

# Testing General Relativity using the growth rate of structure

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**Abstract.** We place tight constraints on the growth index  $\gamma$  by using the recent growth history results of 2dFGRS, SDSS-LRG, VIMOS-VLT deep Survey (VVDS), and *WiggleZ* datasets. Utilizing a standard likelihood analysis, we find that the use of the combined growth data provided by the previous mentioned galaxy surveys, puts the most stringent constraints on the value of the growth index. Assuming a constant growth index, we obtain that  $\gamma = 0.602 \pm 0.055$  for the concordance  $\Lambda$ CDM expansion model. Based on the Dvali-Gabadadze-Porrati gravity model, we find  $\gamma = 0.503 \pm 0.06$  which is lower, and almost  $3\sigma$  away, from the theoretically predicted value of  $\gamma_{DGP} \simeq 11/16$  implying that the present growth rate data disfavor the DGP gravity.

Keywords: cosmology: cosmological parameters

## 1. Introduction

In the current view of cosmology it has been shown that the model that governs the accelerated expansion of the universe is spatially flat and contains a sector of cold dark matter with some sort of dark energy associated with a large negative pressure ([4, 14, 28, 31, 33–36, 58] and references therein). Although the expansion of the universe is attributed to the so called dark energy, its nature and fundamental origin are yet to be revealed. Cosmologists have unfolded two different possible scenarios in order to describe the mechanism of acceleration. The first one involves new fields in nature (scalar fields) and the other one involves some sort of modification of Einstein's General Relativity with the present accelerating stage appearing as a sort of geometric effect (for reviews see [2, 9, 10] and references therein).

The last decade it has been proposed that measuring the growth index  $\gamma$  could help to test the validity of general relativity on cosmological scales because it can provide an efficient way to discriminate between scalar field dark energy (hereafter DE) models which admit to general relativity, and modified gravity models. Linder & Cahn [42] have shown that there is only a weak dependence of  $\gamma$  on the equation of state parameter  $w(z)$ , implying that one can separate the background expansion history,  $H(z)$ , constrained by a large body of cosmological data (SNIa, BAO, CMB), from the fluctuation growth history, given by  $\gamma$ . In this framework, it was theoretically found that for those DE models which adhere to general relativity, the



growth index  $\gamma$  is well approximated by  $\gamma \simeq \frac{3(w-1)}{6w-5}$  (see [42, 47, 55, 65]), which reduces to  $\gamma_\Lambda \simeq 6/11$  for the traditional  $\Lambda$ CDM cosmology  $w(z) = -1$ . On the other hand, in the case of the braneworld model of Dvali-Gabadadze-Porrati [21] (hereafter DGP) the growth index becomes  $\gamma_{DGP} \simeq 11/16$  (see also [22, 42]), while for some  $f(R)$  gravity models we have  $\gamma \simeq 0.41 - 0.21$  for  $\Omega_{m0} = 0.27$  [23, 45]. Indirect methods to determine  $\gamma$  have also been proposed (mostly using a constant  $\gamma$ ), based either on the observed growth rate of clustering [20, 25, 26, 47, 50, 54] providing a wide range of  $\gamma$  values  $\gamma = (0.58 - 0.67)_{-0.11}^{+0.11} {}_{-0.17}^{+0.20}$ , or on massive galaxy clusters [64] and [51] with the latter study providing  $\gamma = 0.42_{-0.16}^{+0.20}$ , or even on the weak gravitational lensing [12]. Gaztanaga et al. performed a cross-correlation analysis between probes of weak gravitational lensing and redshift space distortions and found no evidence for deviations from general relativity.

The scope of the present study is to place constraints on the growth index using a single cosmologically relevant experiment, i.e., that of the recently derived growth data of the 2dFGRS, SDSS-LRG, VVDS, and *WiggleZ* galaxy surveys. We use two reference expansion models, namely flat  $\Lambda$ CDM and DGP, respectively, for the background evolution. The interesting aspect of the latter scenarios is that the corresponding functional forms of the Hubble parameters are affected only by one free parameter, that of the dimensionless matter density at the present time  $\Omega_{m0}$ . The structure of the article is as follows. In section 2, we briefly discuss the background cosmological equations. The theoretical elements of the growth index are presented in section 3. In section 4 we briefly discuss the growth data. In section 5, we perform a likelihood analysis in order to constrain the growth index model free parameters. Finally, the main conclusions are summarized in section 6.

