

# Heavy-quark binary scattering in the quark-gluon plasma

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**Abstract.** I argue that the strong quenching and the sizable anisotropy of heavy quark flavors observed in heavy-ion collisions has a substantial component from binary scattering processes in the quark-gluon plasma. To arrive at this conclusion, I illustrate essential requirements for a more accurate theoretical description by considering first the average collisional energy loss of heavy quarks in a thermal background. On this basis, I describe a consistent transport approach, formulating the in-medium evolution of heavy quarks as a stochastic Markov process, which has been implemented for applications in heavy-ion phenomenology.

## 1. Introduction

Heavy quark flavors are distinguished probes for various properties of the strong interaction. In the context of heavy-ion phenomenology, in a quark-gluon plasma of temperature  $T$ , quarks of mass  $M \gg T$  provide an additional scale, which may help to clarify an ongoing debate on the relevance of different in-medium parton energy loss mechanisms: The relative suppression, or ‘quenching’, of particle yields in nucleus-nucleus collisions has often been attributed *entirely* to radiative energy loss of *light* partons [1, 2] – a picture challenged by the observation that heavy quarks, which radiate less [4], are quenched almost as much as light quarks [5, 6]. This has revived a new interest (see [7] and references therein) for the collisional energy loss as a complementary suppression mechanism, in particular for moderately large momenta.<sup>1</sup> In any case, a reliable theoretical understanding of collisions of heavy quarks with thermal partons is necessary for a well-defined baseline for the analysis of the experimental data.

## 2. Mean energy loss

The average energy loss of a parton subject to binary collisions with gluons and  $n_f$  light quark flavors ( $i = g, q$ ) reads

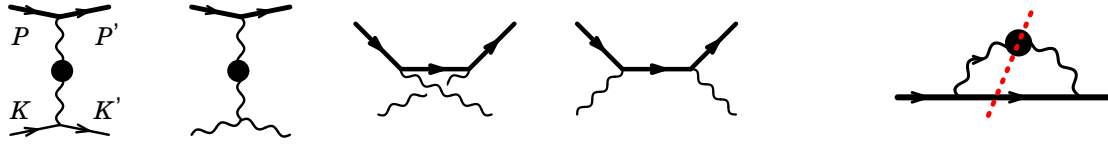
$$\frac{dE}{dx} = \frac{(dv)^{-1}}{2E} \sum_i \int_k \frac{n_i(k)}{2k} \int_{k'} \frac{\bar{n}_i(k')}{2k'} \int_{p'} \frac{1}{2E'} (2\pi)^4 \delta^{(4)}(P+K-P'-K') |\mathcal{M}_i|^2 \omega. \quad (1)$$

Here  $d$  is the degeneracy of the parton ( $d = 6$  for quarks),  $v = p/E$  is its velocity,  $n(k) = 1/(e^{k/T} \mp 1)$  and  $\bar{n} = 1 \pm n$  are the thermal distribution functions of the collision

<sup>1</sup> It seems interesting to recall that Bjorken [8] proposed *collisional* quenching of partons as a signature for the creation of a quark-gluon plasma, based on his adaption of the Bethe-Bloch formula to QCD.



and recoil partners;  $\omega = E - E'$  is the energy transfer. The matrix elements  $\mathcal{M}_i$  of the relevant processes (see Fig. 1) are squared and summed over color/spin states of all particles, and we use a shorthand notation  $\int_k = \int d^3k/(2\pi)^3$ . Bjorken [8] estimated  $dE/dx$  of energetic partons from



**Figure 1.** Heavy-quark scattering off light quarks (thin lines) and gluons (wavy lines). The  $t$ -channel contributions, which require screening (depicted by a blob), can be calculated from the cut *hard thermal loop* resummed self-energy (right diagram).

the two  $t$ -channel processes by

$$\frac{dE}{dx}^{\text{Bjorken}} = \frac{4\pi T^2}{3} \left(1 + \frac{n_f}{6}\right) \alpha^2 \left( \ln \frac{ET}{\mu^2} + c(n_f) + \dots \right), \quad (2)$$

imposing a cut-off  $\mu \sim \sqrt{\alpha}T$  of the order of the Debye mass  $m_D$ . For phenomenological applications the ‘leading’ logarithm is not large, which motivated Braaten and Thoma [9] to calculate the constant  $c(n_f)$ , or in other words the cut-off  $\mu$  in (2), for heavy quarks, and in particular in the limit  $E \gg M^2/T$ , using *hard thermal loop* (HTL) perturbation theory.

