

# Shubnikov-de Haas effect in tilted magnetic fields in wide quantum well

I A Larkin<sup>1</sup>, Sebastian Ujevic<sup>2</sup>, S Wiedmann<sup>3</sup>, N Mamani<sup>4</sup>,  
G M Gusev<sup>5</sup>, A K Bakarov<sup>6</sup> and J C Portal<sup>7</sup>

**Abstract.** We study the magneto-resistance of a two-dimensional electron systems in a wide quantum well. We have considered two occupied subbands subjected to the tilted magnetic field in the regime of the magneto-inter-subband oscillations. We report on both experimental and theoretical studies of such a phenomenon and deduced information on the inter-subband energy gap in the presence of the in-plane magnetic field.

<sup>1</sup> Department of Physics, Minho University, Braga 4710-057, Portugal

<sup>2</sup> Departamento de Ciências Exatas - EEIMVR, Universidade Federal Fluminense, 27255-125, Volta Redonda, RJ, Brazil

<sup>3</sup> Radboud University Nijmegen, Institute for Molecules and Materials, High Field Magnet Laboratory, Toernooiveld 7, 6525 ED Nijmegen, the Netherlands

<sup>4</sup> Instituto de Física de São Carlos, Universidade de São Paulo, 13560-970 São Carlos, SP, Brazil

<sup>5</sup> Instituto de Física da Universidade de São Paulo, CP 66318, São Paulo, SP, Brazil

<sup>6</sup> Institute of Semiconductor Physics, Novosibirsk 630090, Russia

<sup>7</sup> Laboratoire National des Champs Magnétiques Intenses, CNRS-UJF-UPS-INSA, 38042 Grenoble, France; INSA Toulouse, 31077 Toulouse Cedex 4, France; Institut Universitaire de France, 75005 Paris, France

## 1. Introduction

The electron transport of a two-dimensional (2D) electron gas subjected to a magnetic field exhibits different types of magneto-resistance oscillations. Among them, the Shubnikov-de Haas oscillation (SdHO) are periodic in inverse magnetic field  $1/B$  and controlled by the filling factor  $\nu = E_F/\hbar\omega_c$ , where  $E_F$  is the Fermi energy,  $m$  is the electron effective mass and  $\omega_c = eB/mc$  is the cyclotron frequency. SdH oscillations appear only at low temperature, and are strongly damped when thermal broadening of the Fermi distribution exceeds the cyclotron energy. In quantum wells with two occupied 2D subbands, the magneto-resistance exhibits another kind of oscillating behavior, the so-called magneto-inter-subband (MIS) oscillations [1]. These oscillations arise from the periodic modulation of the probability of transitions between the Landau levels belonging to different subbands. The MIS oscillation is periodic in  $\Delta/\hbar\omega_c$ , where  $\Delta$  is the subband separation. Since the origin of the MIS oscillations is related to the alignment between the different Landau levels (LL) of the two subbands and not to the position of LL with respect to the Fermi energy, these oscillations survive at high temperatures when the SdH oscillations are suppressed. The experimental studies of MIS oscillations have been carried out in single quantum wells with two populated 2D subbands [2]. Recently, MIS oscillations with large amplitudes have been observed and investigated in two-subband systems based on



double quantum wells [3], triple quantum wells [3] and wide quantum wells slit into layers electrostatically [4]. MIS oscillations are used to obtain information about electron-electron scattering and quantum life time at high temperatures when SdH oscillations are completely damped in the region of weak magnetic fields.

Another advantage of the MIS oscillations is the possibility of a determination of the inter-subband energy gap  $\Delta$ . Recently it has been predicted that in high Landau levels the tunneling gap oscillates with the in-plane magnetic field [5], which has been confirmed experimentally in double quantum wells [6, 7] in the quantum Hall effect regime. At low magnetic field, the variation of the gap  $\Delta$  has been interpreted as Aharonov-Bohm interference effect between cyclotron orbits in different layers [6, 8]. However, the theoretical model [9] describes the interlayer resistivity in the parallel magnetic field, while in the experiments [7] the longitudinal resistance parallel to the layers has been measured. Therefore the behavior of the SdH and MIS oscillations in the presence of the in-plane magnetic field requires further theoretical study and detailed comparison with the experiments.

