

Diagnostic tool for structural health monitoring: effect of material nonlinearity and vibro-impact process

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Abstract. Numerous techniques are available for monitoring structural health. Most of these techniques are expensive and time-consuming. In this paper, vibration-based techniques are explored together with their use as diagnostic tools for structural health monitoring. Finite-element simulations are used to study the effect of material nonlinearity on dynamics of a cracked bar. Additionally, several experiments are performed to study the effect of vibro-impact behavior of crack on its dynamics. It was observed that a change in the natural frequency of the cracked bar due to crack-tip plasticity and vibro-impact behavior linked to interaction of crack faces, obtained from experiments, led to generation of higher harmonics; this can be used as a diagnostic tool for structural health monitoring.

1. Introduction

The presence of a crack in a structure can drastically change its general dynamic characteristics. The dynamic characteristics are affected by two possible effects occurring near the cracked zone. One of them is the crack-tip plasticity and another is a vibro-impact behavior of the crack, i.e. is the crack-face interaction. Few researchers have tried to address the issue of the effect of crack as discontinuity on dynamics of structures [1] [2]. In [3] it was assumed that the crack changes from a state of fully open to fully closed instantaneously, which gave a rise to bilinear stiffness nonlinearity that was approximated using a higher-order polynomial. Higher-order frequency response functions were defined using Volterra series for systems with polynomial type nonlinearity, and a damage-assessment procedure was suggested, based on a quantitative comparison of these functions. The finite-element scheme was developed for computing the Eigen frequencies for a cracked beam for different degrees of closure [4]. In this study, the finite-element method, the component-mode synthesis method and the linear-elastic fracture mechanics theory are integrated for modeling of the cracked structures. The method used by the authors was benchmarked against earlier results in the literature. A continuous cracked beam vibration theory developed in [5] was used for prediction of changes in transverse vibration of a simply supported beam with a breathing crack. The equation of motion and the boundary conditions of the cracked beam considered as a one-dimensional continuum were used. The Eigen frequency changes due to a breathing edge-crack were shown to depend on the bi-linear character of the system. The associated linear problems were solved over their respective domains of definition, and the solutions were matched through the initial conditions. The changes in vibration



frequencies for a breathing crack were smaller than the ones caused by open cracks. Modal analysis was used to study the nonlinear dynamics of the cantilever beam with a transverse edge crack in [6]. The authors modelled nonlinearity produced by opening and closing of crack as a piece-wise linear system. To define effective natural frequencies for this system they used the idea of bilinear frequency along with a perturbation method to obtain the nonlinear normal modes of vibration and the associated period of motion. It was found that the bilinear frequency formula was a good approximation for the effective natural frequency.

In [7], a study was performed on a single-degree-of-freedom bilinear spring-mass system. Modal expansions were employed to compute the beam's response at any excitation with proper incorporation of continuity of displacement and velocity when the crack was in a state of transition. The Fourier transform of the steady-state time response showed the presence of nonlinearity, which was correlated with the location and depth of the crack. The beam was modeled as an oscillator with a bilinear restoring force and applied bi-spectral analysis to the response to study vibration of a beam with a breathing crack [8]. The bi-spectral analysis showed high sensitivity to the non-linear behavior of the system compared to other techniques. The dynamic behavior of a cantilever beam with a breathing crack was investigated under harmonic excitation theoretically and experimentally in [9]. In this work, a simple model with a single degree of freedom was developed with varying stiffness to simulate the dynamic behavior. The data for simulated and experimental responses were analyzed by applying empirical mode-decomposition and the Hilbert transform. The results obtained indicated that the instantaneous frequency oscillates between the values corresponding to the open and closed states of the crack. The variation of the instantaneous frequency increased with an increasing crack depth following a polynomial law, which could be used to estimate the crack size. The problem of nonlinear dynamics of cracked cantilever beam under harmonic loading using the finite-element technique was investigated in [10]. The study revealed that the frequency did not change with the oscillation amplitude, but the steady-state response obtained was very rich of sub and super-harmonic components. Moreover, within the super-harmonic resonance ranges the phase portraits were characterized by significant wiggles due to impact between crack faces. In [11] numerical evaluation of a forced flexural vibration of a cantilever beam with a transverse surface crack extending uniformly along its width was performed to relate nonlinear resonances to the crack presence, location, and depth. The qualitative characteristics, namely, phase-portrait distortions, sub- and super-harmonic components in the Fourier spectrum, and curved shape of the modal line were considered. It was found that the acceleration record of the beam tip was sufficient to detect the existence of the crack and to identify its depth and site. The finite-element simulation was used to study the longitudinal harmonic loading for assessing damage in a cracked cantilever bar [12]. The nonlinear materials properties were introduced in terms of plastic behaviour. The effect of crack size and position along the beam length was studied. An observed change in the natural frequency was used to characterize the damage state of the component. In [13], [14] longitudinal vibration of the bar with a breathing crack was investigated. The results of simulations included higher-harmonics generation, a frequency shift and their dependency on crack parameters, as well as generation of low-frequency components, which can serve as a diagnostic tool for structural health monitoring.