A principal shortcoming for phenomenological estimates is that neither (2) nor the HTL-improved result specify the value of the QCD coupling  $\alpha$ . The common *prescription* in the literature is to *ad hoc* assign a value given by the running coupling at the order of the lowest Matsubara frequency,  $\alpha \rightarrow \alpha(\sim (2\pi T)^2)$ . This not only introduces sizable uncertainties, it is also inconsistent: Screening, which is required to render  $dE/dx$  finite, arises from resumming thermal contributions of loop corrections to the Born scattering amplitudes. The concurrent vacuum contributions of the loop corrections then make the coupling momentum dependent; the relevant scale(s) of the running coupling in the final result is thus determined unambiguously. I have shown in [10] that in the leading-log contribution from  $t$ -channel scattering in the large- $E$  limit,

$$\left. \frac{dE}{dx} \right|^t \sim \int_k \frac{n(k)}{2k} \int_{-s}^{-m_D^2} dt \frac{d\sigma}{dt}(-t), \quad (3)$$

the differential cross section  $d\sigma/dt \sim \alpha^2/t^2$  has to be evaluated with a running coupling  $\alpha(Q^2)$  at the scale  $Q^2 = t$ . At leading order  $\alpha(t) \sim 1/\ln(|t|/\Lambda^2)$ , which then implies replacing  $\alpha^2 \ln(ET/m_D^2) \rightarrow \alpha(m_D^2)\alpha(ET) \ln(ET/m_D^2)$  in (2). Evaluating one factor of  $\alpha$  at the *soft scale*  $m_D^2$  leads to larger quantitative estimates compared to previous ‘fix-coupling guesses’. Interestingly, in the limit  $E \rightarrow \infty$  the energy loss ceases to grow logarithmically and saturates,  $dE/dx|^t \sim T^2 \alpha(m_D^2) \sim m_D^2$ .<sup>2</sup>

The running coupling calculation has been pushed beyond leading-log accuracy. Our derivation [7, 12] reveals also that the QCD Compton scattering processes (see Fig. 1) actually do contribute already at leading-log order due to a collinear enhancement, which results – besides a ‘new’ log term – in a revised value of the next-lo-leading log constant  $c(n_f)$ ,

$$\begin{aligned} \frac{dE}{dx} = \frac{4\pi T^2}{3} & \left[ \left(1 + \frac{n_f}{6}\right) \alpha(m_D^2)\alpha(ET) \left( \ln \frac{ET}{m_D^2} + c'(n_f) \right) \right. \\ & \left. + \frac{2}{9} \alpha(M^2)\alpha(ET) \ln \frac{ET}{M^2} + \dots \right]. \end{aligned} \quad (4)$$

<sup>2</sup> For a discussion of setting the scale in the running coupling for the Debye mass see [11].

To estimate the running coupling for the time-like Compton processes, we continue the usual 1-loop perturbative expression according to [13],

$$\alpha(Q^2) = \frac{1}{\beta_0} \begin{cases} L_-^{-1} \\ \frac{1}{2} - \pi^{-1} \text{atn}(L_+/\pi) \end{cases} \quad \text{for } Q^2 \gtrless 0, \quad (5)$$

where  $\beta_0 = (11 - \frac{2}{3}n_f)/(4\pi)$  with  $n_f = 3$ , and  $L_{\pm} = \ln(\pm Q^2/\Lambda^2)$ . We use  $\Lambda \simeq 0.2 \text{ GeV}$  [11], and also constrain the coupling strength in the far infra-red,  $\alpha(Q^2) \lesssim 1$ .<sup>3</sup>