In the present paper we study the magneto-resistance in the wide GaAs quantum well subjected to a tilted magnetic field. The charge distribution in a wide single quantum well is more subtle than the one in the double quantum well. Here the Coulomb repulsion of the electrons in the well leads to a soft barrier inside the well, which in turn results in a bilayer electron system [Fig. 1(b)]. We performed analytical and numerical calculations of the symmetric-antisymmetric level separation in the presence of the in-plane magnetic field and make detailed comparison with the behavior of the MIS oscillations in a wide quantum well.

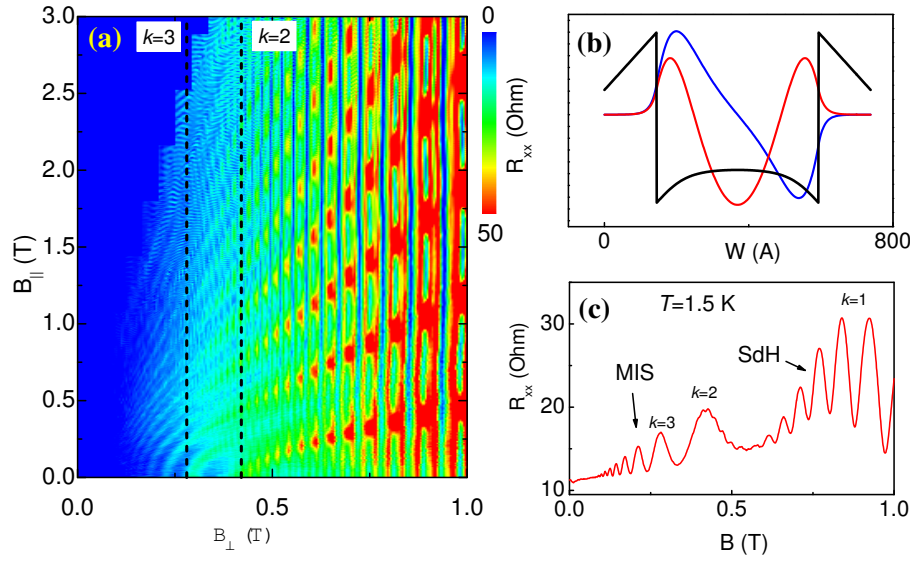
## 2. Experimental results

We have studied wide GaAs quantum wells ( $w=45$  nm) with an electron density of  $n_s \simeq 9.2 \times 10^{11} \text{ cm}^{-2}$  and a mobility of  $\mu \simeq 1.9 \times 10^6 \text{ cm}^2/\text{Vs}$  at low temperatures. Samples in both Hall bar ( $l \times w = 250 \text{ } \mu\text{m} \times 50 \text{ } \mu\text{m}$ ) and van der Pauw (size  $3 \text{ mm} \times 3 \text{ mm}$ ) geometries have been studied. The two lowest subbands are separated by the energy  $\Delta_{12} = E_{AS} - E_S = 1.40 \text{ meV}$ , extracted from MIS oscillation periodicity. Measurements of the longitudinal resistance  $R_{xx}$  have been carried out in a perpendicular magnetic field  $B$  up to 15 T in a dilution refrigerator (base temperature 50 mK).

In Fig. 1(c) we present magneto-resistance for a wide quantum well at  $T=1.5$  K. The figure shows MIS oscillations below 0.4 T. For temperature  $T=50$  mK the MIS oscillations are also superimposed on low-field SdH oscillations. These oscillations are periodic in  $1/B$  and have a maxima under the condition  $\Delta_{SAS} = k\hbar\omega_c$ . The peak at  $B=0.8$  T corresponds to  $k=1$ . Numerous sweeps of the magnetic field sweeps have been taken for different tilt angles. Figure 1(a) shows the resulting phase diagrams, or the plots of the longitudinal resistance  $R_{xx}$ , in the  $B_\perp - B_\parallel$  plane at the balance point in magnetic field below  $B=1$  T in the regime of the MIS oscillations. We recalculate  $B_\perp = B \cos \theta$  and  $B_\parallel = B \sin \theta$ , where  $\theta$  is a tilt angle between the normal to quantum well plane and magnetic fields. The phase diagram shown in Fig. 1(a) is similar to results obtained for double quantum well structures [7,8]. We may see that the vanishing of the resistance minima is correlated with evolution of the MIS oscillations maxima with tilt angle. When the tilt angle increases, even number  $k$  MIS peaks become more pronounced at filling factors  $\nu = 4N + 1$  and  $\nu = 4N + 3$ , while MIS peaks with odd  $k$  number disappear.

