquid. The excellent agreement of predictions of relativistic ideal hydrodynamic models [6, 7] with experimental data [8, 9] led to RHIC to conclude that the QGP behaves like a nearly perfect fluid. The quantity often used to indicate the perfectness, or lack thereof, in relativistic hydrodynamics is the ratio of viscosity to entropy density η/s . For a perfect fluid, its value approaches toward zero. Current consensus, based on viscous hydrodynamic models and experimental data, on the values of this ratio for a quark-gluon plasma is in the range between the conjectured lower bound (the “KSS” bound) of $1/4\pi$ and ~ 0.3 [10]. Note that we are using the units in which $\hbar = k_B = c = 1$. These values of η/s are much smaller than, for example, its value the superfluid liquid helium ($= 0.7$). Anisotropic flow data of produced particles have been traditionally used to estimate η/s . In Ref. [2], Gavin and Abdel-Aziz proposed a different method for estimating this ratio. They argued that viscous diffusion results in broadening the rapidity distribution of transverse momentum correlations and by measuring the change in width between peripheral and central collisions, one can estimate the value of η/s . STAR carried out experimental analysis [3] on this suggestion found that $0.06 < \eta/s < 0.21$. Note that these values are consistent with the values found from flow data.

Our current theory is based on the second order Israel-Stewart viscous hydrodynamics. Using this hydrodynamics and techniques of stochastic methods, we develop a second order deterministic diffusion equation for two-particle transverse momentum correlation. The diffusion coefficient of the equation encapsulates the information on viscosity, more specifically



information on η/s . We use a general temperature dependent η/s based on kinetic theory and latest development in equations of state to construct a general temperature dependent diffusion coefficient. Clearly, our current goal is not to estimate η/s . The goal is to employ the current knowledge of η/s and our theory to compute how a fluctuating initial correlation responds to viscous hydrodynamic expansion. Evolution of correlation observables proposed in Ref. [2] gives a measure of the response. These observables are the same observables measured by STAR [3]. These observables give a measure of contribution of viscous expansion on short range correlations as well as the relevant part of the long range correlations called the ridge.

We organize this paper as follows. We start with a brief discussion of diffusion of transverse momentum fluctuation in Section 2. We then apply the concepts to the two-particle p_t correlations in Section 3 and briefly mention the way we obtain our equations. We also discuss the main observables in that section. Finally, in Section 5, we present our results and compare them with experimental data.

2. Diffusion of Transverse Flow Fluctuations

Equations of dissipative relativistic hydrodynamics are derived from the basic principles of conservations of charges and of energy-momentum. Depending on the order of gradients of flow included in the dissipative part of the energy momentum tensor, relativistic hydrodynamics is said to be of first or second order. The relativistic version of the well known Navier-Stokes hydrodynamics is a first order relativistic hydrodynamics. The second order relativistic hydrodynamics is the also known Israel-Stewart theory. If we take small fluctuations on quantities (like the flow u^μ) over equilibrium values, we can see that transverse modes decouple from the longitudinal modes. The longitudinal modes are propagating sound modes while the transverse modes are solely the diffusing shear modes. As we are more interested in shear viscosity, we focus on the transverse modes. Note that shear viscosity is the most important mode of dissipation in QGP. Starting with relativistic hydrodynamic equations and constitutive relation for shear tensor (see Ref. [11], for example, for such equations and relations) and then linearizing, one arrives at diffusion equations for a transverse momentum current. If δu^y is the fluctuation in transverse flow from the average value, we have $T_{0y} \approx (\epsilon_0 + p_0)\delta u^y$. Here, ϵ_0 and p_0 are the local equilibrium values of energy density and pressure. The first order hydrodynamics leads to

$$\frac{\partial \delta T_{0y}}{\partial t} = \nu \nabla_z^2 \delta T_{0y} \quad (1)$$

Where ν is given by

$$\nu = \frac{\eta}{\epsilon_0 + p_0} = \frac{\eta}{Ts} \quad (2)$$

Note that Eqn. (1) is a diffusion equation and that the diffusion coefficient contains the important ratio η/s . In our work, we refer this regular diffusion equation also as a first order diffusion equation. The diffusion coefficient ν is temperature dependent not just because of the explicit presence of T in Eqn. (2), but also because, as will be elaborated shortly, η/s itself is temperature dependent.