In a recent study [14] we have furthermore investigated the collisional energy loss in a thermal background that flows with a given collective 4-velocity  $u^\mu = \gamma(1, \mathbf{u})$ , as described by Jüttner functions  $n_u(k) = [\exp(K_\mu u^\mu/T) \pm 1]^{-1}$  which replace the thermal distributions in (1). It might be anticipated from the manifest covariant form of intermediate expressions like (3) that we should, in the final result for  $dE/dx$ , replace  $ET \rightarrow P_\mu u^\mu T$ , both in the kinematic logarithms and in the arguments of the running coupling. We have confirmed this expectation by an explicit calculation and find

$$\frac{dE}{dx} = \frac{4\pi T^2}{3} \alpha(P_\mu u^\mu T) \left[ \left(1 + \frac{n_f}{6}\right) \alpha(m_D^2) \ln \frac{P_\mu u^\mu T}{m_D^2} + \frac{2}{9} \alpha(M^2) \ln \frac{P_\mu u^\mu T}{M^2} + \dots \right] \quad (6)$$

at leading-log accuracy. For typical conditions expected in heavy-ion collisions, the flow of the background can have effects of the order of 50% on the mean energy loss of heavy quarks.

### 3. Markov evolution of heavy quarks

Our analytical considerations on the average energy loss illustrate the relevance of two essential requirements for a justified phenomenological understanding of in-medium heavy-quark collisions: taking into account the running of the QCD coupling, and a calculational accuracy beyond leading-log accuracy. On this basis we have developed a transport description of heavy quarks subject to binary collisions in a thermalized quark-gluon plasma. Rewriting their mean energy loss (1) as

$$\frac{dE(p)}{dx} = \int_p \frac{d^3\Gamma(\mathbf{p}', \mathbf{p})}{d(p')^3} \omega, \quad (7)$$

we can first easily obtain the heavy-quark interaction rate  $d^3\Gamma(\mathbf{p}', \mathbf{p})/d(p')^3$ , differential with respect to the outgoing momentum. We calculate this scattering rate beyond leading-log accuracy by using an *effective* mass cut-off [15] for  $t$ -channel scattering,

$$\mu^2(t) = \kappa \cdot 4\pi \left(1 + \frac{n_f}{6}\right) \alpha(t) T^2, \quad (8)$$

which *i*) incorporates the running coupling and *ii*) parameterizes the complex HTL structure of soft electric gluon exchanges by a parameter  $\kappa$ . Its value

$$\kappa = (2e)^{-1} \approx 0.2 \quad (9)$$

is adjusted unambiguously by the requirement that the resulting mean energy loss (7) has to reproduce the analytic formula (4). The value (9) is considerably smaller than unity, which implies that our interaction rates are larger, in proportion, than previous estimates based on a cut-off using the Debye mass in the form  $m_D^2 = 4\pi \left(1 + \frac{n_f}{6}\right) \alpha T^2$ .

With  $d^3\Gamma/d(p')^3$  at hand, we can describe the stochastic propagation of a heavy quark by means of a statistical ensemble. Its momentum distribution evolves as a function of time  $\tau$  according to

$$f(\mathbf{p}', \tau) \rightarrow f(\mathbf{p}', t + \delta\tau) = \int_p \mathcal{T}_{\delta\tau}(\mathbf{p}', \mathbf{p}) f(\mathbf{p}, \tau), \quad (10)$$

<sup>3</sup> The numerical results presented below are largely insensitive to the precise value of the upper bound.

which is a first order Markov process with the kernel

$$\mathcal{T}_{\delta\tau}(\mathbf{p}', \mathbf{p}) = (1 - \delta\tau \Gamma(\mathbf{p})) \delta(\mathbf{p}' - \mathbf{p}) + \delta\tau \frac{d^3\Gamma(\mathbf{p}', \mathbf{p})}{d(p')^3}, \quad (11)$$

where  $\Gamma(\mathbf{p})$  denotes the total rate for scattering out of momentum state  $\mathbf{p}$ . In discrete form, i. e. for binned momenta, (11) becomes a (positive definite) transition matrix satisfying  $\sum_{p'} \mathcal{T}_{p'p} = 1$  for all  $\mathbf{p}$ . Such matrices are called *stochastic*; among their noteworthy properties (discussed in more detail in [15]) we emphasize here only one: They describe a Markov evolution (10) converging to a unique fix point, which in our case – due to detailed balance of the thermal scattering rate – is the thermal equilibrium distribution.