## 3. Numerical and analytical calculations

Electrons subjected to a tilted magnetic field  $(B_\parallel, 0, B_\perp)$  in a quantum well potential are described by the Hamiltonian [10] in the Landau gauge.



**Figure 1.** (Color online)(a) Experimental plot of the resistance in the  $B_{\perp} - B_{\parallel}$  plane for a wide GaAs well at  $T=50$  mK. (b) Calculated confinement potential profile of our wide quantum wells and electron wave functions for the first two subbands. (c) The magneto-resistance  $R_{xx}$  as a function of the magnetic field at  $T=1.5$  K. The magneto-resistance exhibits MIS oscillations together with SdH oscillations.

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{q^2 B_{\perp}^2}{2m} x^2 - \frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V(z) + \frac{q^2 B_{\parallel}^2}{2m} z^2 - \frac{q^2 B_{\perp} B_{\parallel}}{m} xz, \quad (1)$$

where  $q$  is the electron charge and  $m$  is the effective mass. The first four terms describe electrons in a perpendicular magnetic field, where energy levels are Landau Levels in symmetric ( $S$ ) and anti-symmetric ( $AS$ ) sub-bands. The small energy separation and the symmetry of the wave functions for the two lowest subbands show that the corresponding (symmetric and antisymmetric) states are formed as a result of a tunnel hybridization  $t_0 = (E_{AS} - E_S)/2$  of the states in the two quantum wells near the interfaces. The last two terms stem from the in-plane magnetic field.

To diagonalize this Hamiltonian we use the method described in [11]. The matrix elements of  $H$  can be expressed by using the eigen-states in a perpendicular field ( $N, \xi$ ) as a basis set, where  $N = 0, 1, \dots$  and  $\xi_i = S_i, AS_i$  represent the Landau orbit and sub-band indexes respectively. In a perpendicular magnetic field, the eigen-energy of each basis is  $E_{\perp}^{N, S(AS)} = \hbar\omega_{c\perp}(N + 1/2) + E_{S(AS)}$ . When a finite parallel magnetic field is applied, the diagonal matrix elements undergo a diamagnetic shift given by

$$E_d(\xi) = \langle N, \xi | \frac{q^2 B_{\parallel}^2}{2m} z^2 | N, \xi \rangle = \frac{\hbar\omega_{c\parallel}}{2} \langle \xi | \frac{z^2}{l_{B\parallel}^2} | \xi \rangle. \quad (2)$$

More importantly, off diagonal terms arise from the  $xz$  term, since  $x$  couples Landau Levels when the Landau indexes differ by one. As a result, the matrix element between  $(N, \xi_i)$  and  $(N + 1, \xi_j)$  is

$$E_c(\xi_i, \xi_j) = \langle N, \xi_i | \frac{q^2 B_{\parallel} B_{\perp}}{m} xz | N+1, \xi_j \rangle = \hbar \omega_{c\perp} \tan(\theta) \sqrt{\frac{N+1}{2}} \langle \xi_i | \frac{z}{l_{B\perp}} | \xi_j \rangle. \quad (3)$$

For the calculations of this work we have considered 30 Landau Levels and two functions for each  $S$  and  $AS$  sub-band. The Hamiltonian built in this way produces two families of fan energy charts for the system, related to the symmetric and anti-symmetric solutions. We have checked that the accuracy of our calculations is about 1% by increasing the numbers of basis functions.

The magneto-resistance oscillation is then modeled through a function of the form

$$M(B) = C_1 \cos[C_s/B + \phi_s(B)] + C_2 \cos[C_{as}/B + \phi_{as}(B)], \quad (4)$$

where  $C_{1,2}$  are constants, and  $\phi_{s,as}(B)$  the symmetric and anti-symmetric corrections to the overall phase. The arguments  $C_{s,as}$  are given by

$$C_{s,as} = \frac{2\pi m c(\mu - E_{S,AS})}{q\hbar}, \quad (5)$$

with a fixed chemical potential  $\mu$ . Equation 4 synthesizes our numerical magneto-resistance oscillation model.

