Shubnikov-de Haas oscillations in high mobility GaAs quantum wells in tilted magnetic fields

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Abstract. We report on quantum magneto-oscillations in a high mobility GaAs/AlGaAs quantum well at very high ($\geq 87^\circ$) tilt angles. Unlike previous studies, we find that the spin and cyclotron splittings become equal over a *continuous range* of angles, but only near certain, angle-dependent filling factors. At high enough tilt angles, Shubnikov-de Haas oscillations reveal a prominent beating pattern, indicative of *consecutive* level crossings, *all* occurring at the same angle. We explain these unusual observations by an in-plane field-induced increase of the carrier mass, which leads to accelerated, filling factor-driven crossings of spin sublevels in tilted magnetic fields.

1. Introduction

When a two-dimensional electron system (2DES) is subject to a perpendicular magnetic field $B_\perp$, its low temperature magnetoresistivity is known to exhibit Shubnikov-de Haas oscillations (SdHO). SdHO originate from the magnetic quantization of the energy spectrum,

$$E_{N,\mp} = \hbar\omega_c\,(N+1/2) \pm \Delta_s/2\,, \tag{1}$$

where $\hbar\omega_c = \hbar e B_\perp/m^\star$ is the cyclotron energy, $m^\star$ is the effective mass, N is the Landau level index, and Δ_s is the sum of the single particle Zeeman energy Δ_Z and the exchange energy Δ_X. The resistivity of the 2DES senses the density of electron states at the Fermi level, ε_F, and will show a minimum whenever ε_F falls in the middle between the energy levels, see Eq. (1), i.e., when the filling factor $\nu = 2\varepsilon_F/\hbar\omega_c$ acquires an integer value.

The strength of the SdHO at odd and even ν is determined by the *spin splitting*, $\Delta_{odd} = \Delta_s$, and by the *cyclotron splitting*, $\Delta_{even} = \hbar\omega_c - \Delta_s$, respectively. Furthermore, the exchange contribution to the spin splitting becomes important only when there is an appreciable population difference between $(N,+)$ and $(N,-)$ sublevels. The latter turns off rather abruptly at some critical, disorder-dependent filling factor ν_s. The associated disappearance of the odd-ν SdHO minima is referred to as a critical collapse of the exchange-enhanced spin splitting [1, 2, 3, 4, 5]. At odd $\nu \gg 1$, but considerably lower than ν_s, the exchange energy scales with the cyclotron energy $\Delta_X = \alpha_X\hbar\omega_c$ [1, 6], while at even ν, one can set $\Delta_X \approx 0$ and

$\Delta_s \approx \Delta_Z = \alpha_Z \hbar\omega_c$. Since in GaAs $\alpha_Z \ll 1$, $\alpha_X < 1$[1] and, most importantly, since there are no critically collapsing contributions to $\Delta_{\rm even}$, the SdHO at even-ν persist to filling factors much larger than ν_s.[2]

Adding an in-plane magnetic field will increase the Zeeman energy since the latter scales with the total field. On the other hand, one usually assumes that both the cyclotron and the exchange energies depend only on $B_\perp$ [2, 4].[3] As a result, when the magnetic field is tilted by angle θ away from the sample normal, for any given ν, the cyclotron splitting, $\Delta_{\rm even} \propto 1-\alpha_Z/\cos\theta$, will decrease with θ and eventually approach the increasing spin splitting, $\Delta_{\rm odd} \propto \alpha_X + \alpha_Z/\cos\theta$. This approach forms a basis of the *coincidence method* [7], which was successfully used to study the exchange contribution [2, 8]. The "coincidences", manifested as equally strong even-ν and odd-ν SdHO, will occur when[4] $\Delta_{\rm even} = \Delta_{\rm odd}$ or

$$\gamma_i \equiv \frac{1}{\cos\theta_i} = \frac{i-\alpha_X}{2\alpha_Z}\,, \quad i = 1, 3, 5, \dots . \tag{2}$$

Equation (2),[5] dictating that the coincidences should occur *simultaneously* for all ν, was confirmed experimentally [2] for $i = 1$, $\theta_1 \approx 87.2^\circ$ ($\gamma_1 \approx 20$).