## 2. Theoretical approach

In this section we briefly discuss the main points of the background evolution for homogeneous and isotropic flat cosmologies, driven by non relativistic matter and an exotic fluid (DE models) with equation of state (hereafter EoS),  $P_{DE} = w(a)\rho_{DE}$ . In this case the first Friedmann equation:

$$\frac{H^2(a)}{H_0^2} \equiv E^2(a) = \Omega_{m0}a^{-3} + \Omega_{DE,0}e^3 \int_a^1 d\ln y^{[1+w(y)]}, \quad (1)$$

where  $a(z) = 1/(1+z)$  is the scale factor of the universe,  $w(a)$  is the EoS parameter,  $\Omega_{m0}$  is the dimensionless matter density at the present time and  $\Omega_{DE,0} = 1 - \Omega_{m0}$  denotes the DE density parameter.

We can write the EoS parameter in terms of  $E(a)$  [30, 53] using the Friedmann equations:

$$w(a) = \frac{-1 - \frac{2}{3}a \frac{d \ln E}{da}}{1 - \Omega_m(a)} \quad (2)$$

where

$$\Omega_m(a) = \frac{\Omega_{m0}a^{-3}}{E^2(a)}. \quad (3)$$

Differentiating the latter and taking into account eq. (2) we obtain:

$$\frac{d\Omega_m}{da} = \frac{3}{a}w(a)\Omega_m(a)[1 - \Omega_m(a)]. \quad (4)$$

The important clue about this form of the DE EoS parameter is that includes our ignorance regarding the physical mechanism powering the late time cosmic acceleration since the exact nature of the DE is unknown. It is also worth noticing that for  $w(a) = -1$  we have the concordance  $\Lambda$ CDM model.

Interestingly, the above method can be generalized to the context of modified gravity (see [38,40]). Indeed, instead of using the exact Hubble flow through a modification of the Friedmann equation, one may consider an equivalent Hubble flow somewhat mimicking eq.(1). The ingredient here is that the accelerating expansion can be attributed to a kind of “geometrical” DE contribution. Now, due to the fact that the matter density (baryonic+dark) cannot accelerate the cosmic expansion, it is useful to utilize the following parametrization [38, 40]:

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_{m0}a^{-3} + \Delta H^2. \quad (5)$$

It becomes clear that any modification to the Friedmann equation of general relativity is included in the last term of the above expression. Now, using eqs. (2) and (5), one can derive the effective (“geometrical”) dark energy EoS parameter:

$$w(a) = -1 - \frac{1}{3} \frac{d \ln \Delta H^2}{d \ln a} = -1 - \frac{1}{3} \frac{a}{\Delta H} \frac{d \ln H}{da}. \quad (6)$$

In the context of a flat DGP cosmological model, the “accelerated” expansion of the universe can be explained by a modification of the gravitational interaction in which gravity itself becomes weak at very large distances (close to the Hubble scale) due to the fact that our four dimensional brane survives into an extra dimensional manifold (see [16] and references therein). The quantity  $\Delta H^2$  is given by:

$$\Delta H^2 = 2\Omega_{bw} + 2\sqrt{\Omega_{bw}}\sqrt{\Omega_{m0}a^{-3} + \Omega_{bw}} \quad (7)$$

where  $\Omega_{bw} = (1 - \Omega_{m0})^2/4$ . Interestingly, the quantity  $\Delta H^2$  contains only one free parameter,  $\Omega_{m0}$ . From eq.(6), one can check that the geometrical (effective) DE equation of state parameter reduces to:

$$w(a) = -\frac{1}{1 + \Omega_m(a)}. \quad (8)$$

In this model, due to its gravity nature, the effective Newton’s parameter  $G_{\text{eff}}$  is not any more the usual constant  $G_N$  but it takes the following form [44]:

$$G_{\text{eff}}(a) = G_N Q(a) \quad Q(a) = \frac{2 + 4\Omega_m^2(a)}{3 + 3\Omega_m^2(a)}. \quad (9)$$