2. Finite element model: Effect of crack tip plasticity on dynamic behavior of cracked structure

The 2D finite-element model of the un-cracked and cracked cantilever bars was implemented in commercial software MSC Marc. Figure 1 shows the methodology used for performing simulations to study the effect of crack-tip plasticity on the natural frequencies of the cracked bar. The cracked cantilever bar with length L was clamped at the left end and free at the right end. The crack was located at the distance d from the fixed end with d/L being a dimensionless crack position. Different crack depths – 2 mm, 4 mm, 6 mm, 8 mm and 10 mm – were used in analysis. The finite-element model of the cracked cantilever bar (Fig.2a) was meshed using four-node, isoparametric, arbitrary quadrilateral elements, in total, 7500 elements were used. These elements use bilinear interpolation functions and the strain is constant throughout these elements. The stiffness of these elements is

formed using four point Gaussian integration. Table 1 show the main parameters used in finite-element simulations. The first three natural frequencies were obtained for the cracked cantilever bar for two cases: with account for the effect of crack-tip plasticity and without it. Figure 2c presents results for the first case for which elastic-plastic material model was used while Fig. 2b for another one for which elastic material model was used.

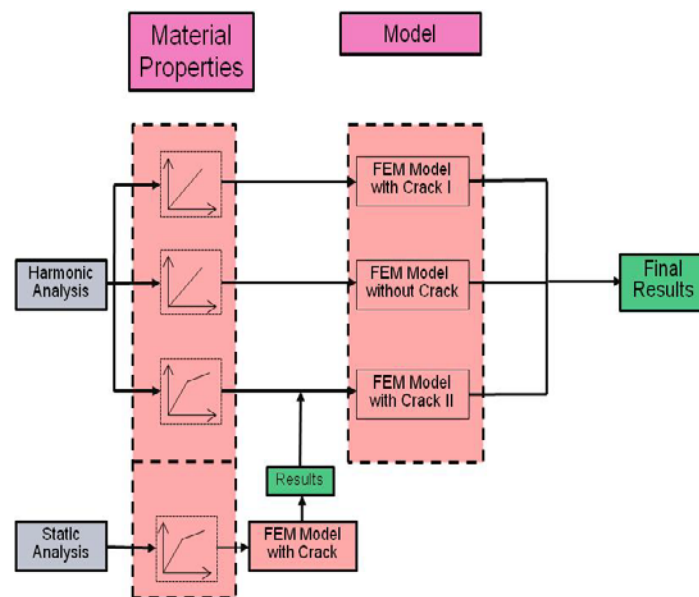
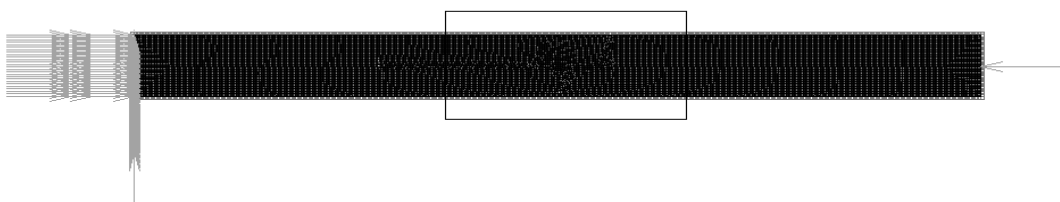