In this work we report on spin-resolved SdHO in an ultra-high mobility GaAs/AlGaAs quantum well at very high tilt angles ($\theta \geq 87^\circ$). We find that the evolution of the SdHO waveform depends sensitively not only on θ, as predicted by Eq. (2), but also on the filling factor. More specifically, we find that coincidence conditions, $\Delta_{\rm even} = \Delta_{\rm odd}$, can be satisfied *over a range of tilt angles* but only at certain filling factors. These filling factors monotonically increase with θ and eventually diverge at $\theta = \theta_i$. At $\theta > \theta_1$, SdHO reveal a beating pattern indicating *cosecutive* ($i = 3, 5$), filling factor-driven coincidences, occurring at the same θ. These findings are in contrast with Eq. (2) and previous experiments [2] and can be explained by an in-plane field-induced increase of the effective mass due to the finite width of our 2DES. This increase leads to a non-monotonic dependence of the cyclotron splitting on ν and its eventual collapse resulting in accelerated crossings of spin sublevels.

2. Experimental details

Our sample is a 200 μm-wide Hall bar fabricated from a 29 nm-wide GaAs-$\mathrm{Al_{0.24}Ga_{0.76}As}$ quantum well, grown by molecular beam epitaxy, with density $n_e \approx 2.85 \times 10^{11}$ cm^{-2} and mobility $\mu \approx 2.4 \times 10^7$ cm^2/Vs . The magnetoresistivity was recorded using a standard low-frequency (a few Hertz) lock-in technique at temperature $T \approx 0.3$ K. The tilt angle θ, controlled *in situ* by a precision rotator, was held constant during each magnetic field sweep.

3. Results and discussion

In Fig. 1 we present the oscillatory part of the resistivity $\delta\rho$ (left axes), obtained from the total resistivity by subtracting a slowly varying background, versus ν at different tilt angles θ, as marked. The corresponding amplitudes, $\delta\rho_{\rm max}$, at even (circles) and odd (squares) filling factors are shown on the right (logarithmic) axes. These amplitudes were evaluated using $\delta\rho$ values at the minima and the average of the neighboring maxima.

[1] According to Ref. [6], $\alpha_X = \ln(k_F a_B)/\pi k_F a_B$, where k_F is the Fermi wave number, and a_B is the effective Bohr radius. In our 2DES we estimate $\alpha_X \approx 0.234$.

[2] While odd-ν minima disappear abruptly at $\nu = \nu_s$, the even-ν oscillations decay roughly exponentially with ν.

[3] While (fully-enhanced) $\Delta_X \propto B_\perp$, ν_s will increase with $\Delta_Z \propto \gamma$ making the exchange contribution relevant at progressively higher ν with increasing θ (Refs. [2], [4]).

[4] Equation (2) for $i = 1$ coincides with Eq. (11) of Ref. [2] if α_X is replaced by $2\alpha_X$. The extra factor appears because in Ref. [2] Δ_X is included in both $\Delta_{\rm odd}$ and $\Delta_{\rm even}$.

[5] Equation (2) should also be approximately valid at even i, corresponding to level crossings, if one takes the exchange part to be only partially ($\simeq 50\%$) enhanced (Ref. [8]).