### 3. The linear growth rate

We now discuss the basic equation which governs the evolution of the matter perturbations within the framework of any DE model (scalar or geometrical). It is very important to notice that at the sub-Hubble scales the DE component is expected to be smooth and, thus, one can use perturbations only on the matter component of the cosmic fluid [13]. In particular, following the notations of [17,37,42,44,57,61,62], we can derive the well known scale independent equation of the linear matter overdensity  $\delta_m \equiv \delta\rho_m/\rho_m$ :

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}}\rho_m\delta_m \quad (10)$$

a solution of which is  $\delta_m(t) \propto D(t)$ , with  $D(t)$  denoting the linear growing mode (usually scaled to unity at the present time) For the scalar field DE models [ $G_{\text{eff}} = G_N$ ,  $Q(a) = 1$ ], the above equation reduces to the usual time evolution equation for the mass density contrast [48], while in the case of modified gravity models (see [23,42,44,61]), we have  $G_{\text{eff}} \neq G_N$  (or  $Q(a) \neq 1$ ). Transforming equation (10) from  $t$  to  $a$  ( $\frac{d}{dt} = H\frac{d}{d \ln a}$ ), we simply derive the evolution equation of the growth factor  $D(a)$

$$\frac{a^2}{D} \frac{d^2 D}{da^2} + \left(3 + a \frac{d \ln E}{da}\right) \frac{a}{D} \frac{dD}{da} = \frac{3}{2} \Omega_m(a) Q(a). \quad (11)$$

**Table 1.** The growth data. The correspondence of the columns is as follows: index, redshift, observed growth rate and references. In the final column, one can find various symbols of the data appearing in Fig.2.

Index	$z$	$A_{obs}$	Refs.	Symbols
1	0.17	$0.510 \pm 0.060$	Song & Percival 2009; Percival et al. 2004	open circles
2	0.35	$0.440 \pm 0.050$	Song & Percival 2009; Tegmark et al. 2006	open circles
3	0.55	$0.750 \pm 0.180$	Song & Percival 2009; Guzzo et al. 2008	open circles
4	0.25	$0.351 \pm 0.058$	Samushia et al. 2012	open triangles
5	0.37	$0.460 \pm 0.038$	Samushia et al. 2012	open triangles
6	0.22	$0.420 \pm 0.070$	Blake et al. 2011	solid circles
7	0.41	$0.450 \pm 0.040$	Blake et al. 2011	solid circles
8	0.60	$0.430 \pm 0.040$	Blake et al. 2011	solid circles
9	0.78	$0.380 \pm 0.040$	Blake et al. 2011	solid circles

Notice that solving eq.(10) for the concordance  $\Lambda$  cosmology<sup>1</sup>, we derive the well known perturbation growth factor (see [48]):

$$D(z) = \frac{5\Omega_{m0}E(z)}{2} \int_z^{+\infty} \frac{(1+u)du}{E^3(u)}. \quad (12)$$

### 3.1. The evolution of the growth index

For any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering [48]:

$$f(a) = \frac{d \ln D}{d \ln a} \simeq \Omega_m^\gamma(a) \quad (13)$$

which implies:

$$D(a) = \exp \left[ \int_1^a \frac{\Omega_m^{\gamma(x)}(x)}{x} dx \right] \quad (14)$$

where  $\gamma$  is the so called growth index (see [40, 42, 44, 47, 65]).

