

Analytical and experimental studies on the strain rate effects in penetration of 10wt % ballistic gelatin

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Abstract. This work concentrates on modeling the super-elastic behavior of 10wt% ballistic gelatin at 4 °C and the mechanical responses at quasi-static and high-speed penetrations. Uniaxial compression and simple shearing experiments were carried out to determine the moduli in Mooney-Rivlin model describing the elastic behavior of gelatin at low strain rates. The failure mode is determined to be elastic fracture as the tensile stretch ratio exceeds a critical value. For high compression strain rates, the available results from the split Hopkinson pressure bar (SHPB) experiments for 10wt% gelatin were carefully examined and assessed. Linear relationship between the moduli and the strain rate is established. Based on these material parameters, an analytic solution of stress for the quasi-static and quasi-dynamic expansion of spherical cavity in gelatin is derived. As a consequence, the work needed to open unit volume of cavity, P_s , which is the key parameter in studying penetration problems, is linearly increasing with the characteristic strain rate. The application of P_s to our quasi-static and high-speed penetration experiments is discussed and assessed.

1. Introduction

As a physical surrogate of the human body to evaluate penetration problems, ballistic gelatin is widely used in ballistic tests. There are several reports on the uniaxial compression or elongation experiments of gelatin [1-3]; all of them, including that done by our research group, showed that gelatin is nonlinearly elastic at large deformation. The stress-strain relation of gelatin is rate sensitive, especially at high strain rates, as demonstrated by the Split Hopkinson Pressure Bar (SHPB) experiments of e.g., Salisbury and Cronin [3]. So viscoelastic behavior of gelatin must be taken into consideration.

To analyze penetration tests, a widely used analytical method, the cavity expansion model, is adopted. This method is to calculate the wall pressure on a spherically expanding cavity in an infinite medium at a constant wall velocity [6-7]. The traditional spherical cavity expansion models consider the hard materials bearing small elastic deformation (metals, rocks, concretes etc.), in which Hookean law for linear elasticity is appropriate. However, for some soft materials such as gelatin the small elastic strain assumption for elastic deformation should be abandoned. Based on Mooney-Rivlin constitutive equation, a cavity expansion model for large elastic deformation is derived and used to analyze our penetration tests.

To characterize the dynamic mechanical responses of gelatin, several experimental methods can be adopted, such as SHPB test [3], indentation (punch) test [4], plate-impact test [5] and ballistic penetration test [1]. In the present investigation we examine the available experimental data by SHPB tests of [3] to determine the elastic moduli and their dependence on the strain rate, as well as our



quasi-static mechanical tests. Finally quasi-static and high-speed penetration tests are carried out to characterize the mechanical responses of 10wt% ballistic gelatin.

2. Mooney-Rivlin model for ballistic gelatin

The constitutive equation of Mooney-Rivlin model ([8]) which is commonly used for large elastic deformation is

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2c_1\mathbf{B} - 2c_2\mathbf{B}^{-1} \quad (1a)$$

$$\boldsymbol{\sigma} = -p\mathbf{I} + G\mathbf{B}, \quad 2(c_1 + c_2) = G \quad (1b)$$

where c_1 and c_2 are material parameters (modulus), \mathbf{I} is the unit tensor, \mathbf{B} the Finger strain tensor and p the isotropic pressure determined by boundary conditions. In the limit of small strain, the model becomes the linear Neo-Hookean model (Equation (1b)), where G is the shear modulus. To determine the parameters, c_1, c_2 and G describing the elastic behaviour of ballistic gelatin, we carried out uniaxial compression and simple shearing tests at low strain rates and examined the SHPB tests of [3] for high strain rates.

2.1. Quasi-static tests

A 50g of bloom 250 collagen powder was mixed with 139g of 20°C water, after 40 minutes, 311g hot water (70°C) was added for dissolution. The solution was then sealed and put into a vacuum oven at the constant temperature of 60°C and was stirred every 15 minutes. 30 minutes later, the solution was poured into aluminium molds with the inner diameter of 40mm and the depth of 40mm, the molds were then placed in a refrigerator for 10 to 12 hours at 4(±0.5)°C before testing. Uniaxial compression tests were carried out on the universal materials tester of Zwick/Roell Z020. The samples were released from the molds and every test was done within 3 minutes. Each sample was compressed up to 60~75% engineering strain at the engineering strain rates from 0.004/s to 0.208/s, defined as the loading rate divided by the original length of the specimen.