I underline that this basic criterion is *not* met by a number of heavy-quark transport approaches used in the literature to analyze heavy-ion experiments. E. g., the interaction rate  $d^3\Gamma(\mathbf{p}', \mathbf{p})/d(p')^3$  is sometimes obtained by means of Fokker-Planck type equations whose validity, however, relies on the dominance of soft momentum exchanges. This is usually justified in non-relativistic situations, while in the present relativistic case momentum exchanges range from soft  $\sim m_D^2$  to hard  $\sim ET$ , as evident e. g. from the argument of the ‘Coulomb logarithm’ in (2). Accordingly, in these Fokker-Planck approaches coefficients have to be ‘*tuned by hand*’ in order to guarantee thermalization of the heavy quarks (after sufficiently long time) in a thermal bath.

#### 4. Phenomenological implications and conclusions

In order to apply our formalism for the analysis of heavy-ion experiments, we need to model the dynamics of the quark-gluon plasma background. For the following results we use specific temperature and flow profiles,  $T(\tau, \mathbf{r})$  and  $\mathbf{u}(\tau, \mathbf{r})$ , extracted either from the QCD Boltzmann cascade [16] or, alternatively, from the (3+1)-dimensional ideal hydrodynamics code [17], both initialized with typical RHIC conditions (see [14] for details). We follow standard procedures by considering charm and bottom quarks ( $M_c \simeq 1.3$  GeV and  $M_b \simeq 4.6$  GeV) with initial spacial distributions according to the Glauber model [18], and with initial momenta sampled by a QCD event generator [19]. We neglect shadowing and the Cronin effect as irrelevant for heavy-quarks with momenta  $p_t \gtrsim 2$  GeV. We also assume, when the local temperature of the background reaches  $T_c \simeq 200$  GeV, a fragmentation [20] of the heavy quarks into heavy mesons, whose subsequent decay into electrons we compute with PYTHIA [21].

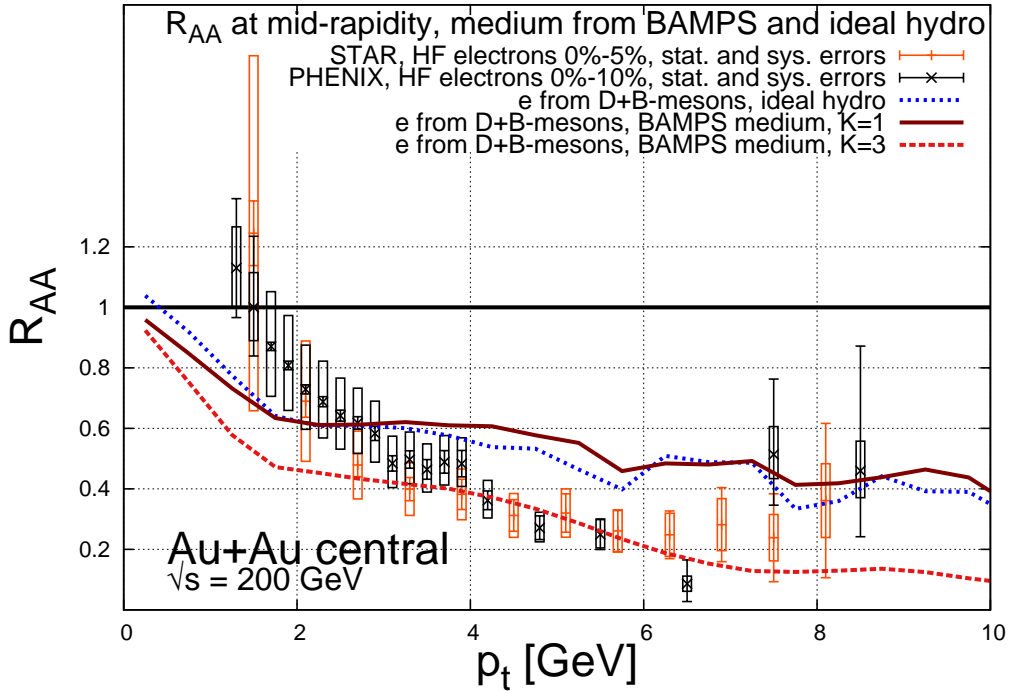
As a first observable we consider the transverse spectrum of electrons from heavy flavor decays, normalized to the corresponding spectrum in proton-proton collisions times the number of binary collisions  $N_{bc}$  according to the Glauber model,