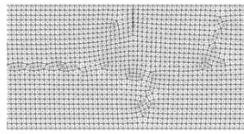
Figure 1. Methodology of simulations

Material	Aluminium (Grade 6082T6)
Length of bar (L) [mm]	300
Width of bar [mm]	10
Height of bar [mm]	25
Depth of crack [mm]	2, 4, 6, 8 and 10
Distance of crack from fixed end (d) [mm]	100, 150 and 250
Density[kg/mm ³]	2.70×10^{-6}
Modulus of Elasticity [N/mm ²]	7×10^4
Yield Strength [N/mm ²]	2.514×10^2
Ultimate tensile strength [N/mm ²]	3.434×10^5

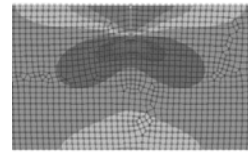
Table 1. Simulation parameters



(a) Entire bar with boundary conditions



(b) Meshing near crack tip
(shown by rectangle in (a))



(c) Stress concentration near crack tip

Figure 2. FEA model of a cracked cantilever bar

3. Simulation results: Effect of crack location on natural frequencies

Figure 3 shows a comparison between the first, second and third natural frequencies with and without plasticity at the crack tip for different d/L ratios and crack depths. Apparently, there was no change in the first natural frequency (Fig. 3a) whereas there were some marginal changes in the second and third natural frequencies as the d/L ratio increased (Figs. 3b and 3c)