This diffusion equation, Eqn. (1), has a well known problem. Its solutions violate causality and is therefore not suitable for describing relativistic physics of quark-gluon plasma. It is a little ironic that the equation obtained from relativistic equation gives acausal solutions. It turns out that the first order gradient corrections contained in the relativistic Navier-Stokes equations are not accurate enough to preserve causality. The situation is resolved by resorting to second order hydrodynamics. For details on causality and diffusion see [11], [12], and the references therein. The second order hydrodynamics, on the other hand, gives us

$$\tau_\pi \frac{\partial^2 \delta T_{0y}}{\partial t^2} + \frac{\partial \delta T_{0y}}{\partial t} = \nu \nabla_z^2 \delta T_{0y} \quad (3)$$

We call this equation a second order diffusion equation. It is often referred to as causal diffusion equation, since it maintains causality. Note that the second order theory introduces the relaxation time τ_π and it is crucial in forcing the causality constraint in solutions to the equation. Note that in the limit $\tau_\pi \rightarrow 0$, we retrieve the first order diffusion equation. Note also that the second order equation has both diffusion and wave parts. The relative strength of the relaxation coefficient compared to the diffusion coefficient determines whether wave or diffusion behavior predominates.

3. Two-particle transverse momentum correlations

Correlation function of two-particle transverse momentum current can be defined as

$$r = \langle T_1^{0y} T_2^{0y} \rangle - \langle T_1^{0y} \rangle \langle T_2^{0y} \rangle \quad (4)$$

where $T_1^{0y} \equiv T^{0y}(\mathbf{x}_1)$ and $T_2^{0y} \equiv T^{0y}(\mathbf{x}_2)$ are momentum currents of particle pairs at points \mathbf{x}_1 and \mathbf{x}_2 , respectively. Considering small fluctuations about the average, $T^{0y} = \langle T^{0y} \rangle + \delta T^{0y}$ and treating them as stochastic quantities, we can write $r = \langle \delta T_1^{0y} \bar{T}_2^{0y} \rangle + \langle \bar{T}_1^{0y} \delta T_2^{0y} \rangle + \langle \delta T_1^{0y} \delta T_2^{0y} \rangle$. The bars over the letters represent the same average - the average over the ensembles of events. Note the inclusion of the last term here. This term becomes important in stochastic methods, and is not dismissed as a second order term. It is a stochastic noise term and vanishes in the absence of noise. Using Eqn. (3) and the expression for r and manipulating the terms we can subtract away the noise term to obtain [13]

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial t^2} + \frac{\partial \Delta r}{\partial t} = \nu (\nabla_{z1}^2 + \nabla_{z2}^2) \Delta r. \quad (5)$$

Here, $\Delta r = r - r_{eq}$, with r_{eq} as the equilibrium value of Eqn. (4). Note that, although diffusion equation for the correlation r contains noise term, the second order diffusion equation for Δr is a deterministic equation.

If one starts with the first order equation for single particle case, Eqn. (1), we obtain a regular deterministic diffusion equation for Δr :

$$\frac{\partial \Delta r}{\partial t} = \nu (\nabla_{z1}^2 + \nabla_{z2}^2) \Delta r. \quad (6)$$

The last equation, Eqn. (5), was first obtained in Ref. [2] and is the basis of the alternative method of estimating η/s of quark-gluon plasma that we mentioned in Section 1. The idea proposed in that reference is to measure the broadening of the width of Δr (strictly, the width of the correlation observable \mathcal{C} in relative rapidity, discussed below).