The results show that the 10wt% ballistic gelatin is highly elastic until failure by cracking, no noticeable plastic deformation was observed, the same observation was reported by Ottone and Deiber[9] and Moy and Tusit [2] for gelatin gels of different concentrations and temperatures. So the failure criterion of gelatin can be described as elastic-crack.

In Figure 1, the uniaxial compression data (negative stress in compression) are presented, as well as the variation of failure engineering stress and strain with the strain rate. Figure 1(b) indicates that the failure stress increases considerably with the increasing strain rate, while the failure engineering strain varies moderately, especially the failure strain increases even more slowly as the strain rate becomes higher. The observation inspires us to use the strain as the failure criterion for gelatin at large deformation. One choice is the maximum tensile stretch ratio in principal directions, $\lambda_m = \max\{\lambda_1, \lambda_2, \lambda_3\}$. We denote the stretch ratio in the compression direction as λ_1 , and other two expansion principal directions as λ_2, λ_3 . $\lambda_1 = 1 + \varepsilon_E$, $\lambda_2 = \lambda_3 = 1/(1 + \varepsilon_E)^{1/2}$. Since the engineering strain at failure is in the range of -0.58~-0.76 from our compression tests, the maximum tensile stretch of the 10wt% gelatin is in the range of $\lambda_m \approx 1.5 \sim 2.0$. And we can infer that at higher strain rates, the value of λ_m would be close to 2.0. For uniaxial compression deformation, the incompressible Mooney-Rivlin model gives the compression stress as,

$$\sigma_1 - \sigma_2 = \sigma_1 = 2(c_1 + c_2 / \lambda_1)(\lambda_1^2 - 1 / \lambda_1) \quad (2)$$

All the stress-strain data of our tests were transformed into the true stress vs. stretch ratio and used to fit the two parameters c_1 and c_2 in Equation (2) based on the least-square approach. See Table 1. For the low strain rate range, the rate dependency seems insignificant compared with that at high strain rates in the SHPB test [3]. So the mean values of the moduli (c_1, c_2 and $G = 2(c_1 + c_2)$) were calculated.

Simple steady shearing tests were carried out on the Gemini200 rheometer (Bolin) by using the cone-plate fixture of 25mm diameter. After the loaded gelatin specimen reached the target temperature of 4°C in the fixture, it was left there for 4 hours aging before the test was started. In Figure 2(a), the four curves of our test results are rather close to each other. Compared with the significant strain rate effects above 1000/s in the SHPB test [3], the strain rate effects can be ignored at low strain rates. For simple shear deformation Mooney-Rivlin model gives linear shear stress τ and quadratic first normal stress difference N_I with the shear strain γ ,

$$\tau = 2(c_1 + c_2)\gamma = G\gamma \quad (3a)$$

$$N_I = 2(c_1 + c_2)\gamma^2 = G\gamma^2 \quad (3b)$$

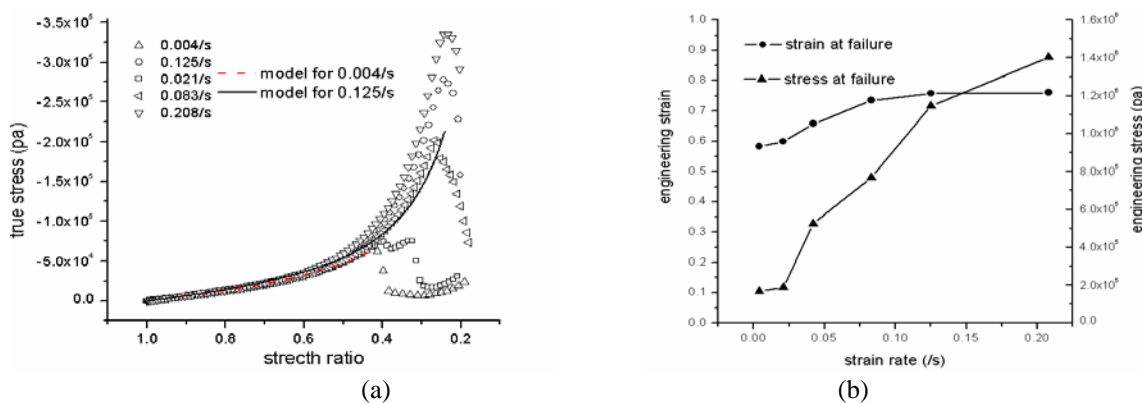


Figure 1. Compression test results of 10wt% ballistic gelatin at 4°C, aging time 12h. (a) Engineering stress-strain curves; (b) Failure engineering stress and strain dependence on strain rate.