$$R_{AA}(p_t, y) = \frac{d^2 N_{AA}/(dp_t dy)}{N_{bc} d^2 N_{pp}/(dp_t dy)}. \quad (12)$$

Figure 2 shows  $R_{AA}$  calculated at mid-rapidity,  $|y| < 0.35$ , for a central AuAu-collision at RHIC with  $\sqrt{s} = 200$  AGeV, compared to experimental findings in the centrality class of 0% – 10%. Obviously, finer details of the background evolution models are not crucial. Our results clearly follow the trend of the data, with some room for effects due to radiative energy loss. Adapting a common attempt to estimate the relative importance of collisional vs. radiative effects we *ad hoc* scale the binary interaction rate by a constant factor which the experimental data constrain by  $1 \lesssim K \lesssim 3$ . A similar picture emerges in Figure 3 when comparing our numerical results for non-central collisions to PHENIX data, with a slightly smaller  $K$ -value which could be a consequence of the different path length dependence for collisional and radiative energy loss.

As a second observable we compute the elliptic anisotropy, defined by the ensemble average

$$v_2(p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle, \quad (13)$$



**Figure 2.** The nuclear modification factor (12) at mid-rapidity for central AuAu-collisions at RHIC; results from our Markov approach [14] vs. RHIC data [5, 22]. The lower dotted line shows the effect of scaling the binary interaction rate by a factor  $K = 3$ , see text.

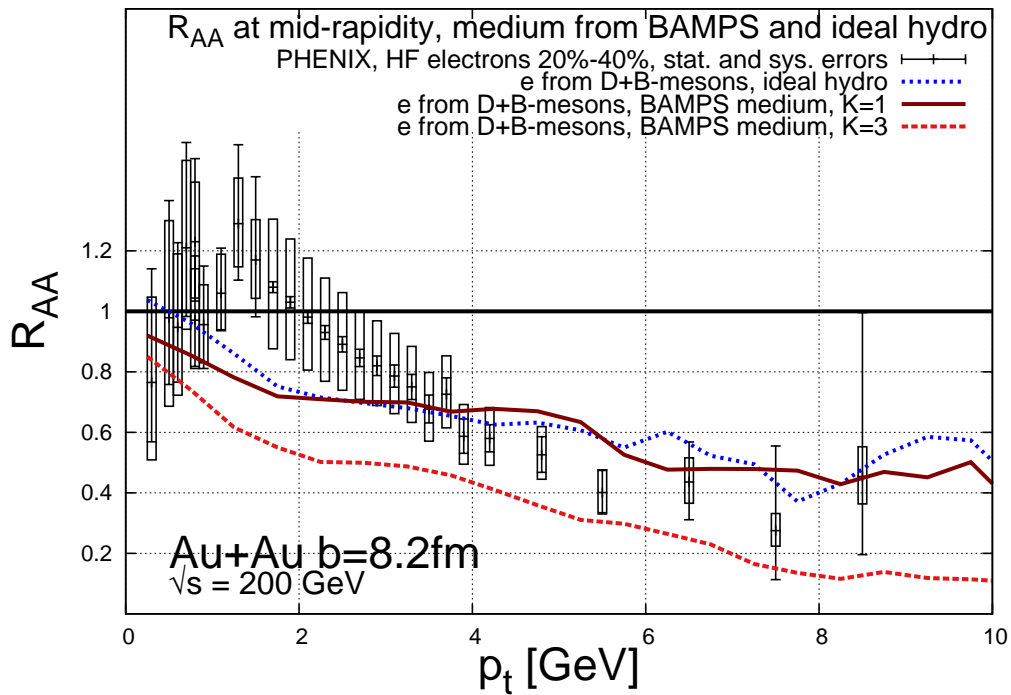
for electrons from heavy-flavor decays, whose transverse momentum dependence is shown in Figure 4. Also in this case the binary interactions alone give a substantial contribution to the experimental results, and again the binary scattering rate had to be scaled by a factor  $K \approx 3$  in order to saturate the data.<sup>4</sup> In this sense we can indeed conclude that collisional effects are of similar importance as radiative effects.