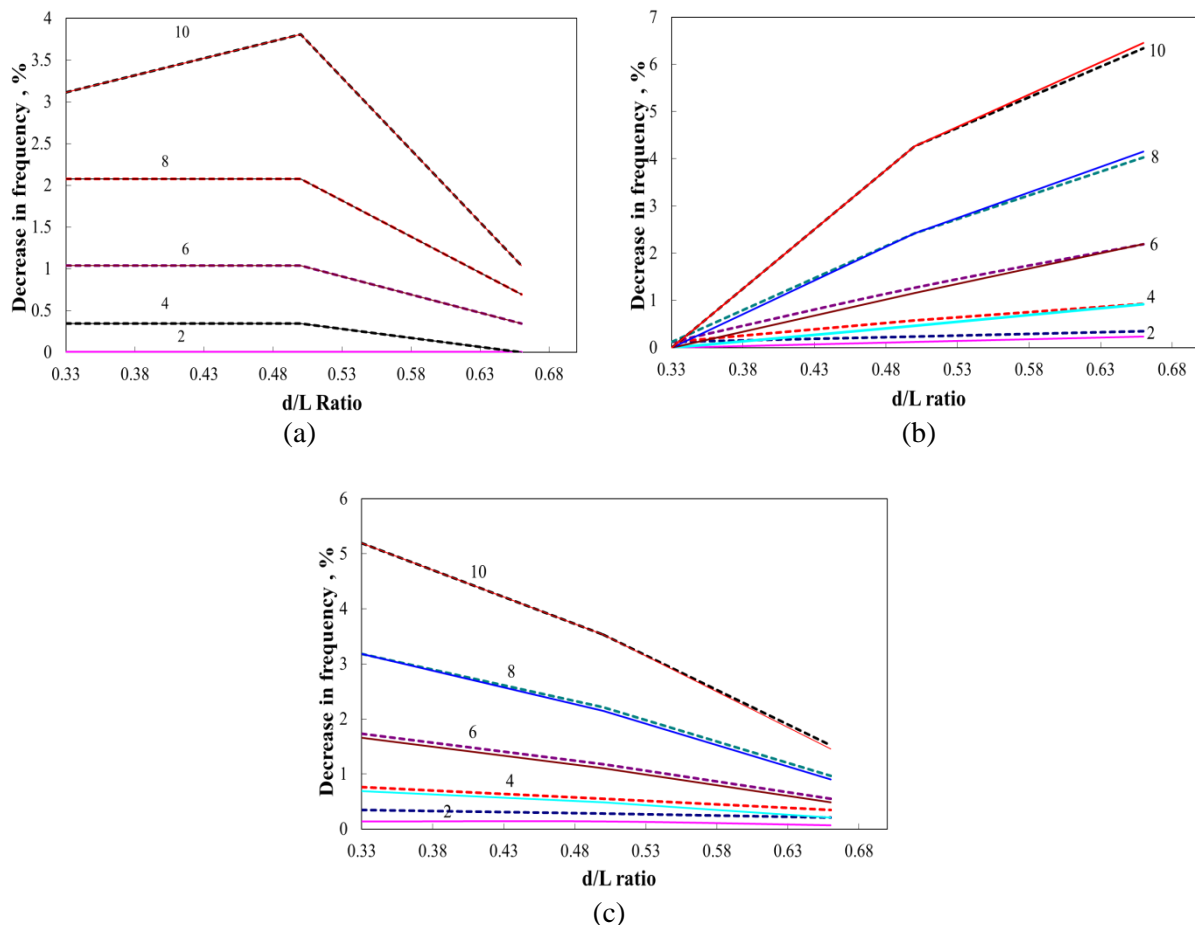


Figure 3. Effect of d/L ratio on frequencies for different crack depths (in mm):
(a) First natural frequency; (b) second natural frequency; (c) third natural frequency
(dotted line -with plasticity, continuous line - without plasticity)

4. Experimental Study: Effect of vibro-impact of behavior of dynamics of cracked bar

An aluminium bar with dimensions 300 mm x 25 mm x 10 mm with a fatigue crack at its centre, shown in Fig.4, was used in experiments. This aluminium bar was fixed at the left end and the other

end of bar was excited in longitudinal direction by harmonic load using a shaker, which was driven by an oscillator (wave form generator). Piezoelectric strain gauges were glued at four different locations on the cracked bar to measure its dynamic response. Two of them were glued near the crack, one near the free end and the last near the fixed end of the bar. The frequency sweep was performed to obtain the resonant frequency of the bar. Then, the bar was excited by the harmonic load of 2.8 N using a shaker at this resonant frequency, and the responses were obtained at the four strain gauges in the longitudinal direction. These responses were processed using a picoscope taking into account the Nyquist criteria to avoid aliasing errors and the Hanning window to minimize a leakage error in the signal acquired.

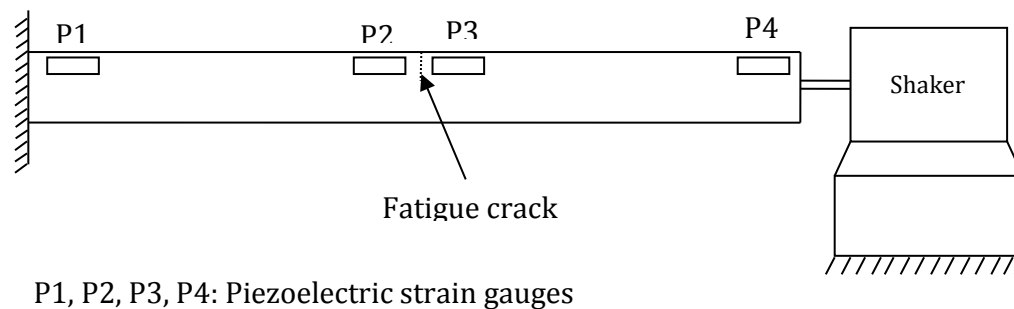


Figure 4. Schematic of experimental setup

5. Experimental results

From the frequency sweep it was found that the resonance occurred at the frequency of 2821 Hz. At this resonant frequency, the cracked bar was excited at the vibration amplitude of 2.8 N. It was found that the responses (Figs. 5b and 5c) obtained near the crack showed a higher harmonic of the forcing frequency at 8464 Hz. Similarly, the responses obtained far away from the crack showed only the resonant frequency of excitation. This gives an indication that the crack-induced nonlinearity has the localized effect on dynamics of the cracked bar leading to generation of additional perturbations in it.

6. Conclusions

This paper was focused on the effect of crack tip plasticity and the vibro-impact behavior of crack on dynamics of the cracked cantilever. The 2D finite-element simulations indicated that the presence of plasticity at the crack tip had no effect on the first natural frequency. On the contrary, as the d/L ratio increased, the second and third natural frequency of beams with different crack depths were affected. The results obtained from experiments revealed that the vibro-impact behavior of crack led to generation of higher harmonics in the response measured near the crack. It indicates that crack-induced nonlinearity had the localized effect on the dynamic behavior. The results obtained from simulations and experiments can be used as a good diagnostic tool for structural health monitoring.