Table 1. Material parameters in Equations (1,2) fitted with uniaxial compression experimental data.

Strain rate	0.004	0.021	0.083	0.125	0.208	mean
G	19600	18100	20300	21100	23600	20000
c_1	6200	4400	6600	5700	8600	6000
c_2	3600	4700	3500	4900	3200	4000

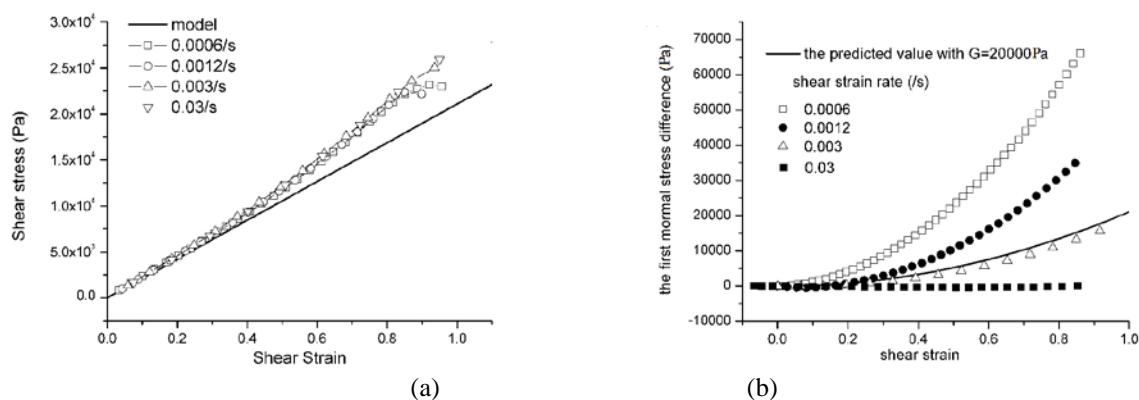


Figure 2. Mooney-Rivlin model describes the simple shear deformation of gelatin with $G=20000(Pa)$

The fitted value of $G=20000Pa$ from the compression tests is used in Equation (3) to predict the τ vs. γ and N_I vs. γ , they are compared with experimental results in Figure 2. Mooney-Rivlin model predicts linear shear stress in simple shear deformation. Figure 2 shows that the curves of shear stress are very close to each other, while the first normal stress difference is very rate sensitive. Rather peculiar is that

the first normal stress difference decreases as the strain rate increases, the opposite behaviour to the common viscoelastic fluids. At the present stage this peculiar behaviour is still under investigation.

2.2. High strain rate tests

Material parameters of gelatin at high loading rates are scarce and the available experimental data by SHPB show rate-sensitive stress-strain behaviour. Here we choose to let the parameters (the moduli c_1 and c_2) of Mooney-Rivlin model be the functions of strain rate to describe the strain-rate dependent response of ballistic gelatin. We read the true-stress vs. true-strain data in Figure 12 of Salisbury and Cronin [3] and transformed them into the true stress vs. stretch ratio curves as shown in Figure 3(a). The transformation from the true strain to the stretch ratio is $\varepsilon_T = \ln(\lambda)$.

In Figure 3(a), the data show modest stress increasing for the nominal (engineering) strain rates from 1000/s to 1350/s and a sudden stress increase at 1550/s. The stress strain curve at 1350/s was chosen to obtain the parameters c_1 and c_2 with Equation (2). Then the parameters at 1000/s, 1250/s and 1550/s were obtained by linear interpolation. The values of c_1 and c_2 are shown in the third to sixth columns of Table 2 and in Figure 3(b).