If in our Hamiltonian we leave only one  $S$  and one  $AS$  subband then energy spectrum reduces to [6],

$$E_{AS,S} = E_0 + \frac{md^2\omega_{c\parallel}^2}{8} + \frac{\hbar^2}{2m}(k_x^2 + k_y^2) \pm \sqrt{t_0^2 + \frac{\hbar^2 d^2 \omega_{c\parallel}^2 k_y^2}{4}}, \quad (6)$$

where  $d$  is the distance between the centers of the electrons wave functions at the different sides of the quantum well. The sign  $+$  in the last term corresponds to the antisymmetric state and  $-$  to the symmetric one. In quasi-classical approximation, the Bohr-Sommerfeld quantization rule reads

$$\oint dk_y \sqrt{\frac{2m}{\hbar^2} \left( E \pm \sqrt{t_0^2 + \frac{\hbar^2 d^2 \omega_{c\parallel}^2 k_y^2}{4}} \right) - k_y^2} = \frac{2\pi(n + 1/2)qB_{\perp}}{\hbar}, \quad (7)$$

where  $n$  is the level number. Assuming that the Fermi energy of the 2D electron gas is much larger than the difference between the symmetric and antisymmetric energies, the integral can be reduced to

$$\pi k_F^2 \pm \frac{2m}{\hbar^2} t_0 \mathbf{E} \left( -\frac{\hbar^2 d^2 \omega_{c\parallel}^2 k_F^2}{4t_0^2} \right), \quad (8)$$

where  $k_F$  is the Fermi wave vector of the 2DEG and  $\mathbf{E}(\dots)$  is the full elliptic integral of second kind. At small magnetic field, the symmetric-antisymmetric level splitting  $\Delta_{SAS}$  is

$$\Delta_{SAS} = 2t_0 \left( 1 + \frac{\hbar^2 d^2 \omega_{c\parallel}^2 k_F^2}{4t_0^2} \right), \quad (9)$$

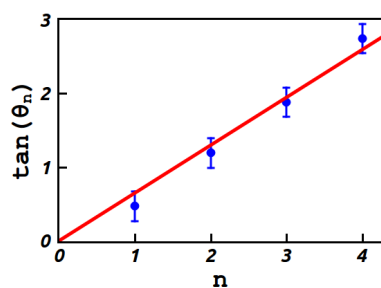
and increases slightly at small  $B_{\parallel}$ . At large  $B_{\parallel}$ , when  $\hbar^2 d^2 \omega_{c\parallel}^2 k_F^2 \gg t_0^2$ , we obtain

$$\Delta_{SAS} = \frac{d\hbar k_F}{\pi m} qB_{\parallel}. \quad (10)$$

As a result, the SdHOs experience a modulation that depends only on the ratio  $B_{||}/B_{\perp}$  and the distance  $d$ . In this way, the longitudinal resistivity  $\rho_{xx}$  can then be written as

$$\rho_{xx} = \rho_0 \left( 1 - 4D(T) \cos \left[ k_F^2 \frac{\pi \hbar}{q B_{\perp}} \right] \cos \left[ k_F d \frac{B_{||}}{B_{\perp}} \right] \exp[-\pi m / q B_{\perp} \tau] \right), \quad (11)$$

where  $\tau$  is the relaxation time and  $D(T)$  the temperature damping factor of the SdHOs. Equation (11) shows that the amplitude of the SdHOs has maximums at  $k_F d \tan \theta_n = \pi n$ , where  $n$  is a integer number. In experiment, the amplitude maximums were observed at  $\theta=25^\circ, 50^\circ, 62^\circ, 70^\circ \dots$  at high magnetic field. This allows us to deduce a factor  $k_F d \approx 4.6$  and distance  $d \approx 39$  nm that agree well with the size of the well and with the thickness of the wave functions.



**Figure 2.** (Color on line) Blue points is the experimental data of  $\tan \theta_n$  at which amplitude of SdHOs is maximal. Error bars correspond to half width of the maxima. Red line is a linear fit of the data points. The slope of the line is  $\pi/k_F d$ .

#### 4. Conclusion

In conclusion, we have studied the magneto-resistance in a wide GaAs quantum well subjected to a tilted magnetic field. We have developed a very effective algorithm to calculate the spectrum of electrons in a quantum well in tilted magnetic field. We performed numerical and analytical calculations of the symmetric-antisymmetric level separation in the presence of a magnetic field, and compared it with the amplitude variation of the SdHO in a wide quantum well.

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