We have developed the diffusion equations in such a way as to focus on the longitudinal expansion with Bjorken boost invariance. This is, clearly, a simplified part of a more general three dimensional problem. The RHIC and LHC experiments have provided us complex two-particle correlation profiles consisting of ridges, valleys and peaks. The goal of the future extended version of our current theory is to understand the contribution of viscous hydrodynamic expansion in those correlation profiles. Currently, we limit ourselves to the correlation profiles only in rapidity window, not the correlations in azimuthal angle. However, as we will point out below that, as far as our integral correlation observables are concerned, we can leave out transverse expansion and consider only the longitudinal expansion. It turns out that the transverse expansion integrates out and becomes redundant.

Bjorken longitudinal boost invariance can be applied in the simplest way by writing equations in terms of rapidity and proper time. Bjorken boost invariant velocity $v_z = z/t$ is like one diminutional Hubble expansion along z direction. The suitable coordinate is the Milne

coordinates $x^\mu = (\tau, x, y, \eta)$. Here τ and η are the longitudinal proper time and spacetime rapidity respectively: $\tau = \sqrt{t^2 - z^2}$ and $\eta = (1/2) \ln((t+z)/(t-z))$.¹ For longitudinal Bjorken expansion, fluid rapidity reduces to spacetime rapidity. In Minkowski coordinates $x^\mu = (t, x, y, z)$, Bjorken boost invariant flow is given by $\bar{u}^\mu = \gamma(1, 0, 0, z/t) = (t/\tau, 0, 0, z/\tau) = (\cosh \eta, 0, 0, \sinh \eta)$. In Milne coordinates, it becomes $u^\mu = (1, 0, 0, 0)$. Note that expression of Bjorken flow cannot get any simpler. A fluid particle expanding longitudinally with Bjorken flow is at rest in Milne coordinates. The affine connections (Christoffel symbols), however, are not all zero. The non-zero elements are $\Gamma_{\eta\tau}^\eta = 1/\tau$ and $\Gamma_{\eta\eta}^\tau = \tau$. Now we can use this to find the expansion rate $\nabla_{;\mu} u^\mu \rightarrow \nabla_{;\mu} u^\mu = \partial_{;\mu} u^\mu = \partial_\mu u^\mu + \Gamma_{\mu\nu}^\mu u^\nu = \Gamma_{\eta\tau}^\eta u^\tau = 1/\tau$, for example. We can use this coordinate system to derive our diffusion equations in terms of proper time and spacetime rapidity. The second order equation, Eqn. (5), takes the form

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial \tau^2} + \frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2} (\nabla_{\eta_1}^2 + \nabla_{\eta_2}^2) \Delta r. \quad (7)$$

Clearly, η_1 and η_2 are respective spacetime rapidities for particle 1 and particle 2. Recall that we are not including the transverse expansion. Similarly, the first order equation takes the form

$$\frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2} (\nabla_{\eta_1}^2 + \nabla_{\eta_2}^2) \Delta r \quad (8)$$

The last two equations, Eqns. (8) and (7), are solved numerically and the solutions Δr 's are used to compute the observables. The main observable is the correlation profile \mathcal{C} given by (see Ref. [2] for detail):

$$\mathcal{C} = \langle N \rangle^{-2} \int \Delta r(\mathbf{x}_1, \mathbf{x}_2) d^3 x_1 d^3 x_2 \quad (9)$$

Again, following the Ref. [2], this observable is related to experimentally measurable p_t covariance:

$$\mathcal{C} = \langle N \rangle^{-2} \left\langle \sum_{i \neq j} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2 \quad (10)$$

The last two relations give the connection between our theoretically computed Δr to experimentally measurable two-particle transverse momentum correlations.

The other observable is the width of \mathcal{C} . STAR has reported \mathcal{C} and its widths for various centralities [3]. In Section 5, we present the results of our computation and compare them with the STAR data. Before that, we would like to elaborate on the temperature dependent η/s in the following section.