Combining eq.(13), eq.(11) and eq.(2), we find that:

$$a \frac{df}{da} + f^2 + X(a)f = \frac{3}{2} \Omega_m(a) Q(a), \quad (15)$$

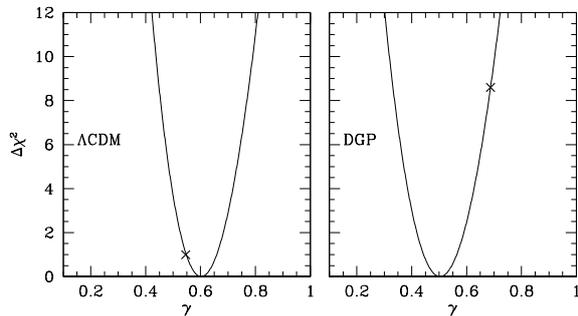
where

$$X(a) = \frac{1}{2} - \frac{3}{2} w(a) [1 - \Omega_m(a)]. \quad (16)$$

If we change variables in eq.(15) from  $a$  to redshift [ $\frac{d}{da} = -(1+z)^{-2} \frac{d}{dz}$ ] and utilizing eqs.(13) and (4), then we can derive the evolution equation of the growth index  $\gamma = \gamma(z)$  (see also [50]):

$$\begin{aligned} & -(1+z)\gamma' \ln(\Omega_m) + \Omega_m^\gamma + 3w(1-\Omega_m)\left(\gamma - \frac{1}{2}\right) + \frac{1}{2} \\ & = \frac{3}{2} Q \Omega_m^{1-\gamma}, \end{aligned} \quad (17)$$

<sup>1</sup> For the usual  $\Lambda$ CDM cosmological model we have  $w(a) = -1$ ,  $\Omega_\Lambda(a) = 1 - \Omega_m(a)$  and  $Q(a) = 1$ .



**Figure 1.** *Left Panel:* The variance  $\Delta\chi^2 = \chi^2 - \chi_{min}^2$  around the best-fit  $\gamma$  value for the  $\Lambda$  cosmology. Note that the cross corresponds to  $(\gamma_\Lambda, \Delta\chi_{1\sigma}^2) = (\frac{6}{11}, 1)$ . *Right Panel:* The statistical results in the case of the DGP model. The corresponding cross is  $(\gamma_{DGP}, \Delta\chi_{3\sigma}^2) = (\frac{11}{16}, 9)$ .

Evaluating eq.(17) at  $z = 0$ , we have:

$$\begin{aligned} & -\gamma'(0)\ln(\Omega_{m0}) + \Omega_{m0}^{\gamma(0)} + 3w_0(1 - \Omega_{m0})[\gamma(0) - \frac{1}{2}] + \frac{1}{2} \\ & = \frac{3}{2}Q_0\Omega_{m0}^{1-\gamma(0)}, \end{aligned} \tag{18}$$

where  $Q_0 = Q(z = 0)$  and  $w_0 = w(z = 0)$ .

It is interesting to mention here that the last few years there have been many theoretical speculations concerning the functional form of the growth index and indeed various candidates have been proposed in the literature. In this work, we decide to phenomenologically treat the functional form of the growth index  $\gamma(z)=\text{constant}$ .

#### 4. The Growth data

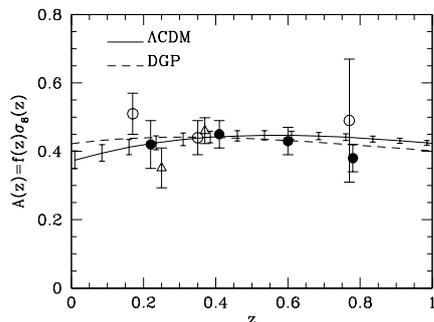
The growth data that we utilize in this work, is based on 2dF, SDSS, and *WiggleZ* galaxy surveys, for which their combination parameter of the growth rate of structure,  $f(z)$ , and the redshift-dependent rms fluctuations of the linear density field,  $\sigma_8(z)$ , is available as a function of redshift,  $f(z)\sigma_8(z)$ . The  $f\sigma_8 \equiv A$  estimator is almost a model-independent way of expressing the observed growth history of the universe [56]. In particular, we use:

- The 2dF [49], SDSS-LRG [59] and VVDS [26], based growth results as collected by Song & Percival [56]. This sample contains 3 entries.
- The SDSS (DR7) results (2 entries) of Samushia et al. [54], based on spectroscopic data of  $\sim 106000$  LRGs in the redshift bin  $0.16 < z < 0.44$ .
- The *WiggleZ* results of Blake et al. [8], based on spectroscopic data of  $\sim 152000$  galaxies in the redshift bin  $0.1 < z < 0.9$ . This dataset contains 4 entries.