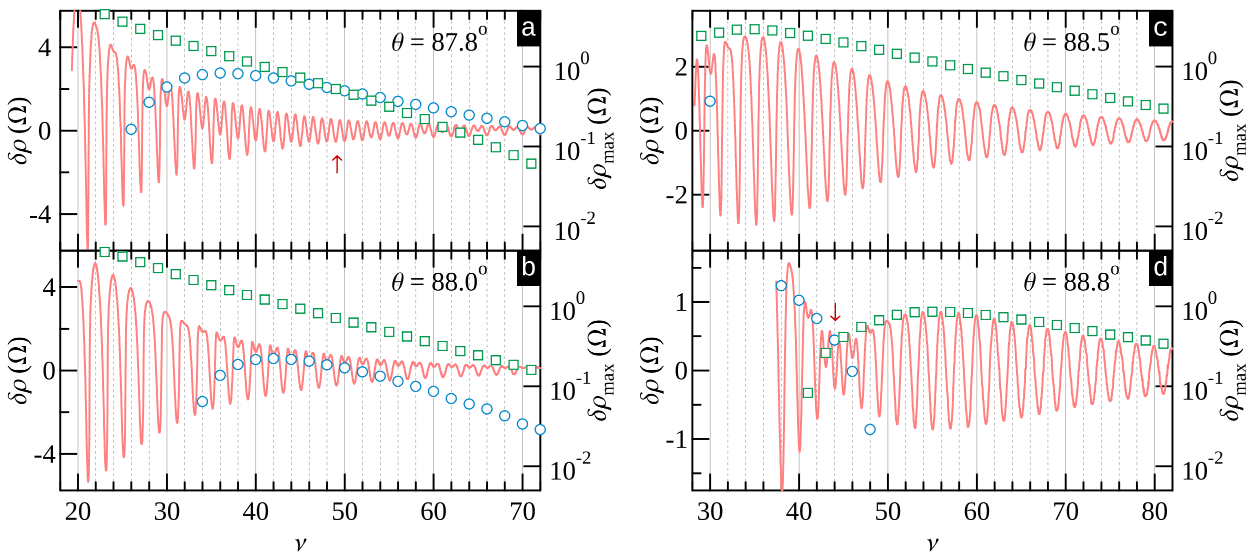

Figure 1. Oscillatory part of the resistivity $\delta\rho$ (left axis) and amplitude $\delta\rho_{\max}$ (right axis) versus the filling factor ν measured at tilt angles (a) $\theta \approx 87.8°$, (b) $\theta \approx 88.0°$, (c) $\theta \approx 88.5°$, and (d) $\theta \approx 88.8°$. Even-ν (circles) and odd-ν (squares) amplitudes become equal at $\nu = \nu_1$ (cf. $\uparrow$) and then again at $\nu = \nu_3$ (cf. $\downarrow$).

At the lowest θ, Fig. 1(a), we observe that the amplitudes at odd-ν and even-ν become equal at $\nu_1 \approx 49$ (cf. $\uparrow$). The data at lower and somewhat higher θ (not shown) reveal that ν_1 increases with θ. At lower filling factors, $\nu < \nu_1$, the odd-ν amplitude continues to grow but the even-ν amplitude first saturates, then starts to *decrease*, and eventually disappears at $\nu = \nu_c \approx 24$.

The observed amplitude crossing indicate that at $\nu = \nu_1$ the spin splitting becomes equal to the cyclotron splitting. On the other hand, the vanishing of the even-ν amplitude at $\nu = \nu_c$ signals a collapse of the cyclotron gap, which is a precursor of the crossing of spin sub-levels from the neighboring Landau levels. Such crossing would occur when $\Delta_{\rm even} = \hbar\omega_c - \Delta_Z = 0$, so the observed collapse indicates that Δ_Z is growing faster than $\hbar\omega_c$ with decreasing ν. We further notice that the exchange energy at odd filling factors should be close to its maximum value, $\alpha_X \hbar\omega_c$ since, at $\nu < \nu_1$, the separation between spin-split maxima is close to unity and the odd-ν amplitude shows excellent exponential dependence. Finally, $\gamma\alpha_Z$ is no longer a small fraction and Δ_Z gives a significant contribution to the spin splitting.