4. Viscosity and entropy density

Viscosity and relaxation coefficients are determined from kinetic theory. In Ref. [14], Hirano and Gyulassy have combined the latest information on η : results from perturbative QCD, kinetic theory, and $\mathcal{N} = 4$ Super Yang-Mills (SYM) at infinite coupling strengths, in the respectively applicable temperature ranges. They also propose that low values of η/s for quark-gluon plasma is not because viscosity suddenly drops in QGP phase, but because the entropy density increases when the system makes a transition to QGP from hadronic phase. We use the expressions for shear viscosity given in that reference. As for the entropy density, we make use of the values calculated using lattice QCD. Specifically, values given in s95p-v1 from Ref. [15, 16] have been used in this work. We refer to this equation of state as EOS I here. Another equation of state we use here is the standard equation of state based on the bag model, also summarized in [14].

¹ Note that the same letter η is used for shear viscosity and for the longitudinal spacetime rapidity here. The meaning should be clear from the context of its use.

With the information on $\eta(T)$ and $s(T)$, we construct the ratio of viscosity to entropy density $\eta/s(T)$. This gives us the temperature dependent diffusion coefficient $\nu(T)$. In order to get ν as a function of proper time, necessary to make our correlation Δr evolve, one needs entropy density as a function of time. We employ the entropy production equations for this purpose. It should be noted that entropy production equations are obtained from the same hydrodynamic equations and, therefore, can be of first or second order. The general equation, with Bjorken longitudinal boost invariance, can be written as [11]:

$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\pi}{T\tau}, \quad (11)$$

where π is the entropy production current, which comes from shear viscosity. In Milne coordinates: $\pi = \pi_\eta^\eta$ (or $\pi = \pi_0^0 - \pi_z^z$ in Minkowski coordinates). For an ideal fluid, $\pi = 0$, and we obtain the well known Bjorken relation $s(\tau) = s_0\tau_0/\tau$. For Navier-Stokes or first order theory, $\pi = 4\eta/3\tau$ and for the second order theory π is given by the differential equation

$$\tau_\pi \frac{d\pi}{d\tau} + \left(1 + \frac{\tau_\pi}{2\tau} + \frac{1}{2}\eta T \frac{d}{d\tau} \left(\frac{\tau_\pi}{\eta T}\right)\right) \pi = \frac{4\eta}{3\tau}. \quad (12)$$

Since the second order diffusion and entropy production equations are obtained from the same second order Israel-Stewart hydrodynamics, we are speaking of the same relaxation time.

Like viscosity, relaxation time is obtained from kinetic theory. The latest and comprehensive treatment on kinetic theory estimate of relaxation times are in Ref. [17]. This reference gives $\tau_\pi = \beta \frac{\eta}{T s}$ with $\beta = 5 - 6$. The value of β suggested in Ref. [18] is 6.32. These values of β are significantly higher than those which are used in hydrodynamic calculations by some other groups (for example, $\beta = 3$ in Ref. [19]). We, however, adopt the kinetic theory values. In our work here, we have used $\beta = 6$.

Before we present our results in Section 5, we would like to discuss the redundancy of transverse expansion for our observables. Here is a simple, but general, argument based on conservation of energy-momentum, $\partial T^{\mu\nu} = 0$. Integrating it over \mathbf{r}_\perp , which is, effectively, the same as integrating over the azimuthal angle ϕ , we obtain

$$\int d\mathbf{r}_\perp \partial_0 T^{0\nu} + \int d\mathbf{r}_\perp \partial_i T^{i\nu} = 0 \quad (13)$$

Let us take y as a transverse component, as before ($\nu = y$). We get

$$\begin{aligned} 0 &= \int dx dy \partial_0 T^{0y} + \int dx dy \partial_i T^{iy} \\ &= \int dx dy \frac{\partial T^{0y}}{\partial t} + \int dx dy (\partial_x T^{xy} + \partial_y T^{yy}) + \int dx dy \frac{\partial T^{zy}}{\partial z} \end{aligned} \quad (14)$$

The integral of the middle term in the second line is a surface integral of a gradient. If we take the surface to “infinity”, i.e., to a large r_\perp , this term vanishes. Note that this is the very term that contains the transverse expansion. The last one is the longitudinal expansion term. Upon linearization and using $T_{zy} = -\eta \partial_z v^y$, we obtain a diffusion equation. Thus, a three dimensional diffusion equation when averaged over \mathbf{r}_\perp gives our longitudinal diffusion equations.