In Table 1 we list the precise numerical values of the data points with the corresponding errors bars.

#### 5. Fitting Models to the Data

In order to quantify the free parameters of the growth index, we perform a standard  $\chi^2$  minimization procedure between  $N = 9$  growth data measurements,  $A_{obs} = f_{obs}(z)\sigma_{8,obs}(z)$ , with the growth values predicted by the models at the corresponding redshifts,  $A(\mathbf{p}, z) = f(\mathbf{p}, z)\sigma_8(\mathbf{p}, z)$  with  $\sigma_8(\mathbf{p}, z) = \sigma_{8,0}D(\mathbf{p}, z)$ . The vector  $\mathbf{p}$  represents the free parameters of



**Figure 2.** Comparison of the observed and theoretical evolution of the growth rate  $A(z) = f(z)\sigma_8(z)$ . The solid and dashed lines correspond to  $\Lambda$ CDM ( $\gamma = 0.602$ ) and DGP ( $\gamma = 0.503$ ) expansion models respectively. The different growth datasets are represented by different symbols (see Table 1 for definitions).

the model, depending on the model. In particular, the essential free parameters that enter in the statistical vector are:  $\mathbf{p} \equiv (\gamma, \Omega_{m0})$ . We will restrict our present analysis to the choice  $(\Omega_{m0}, \sigma_{8,0}) = (0.273, 0.811)$  provided by WMAP7 [34]<sup>2</sup>. Note that we sample  $\gamma \in [0.1, 1.3]$  in steps of 0.001 and  $\Omega_{m0} \in [0.1, 1]$  in steps of 0.001. The  $\chi^2$  function<sup>3</sup> is defined as:

$$\chi^2(z_i|\mathbf{p}) = \sum_{i=1}^N \left[ \frac{A_{obs}(z_i) - A(\mathbf{p}, z_i)}{\sigma_i} \right]^2 \quad (19)$$

where  $\sigma_i$  is the observed growth rate uncertainty. A numerical summary of the statistical analysis is shown in Table 2. In general, we find that our results are in agreement, within  $1\sigma$  uncertainties, with previous studies [3, 19, 20, 22, 25, 47].

In the left panel of Fig. 1, we show the variation of  $\Delta\chi^2 = \chi^2(\gamma) - \chi^2_{min}(\gamma)$  around the best-fit  $\gamma$  value for the concordance  $\Lambda$  cosmology. We find that the likelihood function of the growth data peaks at  $\gamma = 0.602 \pm 0.055$  with  $\chi^2_{min} \simeq 7.06$  for 8 degrees of freedom. Alternatively, considering the  $\Lambda$ CDM theoretical value of  $\gamma$  ( $\equiv 6/11$ ) and minimizing with respect to  $\Omega_{m0}$ , we find  $\Omega_{m0} = 0.243 \pm 0.034$  (see also [47]) with  $\chi^2_{min}/dof \simeq 7.37/8$ . Our growth index results are in agreement within  $1\sigma$  errors, to those of Samushia et al. [54] who found  $\gamma = 0.584 \pm 0.112$ . However, our best-fit value is somewhat greater and almost  $1\sigma$  ( $\Delta\chi^2_{1\sigma} \simeq 1$ ) away, from the theoretically predicted value of  $\gamma_\Lambda \simeq 6/11$  (see cross in the left panel of Fig. 1). It is interesting to mention here that such a small discrepancy between the theoretical  $\Lambda$ CDM and observationally fitted value of  $\gamma$  has also been found by other authors. For example, Di Porto & Amendola [18] obtained  $\gamma = 0.60^{+0.40}_{-0.30}$ , Gong [25] measured  $\gamma = 0.64^{+0.17}_{-0.15}$  while Nesseris & Perivolaropoulos [47] found  $\gamma = 0.67^{+0.20}_{-0.17}$ . Recently, Basilakos [3] using a similar analysis shows that  $\gamma = 0.616^{+0.088}_{-0.083}$ .