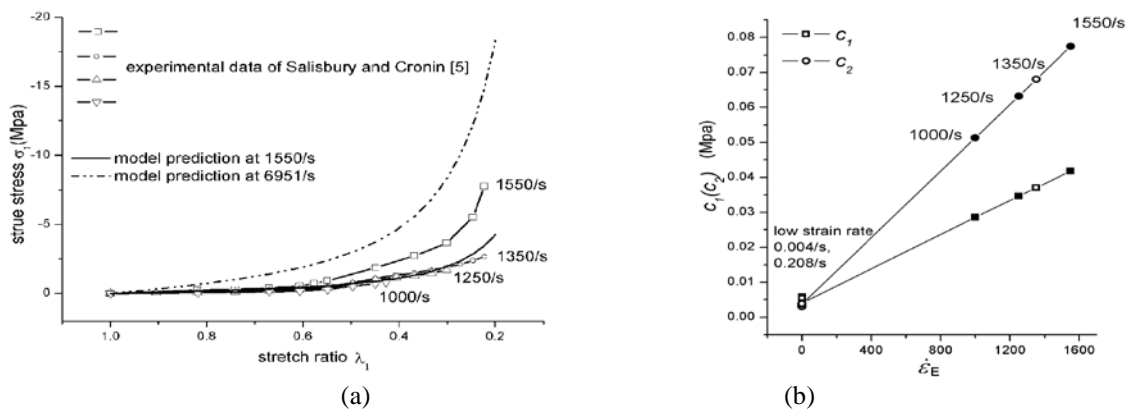


Figure 3. Dependence of c_1 and c_2 on strain rate

The predictions for the true-stress vs. stretch-ratio by Mooney-Rivlin model at 1550/s are also shown in Figure 3(a). For the stretch ratios below 0.6 and at 1550/s the experimental stresses are suddenly increased and considerably larger than the model predictions. The true strain rate, defined as the compression velocity divided by the instant length of the specimen, increased as the strain increased during the compression test. Actually, for the nominal 1550/s compression test, the true strain rate could be as high as 6951/s at the end of compression. Taking this into consideration, the experimental data are reasonably between the predicted values for 1550/s and 6951/s as shown in Figure 3(a).

3. Application to penetration problems

One approach to the problem of deep penetration into semi-infinite target is to modify the analysis of the static expansion of a spherical cavity to determine the normal stress at the surface of the penetrator. We developed a cavity expansion model for soft materials like ballistic gelatin considering large elastic deformation. The elastic behaviour is modelled by the Mooney-Rivlin constitutive equation and the failure criterion by elastic fracture at the largest elastic stretch ratio λ_m . Suppose a spherically symmetric cavity expands from initial zero radius in an infinite medium, it will produce elastic and cracked response regions. The radius of the cavity is denoted by a , and the radius of the cracked region surrounding it by c . The pressure on the cavity wall derived by Liu et al. ([10] to be published) for incompressible mediums is,

$$-P_s = \sigma_r|_{r=a} = \left(c_1 \left[\left(1 - (a/c)^3 \right)^{1/3} \left(5 - (a/c)^3 \right) - 5 \right] - 2c_2 \left[\left(1 + (a/c)^3 \right) \left(1 - (a/c)^3 \right)^{-1/3} - 1 \right] \right) (c/a)^2 \quad (4a)$$

$$a/c = (1 - \lambda_m^{-3})^{1/3} \quad (4b)$$

This solution is radial position self similar, and the pressure is equivalent to the energy needed to open unit volume of the cavity.

For the ballistic gelatin the parameter c_1 and c_2 are linear functions of the characteristic strain rate (Table 2). For low strain rates the maximum stretch ratio $\lambda_m=1.5$, while for large strain rates $\lambda_m=2.0$. The cavity wall pressure P_s is a linear function of the strain rate since the parameters c_1 and c_2 are linearly rate-dependent on the strain rate.

$$P_s = P_0 + \beta \cdot \dot{\epsilon} \quad (5)$$

With $P_0=34000$ Pa and $\beta=360$ Pa/s, Equations (4) yield the pressures on the expanding cavity wall, P_s , which are listed in Table 2.