7 Acknowledgement

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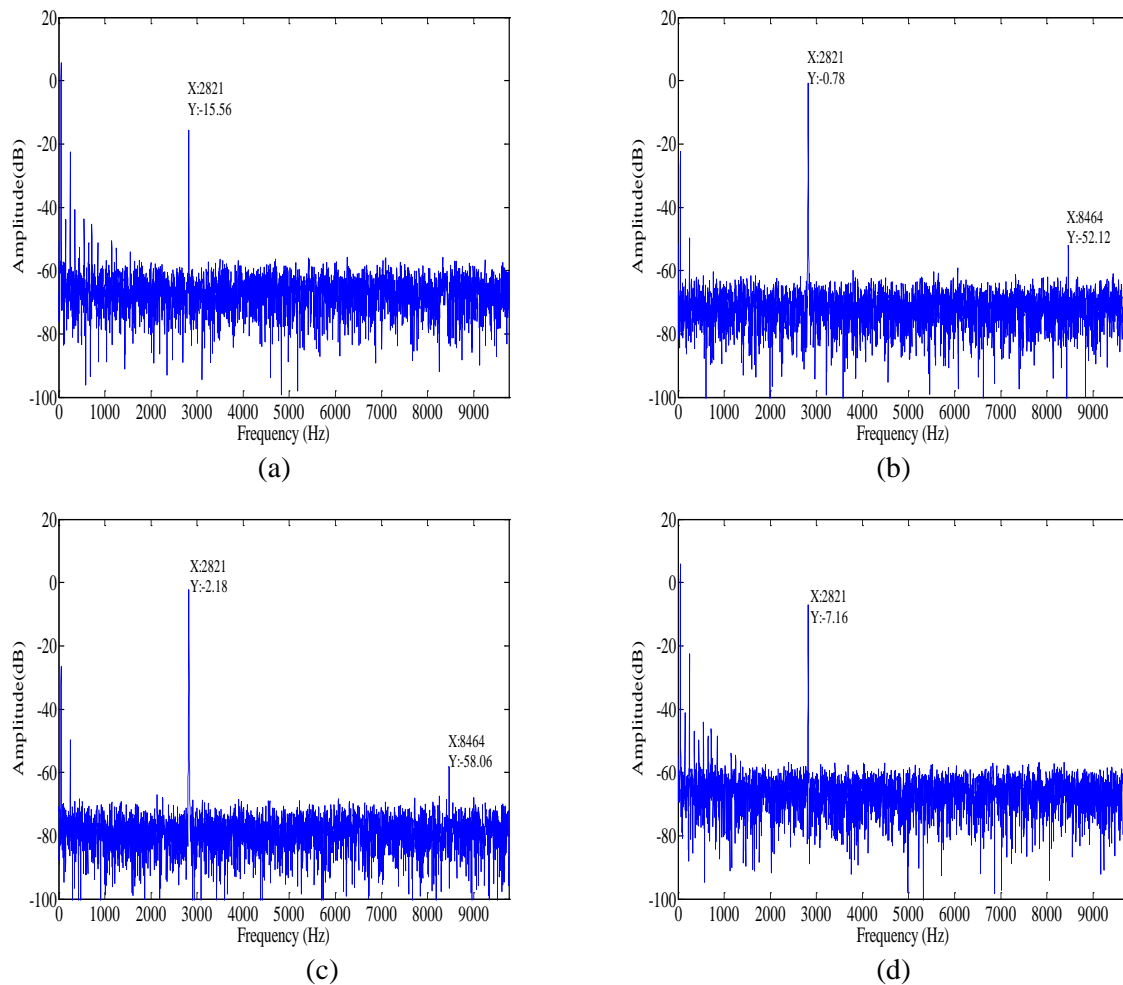


Figure 5. Response at resonant frequency for low excitation amplitude for various sensors:
(a) P1; (b) P2; (c) P3; (d) P4

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