Concerning the DGP model (see the right panel of Fig. 1), the best-fit value is  $\gamma = 0.503 \pm 0.06$  with  $\chi^2_{min}/dof \simeq 5.30/8$ . If we fix the value of  $\gamma$  ( $\equiv 11/16$ ) to that predicted by the DGP model we find a quite large value of the dimensionless matter density at the present time,  $\Omega_{m0} = 0.380 \pm 0.042$  with  $\chi^2_{min}/dof \simeq 5.38/8$ . It becomes clear that the best-fit  $\gamma$  value is much lower and almost  $3\sigma$  ( $\Delta\chi^2_{3\sigma} \simeq 9$ ) away, from  $\gamma_{DGP} \simeq 11/16$  (see cross in the right panel of Fig. 1) implying that the growth data disfavor the DGP gravity. We would like to stress here that the

<sup>2</sup> For the DGP model, Gong [25] found  $\Omega_{m0} = 0.278$ .

<sup>3</sup> Likelihoods are normalized to their maximum values. In the present analysis, we always report  $1\sigma$  uncertainties on the fitted parameters. Note that the uncertainty of the fitted parameters will be estimated, in the case of more than one such parameters, by marginalizing one with respect to the others.

**Table 2.** Statistical results for the combined growth data (see Table I): The 1<sup>st</sup> column indicates the expansion model, the 2<sup>nd</sup> column corresponds to the best values of  $\gamma$  for each cosmological model and the third column presents the reduced  $\chi_{min}^2$ .

Expansion Model	$\gamma$	$\chi_{min}^2/dof$
$\Lambda$ CDM	$0.602 \pm 0.055$	7.10/7
DGP	$0.503 \pm 0.060$	5.32/7

above observational DGP constraints are in excellent agreement with previous studies. Indeed, Gong [25] and Dosset et al. [20] found  $\gamma = 0.55^{+0.14}_{-0.13}$  and  $\gamma = 0.483^{+0.113}_{-0.088}$ , respectively. In Fig. 2, we plot the measured  $A_{obs}(z)$  with the estimated growth rate function,  $A(z) = f(z)\sigma_8(z)$  [see  $\Lambda$ CDM - solid line and DGP - dashed line].

The goal from the above discussion is to give the reader the opportunity to appreciate the relative strength and precision of the different methods used in order to constrain the growth index. It becomes evident that with the combined high-precision  $f\sigma_8$  growth rate data of Song & Percival [56], Samushia et al. [54] and Blake et al. [8], we have achieved to place quite stringent constraints on  $\gamma$ . Furthermore, notice that using the cosmological parameters ( $\Omega_m, \sigma_8$ ) from Planck [1], our results remain almost the same within  $1\sigma$  errors.

Finally, as we have already mentioned in Table 2, one may see a more compact presentation of our statistical results.

## 6. Conclusions

It is well known that the so called growth index  $\gamma$  plays a key role in cosmological studies because it can be used as a useful tool in order to test Einstein's general relativity on cosmological scales. We have utilized the recent growth rate data provided by the 2dFGRS, SDSS-LRG, VVDS, and *WiggleZ* galaxy surveys, in order to constraint the growth index. Performing a likelihood analysis for various  $\gamma(z)$  parametrizations, we argue that the use of the above combined growth data places the most stringent constraints on the value of the growth index. Overall, considering a  $\Lambda$ CDM expansion model, we find that the observed growth index is in agreement, within  $1\sigma$  errors, with the theoretically predicted value of  $\gamma_\Lambda \simeq 6/11$ . In contrast, for the DGP expansion model we find that the measured growth index is almost  $3\sigma$  away from the corresponding theoretical value  $\gamma_{DGP} \simeq 11/16$  which implies that the present growth data can not accomodate the DGP gravity model.

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