3.1. Quasi-static penetration test

The liquid gelatin at 60°C was put into aluminium molds (120mm diameter and 50mm depth) and after solidification, the molds were sealed and put into a refrigerator for aging at 4°C for 16~18h. The quasi-static penetration tests were carried out on the universal materials tester. Steel rods with conical head of various cone angles (20°, 30°, 40°) were stick on the upper loading surface of the tester (Figure 4). As the cone penetrates into the gelatin block, a “cavity” is opened. Suppose the normal stress σ and frictional stress τ are uniform on the conical head, the resistance force on the rod is,

$$F = \pi z^2 (\sigma \tan^2 \theta + \tau \tan \theta) = Kz^2 \quad (6)$$

where θ is the semi-cone angle and z the penetration depth. So the resistance force F is proportional to the square of the penetration depth z (Figure 5(a)) and the factor K is dependent on the semi-cone angle θ (Figure 5(b)),

$$K / (\pi \tan \theta) = \sigma \tan \theta + \tau \quad (7)$$

Table 2. Material parameters of gelatin and calculated values of P_s by cavity expansion model of Equations (4)

strain rate $\dot{\epsilon}$ (/s)	0.004~ 0.208	1000	1250	1350	1550
c_1 (MPa)	0.006	0.029	0.035	0.037	0.042
c_2 (MPa)	0.004	0.051	0.063	0.068	0.077
λ_m	1.5	2.0	2.0	2.0	2.0
P_s (MPa)	0.03	0.4	0.49	0.53	0.6

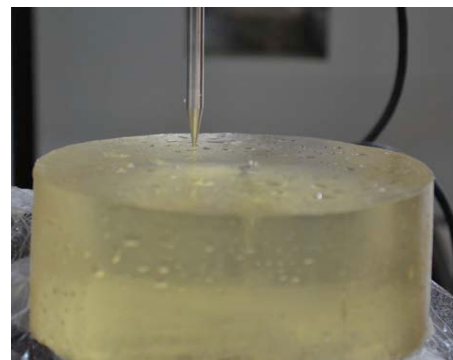


Figure 4. Quasi-static penetration test

This linear relationship between $K/(\pi \tan \theta)$ and $\tan \theta$ can be analyzed to determined the slop of σ and the intercept τ . The normal stress σ is the pressure needed to open a cavity quasi-statically. The results show that σ is in the range of 0.019~0.051 MPa at low expansion rate (0.03~1.2 mm/s) which is defined as the penetration velocity multiplied by $\tan \theta$; while the value of P_s is 0.03 MPa in the low strain rate range of Table 1. Considering the experimental errors, the agreement supports our model's estimation.

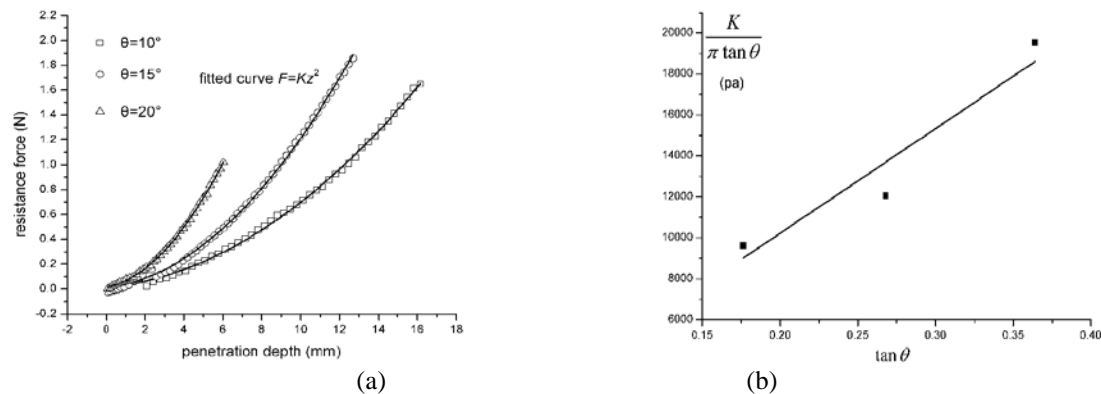


Figure 5. Fitting results of quasi-static penetration tests: (a), resistance force vs. penetration depth; (b) determination of the normal stress and frictional stress.