At a higher tilt angle, Fig. 1(b), we find that the even-ν amplitude remains considerably smaller than the odd-ν amplitude over the whole range of ν and the crossing never takes place. At still higher angle, Fig. 1(c), the even-ν SdHO disappear completely indicating that the condition for the first level crossing is approximately satisfied for all filling factors under study. However, closer examination of Fig. 1(c) reveals a maximum of the odd-ν amplitude at $\nu \approx 35$, and the re-emergence of the even-ν SdHO to the left of this maximum, indicating that at these filling factors the crossing of spin sublevels has already occurred.

The data at the next tilt angle, Fig. 1(d), reveal a node which occurs at $\nu = \nu_3$ (cf. $\downarrow$). Similar to the "coincidence" at $\nu = \nu_1$, this node moves to progressively higher ν with increasing θ. Passing through the node from high to low filling factors, odd-ν SdHO decay away while the even-ν SdHO set in. As a result, at $\nu \approx \nu_3$ the amplitudes of the odd-ν and even-ν SdHO again become close to each other, reflecting equal gaps, similar to the situation at $\nu \approx \nu_1$ (cf. Fig. 1(a)).[6] While the SdHO shown in Fig. 1(d) differs in detail from those shown in Fig. 1(a)

[6] The data at higher angles [9] reveal a node at $\nu = \nu_5$, which signals the next crossing.

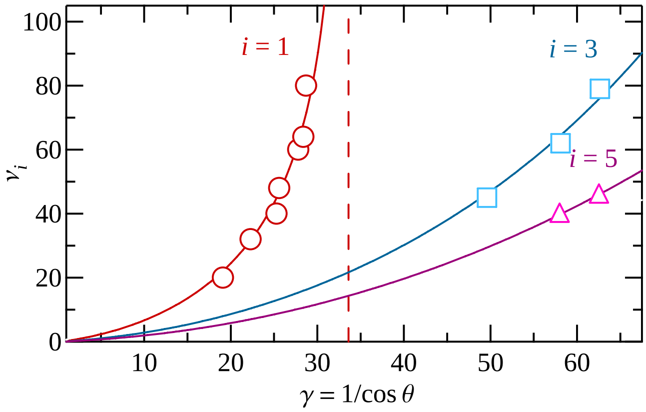

Figure 2. Filling factors ν_i for $i = 1$ (circles), $i = 3$ (squares) and $i = 5$ (triangles) corresponding to crossings of the even-ν and odd-ν amplitudes as functions of $1/\cos\theta$. Solid lines are fits to the data with Eq. (4). The dashed vertical line is the asymptote of Eq. (4) for $i = 1$ coincidence at large ν. This line also corresponds to θ_1 given by Eq. (2). Adapted from Ref. [9].

and Fig. 1(b), as we show next, all observations can be attributed to subsequent level crossings. Clearly, the observed behavior cannot be described by Eq. (2), which is independent of ν.

To explain our findings we consider finite thickness effects which, in combination with $B_\parallel$, can lead to an increase of the effective mass. This increase was studied theoretically [10, 11, 12, 13, 14, 15, 16] and confirmed in experiments examining the temperature damping of the SdHO [17] and the shift of magnetoplasmon resonances [18, 19]. Following Ref. [19], we use the expression for a parabolic potential [10, 16] and substitute the intersubband splitting of our quantum well, ε_{10}, for the intersubband spacing in the parabolic well. At $\gamma \gg 1$, the effective mass increases with respect to its value at $\theta = 0$ by a factor $1/\alpha_\gamma$, which is given by [10, 16]

$$\frac{1}{\alpha_\gamma} = \frac{m^\star_\theta}{m^\star_0} \approx \sqrt{1 + \left(\frac{\beta\gamma}{\nu}\right)^2}, \tag{3}$$