Thus, even if we do not include the transverse expansion, our results on the observable \mathcal{C} and its widths are general in nature, as long as the conditions for the vanishing surface terms are valid.

5. Results and discussion

We now present the results of our computation. Before that let us mention some of the parameters used in the theory. It is already mentioned that we have used $\beta = 6$. The initial or thermalization time is taken to be $\tau_0 = 1$ fm/c. Initial Δr is taken to be a gaussian in relative rapidity $\Delta\eta$, with initial width $\sigma_0 = 0.54$, making it consistent with experimental data for the most peripheral collisions. We stop the hydrodynamic evolution at a constant temperature of 150 MeV. Glauber model has been used to connect impact parameters and multiplicity of the collisions. The freezeout proper time is assumed as $\tau_F - \tau_0 \propto (R - R_0)^2$, with R as the rms participant radius. Freezeout time for most central collisions is taken to be 9 fm/c. The computed σ and \mathcal{C} are compared with the values measured recently by STAR [3].

Figure 1 shows widths versus centralities computed using both first and second order theory with two different equations of state - EOS I and EOS II (equations of state used in our computations are discussed in Section 4). We observe that second order calculations agree very well with the experimental data. We also note that the choice of EOS makes a very small difference. However, there is significant difference between the first and second order diffusion, except for a few most central cases. The deviation of the first order results from data relates to how the relaxation time compares with the evolution time [13]. This clearly shows that one needs causally constrained hydrodynamic evolutions to better explain the experimental data. Figure 1 also compares results with NEXSPHERIO calculations [20]. NEXSPHERIO uses inviscid hydrodynamics in evolution of initial correlations. It reproduces most qualitative observed features of the two-particle correlations. It, however, does not reproduce the broadening of the rapidity width with increasing centrality, as one can see in Fig. 1. We attribute that to the absence of viscosity. Other explanations involving collective initial state behavior are also possible. One important reason for comparing with the NEXSPHERIO results is to look for any broadening effect of resonances, which is included in that computation. Since we do not observe any broadening in NEXSPHERIO results, we infer that resonances, a source of short range correlations, do not have important, if any, effect in our observables.

Figure 2 displays the observable \mathcal{C} vs $\Delta\eta$ for various centralities from our numerical calculations and the values measured by STAR [3]. The STAR measured values have error bars in the offsets (ridge), which we have not shown here. This figure shows a good agreement of the numerical results with the experiments. The single point at $\Delta\eta = 0$ in the most central case has been attributed to the track merging and is just a detector artifact. One important observation we make here is the flattening of the peak (and to some extent double humps) in \mathcal{C} . The reason for the double hump (or rather the flattened peak) is a second order diffusion effect. We note that the second order diffusion equation has propagating wave part as well. Thus there are competing wave and diffusion effects, depending on the size of τ_π and ν . Wavefronts propagate in opposite directions and diffusion fills in the space in between. This can be seen more effectively in coordinate space (not shown here). In rapidity space, width saturates since the effective diffusion coefficient ν/τ^2 in Eqn. (7) decreases rapidly with time. The first order diffusion obviously does not show such humps and flattening - there are no propagating waves. Figure 3 compares \mathcal{C} obtained from the solutions of the first and second order theory. It can be observed that STAR data for other centralities [21] show the humps exactly at the same centralities. Shapes show fair agreement with the calculations. Here we have restricted ourselves with the published data only for the three centralities shown in Fig 2. Finally, we just want to mention that the order of entropy production equations does not make any discernible difference in our results. Note that we have relatively large β in $\tau_\pi = \beta\eta/Ts$.

It would be interesting to measure the observable \mathcal{C} from the p-A [22] and Pb-Pb [23] collisions data from LHC. In the pA case, it would be interesting to see whether there is a broadening with respect to pp. Experiments have identified a ridge in pA [22]. If hydrodynamics is applicable in the pA system, then viscous diffusion would broaden the rapidity width of \mathcal{C} .