3.2. High-speed penetration test

For soft materials, the radial acceleration of the target material under penetration causes considerable free expansion of cavity. Since the parameter P_s characterizes the energy needed to open unit volume of cavity, we have $\Delta E = P_s V_m$, where ΔE is the total energy transferred from the projectile to the target and V_m the maximum volume of the temporary cavity. Sellier [11] proposed an empirical formula to estimate the maximum cavity volume V_m in terms of the total energy ΔE transferred to the target: $V_m = \mu \Delta E$. This is a rough estimate based on ballistic experiments. Sellier gives $\mu = 1.26 \text{ cm}^3/\text{J}$ for 10wt% gelatin. In the present terminology, $P_s = 1/\mu = 0.794 \text{ MPa}$, this value is in the same order of magnitude as those in Table 1 obtained by using the material parameters of 10wt% gelatin and Mooney-Rivlin model.

In our penetration experiments, 10wt% gelatin blocks ($30 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm}$) stored at 4°C in a refrigerator for 24 hours were used. We carried out experiments by firing fragments of various shapes: spheres, cylinders, rhombus and a type of bullet into the gelatin blocks. The penetration velocity varied from 600 m/s to 1100 m/s . A high-speed video camera recorded the motion of the penetrators and the temporary cavity's wall-variations with time. The parameter P_s was estimated by $\Delta E/V_m$, where ΔE is the energy lost of the penetrators and V_m the maximum volume of the cavity produced. The experimental values of P_s are in the range of $0.2 \sim 0.6 \text{ MPa}$ as shown in Table 2. This range is consistent with that in Table 2.

Table 2. Calculated value of P_s from penetration tests by different shaped fragments

	spheres				Rhombus	Cylinders		5.8 caliber Bullet	
$\rho_p (\text{kg/m}^3)$	7800				7800	7800		-	
$m_p (\text{g})$	0.11	0.26	0.45	0.45	0.25	0.92	0.92	4.15	4.15
$v_p (\text{m/s})$	644	658	725	856	651	1032	723	820	738
$V_m \times 10^5 (\text{m}^3)$	8.05	17.7	31.2	49.5	19.9	83.5	66.8	523	406
$\Delta E (\text{J})$	22.7	56.0	117.9	164.7	53.0	488.6	239.6	1108.0	817.7
$P_s (\text{MPa})$	0.28	0.32	0.38	0.32	0.27	0.59	0.36	0.21	0.20

4. Conclusions

Uniaxial compression and simple shearing tests were carried out for 10wt% ballistic gelatin at low strain rates; the elastic moduli c_1, c_2 in Mooney-Rivlin model were obtained and the main failure mode is determined to be elastic fracture as the maximum tensile stretch ratio exceeds a critical value.

Gelatin's mechanical behavior shows rate-sensitivity, especially at high strain rates. We fit Mooney-Rivlin model with the data of Salisbury and Cronin [3] and establish linear variations of the moduli with the strain rate. Consequently, the key parameter P_s in cavity expansion model, that is, the

energy needed to open unit volume of cavity in 10wt% gelatin increases linearly with the strain rate. The prediction of P_s is compared favorably with the normal stress in quasi-static penetration tests and the free cavity expansion in 10wt% gelatin ballistic tests by fragments of various shapes.

References

- [1] Cronin D S and Falzon C 2011 *Exp Mech* **51** 1197-1206
- [2] Moy P and Tusit W *Proc. XIth Int. Congr. and Expo* June 2-5 2008 Orlando, Florida USA
- [3] Salisbury C.P. and Cronin D.S.. *EXP MECH* 2009; **49** 829-840
- [4] Foster M et al. *Dynamic Behavior of Materials*, 1, Conf. Proc. of the Society for Experimental Mechanics Series 99
- [5] Shepherd C J et al. 2009 *AIP Conf. Proc.* **1195** 1399-1402
- [6] Forrestal M J, Brar N S and Luk V K *J. Appl. Mech.-T ASME* **58** (1) 7-10
- [7] Forrestal M J and Tzou D Y 1991 *Int. J.Solids Struct.* **34** (31-32) 4127-4146
- [8] Macosko C W 1994 *Rheology principles, measurements, and applications* (New York:Wiley)
- [9] Ottone M L and Deiber J A 2005 *Polymer* **46** 4928-4937
- [10] Liu L et al. A spherical cavity expansion model for ballistic gelatin and its application to penetration problems. To be published in the *International J. Impact Engng*
- [11] Sellier K G 1994 *Wound ballistics and the scientific background* (Amsterdam: Elsevier)