To summarize: We have reviewed analytic results for the mean energy loss of heavy flavors due to binary collisions in a thermalized quark-gluon plasma. We have outlined a consistent transport description within a framework that takes into account the momentum dependence of the QCD coupling and screens soft interactions by a next-to-leading log adjusted cut-off. We have demonstrated that resulting collisional effects are quantitatively as important as radiative effects and thus need be thoroughly considered in heavy-ion phenomenology.

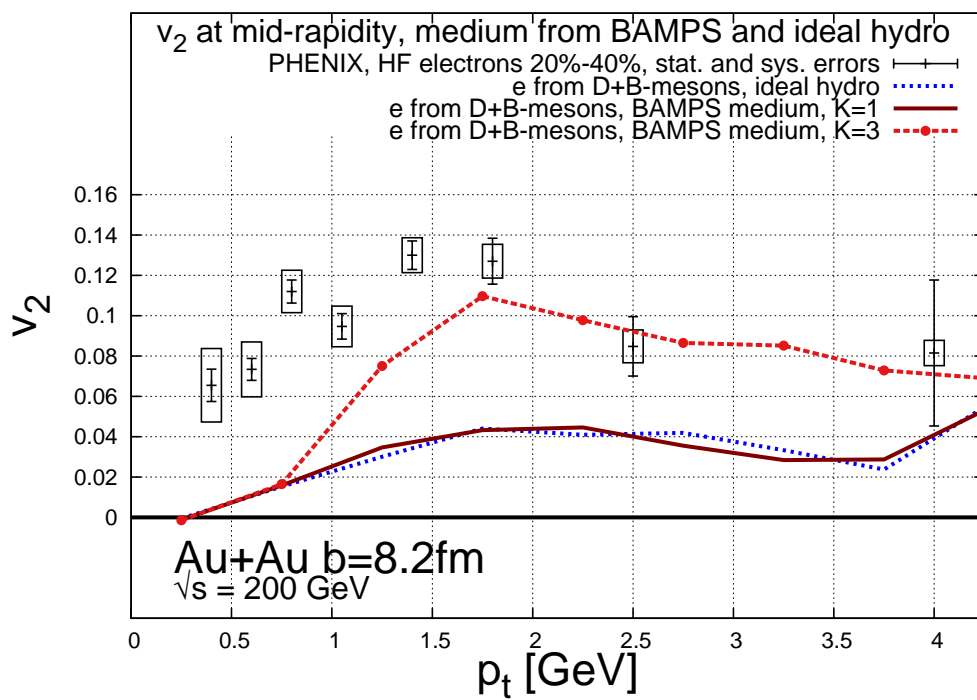
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<sup>4</sup> We note that our approach, namely the value (9) for the screening parameter, is justified for larger heavy quark energies,  $E \gtrsim M^2/T$ , i.e possibly a few GeV for charm quarks. Results at smaller momenta should thus be understood as extrapolations (which may still give reasonable estimates).



**Figure 3.** Same as Figure 2, but for non-central collisions with impact parameter  $b = 8.2$  fm vs. PHENIX data for the centrality class 20%–40%.



**Figure 4.** Same as Figure 3, but for the elliptic flow.

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