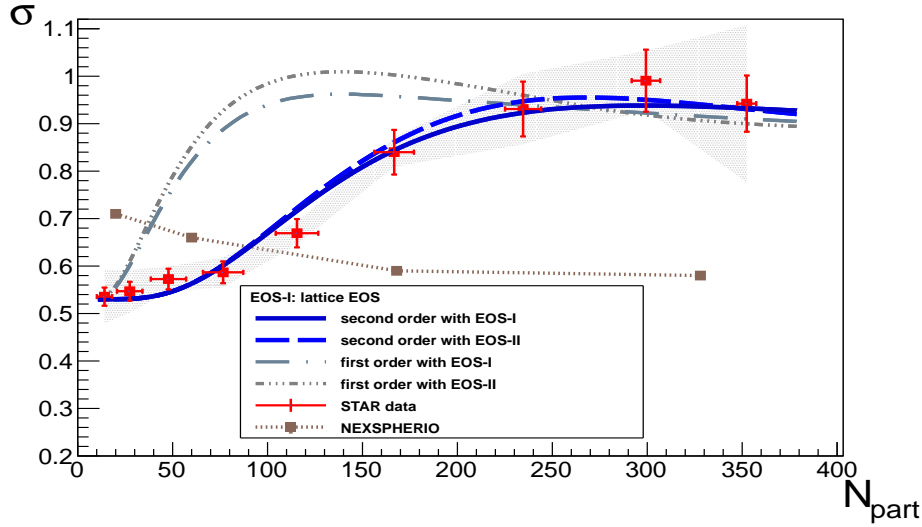


Figure 1. Second order, first order, with both EOS [13] and NeXSPHeRIO [20] calculations of width vs STAR [3] measurements.

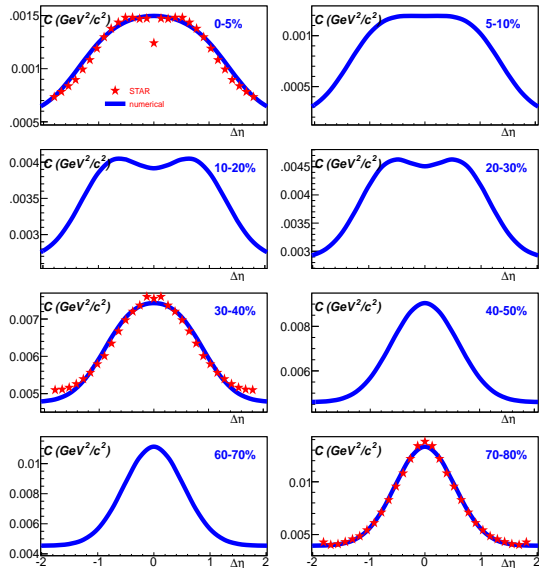


Figure 2. \mathcal{C} vs $\Delta\eta$. Comparison with STAR data at different centralities. [3].

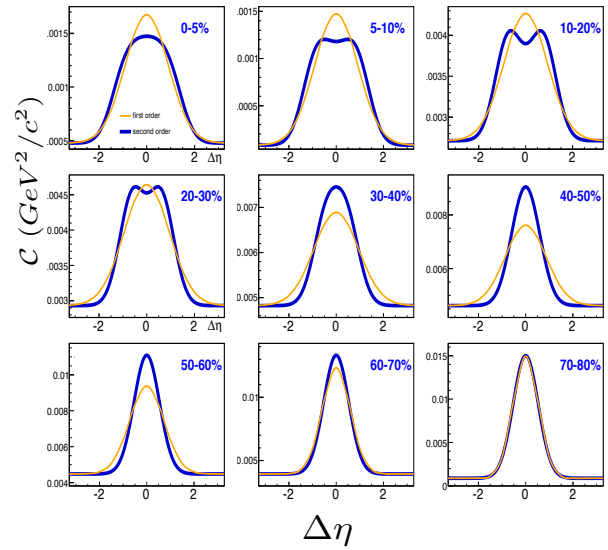


Figure 3. Profiles of correlation observable \mathcal{C} obtained from the solutions of the first and second order diffusion equations.

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