where $\beta = 2\varepsilon_F/\varepsilon_{10}$ and ε_F is evaluated at $\theta = 0$. Since β is of the order of unity,[7] a significant mass increase is expected at $\nu < \gamma$. This increase will result in a sublinear dependence of the cyclotron energy on $1/\nu$ and its eventual saturation at $\hbar\omega_c = \varepsilon_{10}/\gamma \ll \varepsilon_{10}$, in the limit of $\nu \ll \gamma$. As a result, at $\theta \neq 0$, the cyclotron splitting, $\Delta_{\rm even} \equiv \hbar\omega_c - \Delta_Z \propto [\alpha_\gamma(\nu) - \alpha_Z\gamma]/\nu$, is no longer a monotonic function, giving rise to the ν-dependence of the even-ν SdHO amplitude, see Fig. 1. Coincidence conditions, which take into account the mass increase, Eq. (3), can now be obtained by replacing i with $i\alpha_\gamma$ in Eq. (2) and solving for ν, which results in

$$\nu_i \approx \frac{\beta\gamma}{\sqrt{i^2/\left(2\alpha_Z\gamma + \alpha_X\right)^2 - 1}}. \tag{4}$$

Tilting the sample with respect to the magnetic field is also known to affect α_Z due to the anisotropy of the g-factor. Recent experiments on electron-spin resonance have shown that $g_0 = \sqrt{g_\perp^2 \cos^2\theta + g_\parallel^2 \sin^2\theta}$, where $g_\perp = 0.414$ and $g_\perp = 0.340$ [20]. At $\gamma \gg 1$, we obtain $g_0 \approx g_\parallel = 0.340$. It is also known that at high magnetic fields quadratic in B corrections to Δ_Z become important [20, 21, 22]. These terms can be incorporated into the g-factor which acquires linear B dependence, $g = g_0 - aB$. At $\theta \neq 0$, $a \approx (\cos\theta/g)(g_\perp a_\perp \cos^2\theta + g_\parallel a_\parallel \sin^2\theta)$, where $a_\perp \sim a_\parallel \sim 0.01$ T^{-1} [20]. At $\gamma \gg 1$, $a \approx a_\parallel/\gamma \ll a_\parallel$, and the correction to the g-factor, aB, remains less than 1% in our experiment even at the highest magnetic fields. We will therefore assume a B- and θ-independent g-factor, $g = g_0 = 0.34$ in our analysis.

To demonstrate that our findings can be described by Eq. (4), we present in Fig. 2 the filling factors ν_1 (circles), ν_3 (squares), and ν_5 (triangles) corresponding to crossings of the even-ν

[7] Using infinite square well approximation for our 2DES gives $E_{10} \approx 225$ K and $\beta \approx 1.1$ ($E_F \approx 125$ K).

and odd-ν amplitudes as functions of γ. Solid lines are fits to the data using Eq. (4) with $\alpha_Z \equiv g_0 m^\star/2m = 0.0114$, $\alpha_X = 0.234$, and β as a fitting parameter. For ν_1 the fit generates $\beta \approx 1.3$, which is close to our estimate of $\beta = 1.1$. The vertical line is drawn at $\theta = \theta_1$, calculated using Eq. (2). For ν_3 and ν_5 the fits produce $\beta \approx 1.8$ and $\beta \approx 2.1$, respectively. These values are somewhat higher than our estimate, which likely reflects the limitations of our model.

4. Summary

In summary, we have shown that in a typical high mobility GaAs/AlGaAs quantum well, the Shubnikov-de Haas oscillations in tilted magnetic fields cannot be described by a simple series of level crossings at a set of angles defined by Eq. (2). Instead, multiple level crossings can be realized for a given angle, but only near some characteristic filling factors, see Eq. (4) and Fig. 2. These findings can be explained by finite thickness effects, which give rise to an effective mass increase induced by an in-plane magnetic field. Finally, we mention that an in-plane magnetic field can also lead to the overall suppression of the Shubnikov-de Haas oscillations [23].

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