Effect of crack induced nonlinearity on dynamics of structures: application to structural health monitoring

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Abstract. In recent years, there has been a growing interest in vibration-based structural health-monitoring techniques. At present, most of the vibration-based techniques consider an open crack, which is far from realistic situations. In real-life scenarios crack can open and close under the dynamic loading. This bilinear behavior of crack has a significant effect on dynamics of structures which was investigated by performing finite-element simulations of a beam with a breathing crack excited longitudinally. Its response measured at different locations with regard to the crack indicated that the crack-induced nonlinearity had a localized effect on dynamics of the beam. Measurements obtained near the crack revealed complex spectra with higher harmonics.

1. Introduction

A growing complexity of modern civil, aerospace and power-plant structures led to development of more strict safety regulations. To meet these strict standards of safety, there is a need for cost-effective and advanced reliable damage-detection techniques. At present, there are various damage-detection techniques such as visual inspection, radiographic testing, linear ultrasonic techniques etc. Most of these techniques are time-consuming and used for specific purpose. Some of the most upcoming and popular techniques are vibration-based structural health-monitoring techniques.

It was well established that damage in structures such as open/breathing cracks influence their natural characteristics such as frequency, damping and mode shapes. So far several researchers have addressed the issue of the effect of an open crack on dynamics of the structure using finite-element studies and validated the results through experimental analysis. They made an assumption of open crack to avoid the problem of nonlinearities related to its closure [1]. Some researchers have investigated the effect of the breathing crack on the dynamics of the beam. Their investigation revealed presence of higher harmonics in a frequency response [2,3].

1.1. Nonlinear vibrations in Structural Health Monitoring (SHM) applications

Physical basis of crack-induced nonlinearities has been investigated for both vibration- and ultrasonic-based techniques. In its simplest form, the breathing crack might be modelled as bi-linear spring, with different elastic moduli for ‘open’ and ‘closed’ crack. More complex models have to deal with a nonzero velocity (that leads to impacts) and roughness of crack faces during the contact. It was shown that not only contact of crack faces, but also temperature gradients near crack tips play a significant role in introducing the nonlinearity [4,5].

Numerous papers addressed the problem of vibrations of cracked structures and NDT (Non-Destructive Testing) methods based on crack–induced nonlinearity [6-9]. It is a well-known fact, that a crack affects the natural frequencies of the structure. For a breathing-crack model, a measured frequency drop was shown to be smaller than in case of the open-cracks model [9].The smaller frequency drop makes the closing crack more difficult to detect than open cracks by using the 'linear' methods. The problem of natural frequencies of a beam with the breathing crack was solved by introducing 'bilinear frequency', based on two linear configurations (with the open and closed cracks) [10]. Numerous investigations are focused on higher and sub-harmonic frequencies in the output spectra [7]. Sundermeyer and Weaver [11] presented a model, which takes into consideration appearance of difference of two driving frequencies in presence of nonlinearity. Bi-spectral analysis was proposed as efficient signal-processing method for detection of bi-linear stiffness in the structure [12]. More complex signal-processing methods are based on instantaneous frequency calculated via Hilbert Transform [13]. The methods employing modulation of an ultrasonic wave by vibrations were also suggested. In cracked specimens, the modulation might be observed in the form of sidebands around the ultrasonic wave frequency peak [14].

A 2D FE model of a cracked beam was investigated by Andreaus *et al.* [15]. The beam was excited with harmonic signal at different frequencies (multiplies of the first natural frequency) revealing significantly different spectra. The model of the system was simplified with the SDOF (Single Degree of Freedom) system. These studies were extended in [16] by taking the depth and position of the crack into consideration; a method for crack detection was proposed. In [17] a finite-element simulation was used to study the longitudinal harmonic loading to assess damage in a cracked cantilever bar. For nonlinear materials, the properties were introduced in terms of their plastic behaviour. The effect of crack size and position along the beam length was studied. It was shown that observed change in the natural frequency can be used to characterize the damage state of the component.

1.2. Earlier work

The longitudinal vibration of the bar was investigated by Hiwarkar *et al.* [18,19]. The expression for dynamic compliance (receptance) was obtained in the following form [18]:

$$L_l(l,j\omega) = \frac{l}{ES\zeta}\left[\tan\zeta - \frac{j\mathrm{X}}{4\pi}\frac{\zeta + \frac{1}{2}\sin 2\zeta}{\cos^2\zeta}\right] \quad (1)$$

where $\zeta = \frac{\omega l}{\gamma}$, $\gamma = \left(\frac{E}{\rho}\right)^{\frac{1}{2}}$, l, S and E are the length, cross-sectional area and modulus of elasticity of the bar, respectively, ω is frequency of excitation and X stands for absorption coefficient. The equation was transformed using a general linear theory of integral equations for straight rods, to enable its numerical simulation in Matlab-Simulink environment [18]. The results of simulations include generation of a higher harmonics, frequency shift and their dependency on crack parameters, as well as generation of low-frequency components.

1.3 Aim and scope

In the present paper, simulations with a two-dimensional finite-element model are presented. The investigations are focused on the influence of a breathing crack on dynamics of the beam, in particular, of the impacts of crack faces. The beam was excited with a longitudinal force. Longitudinal vibrations were measured and compared at different distances from the crack.

2. Finite-element simulations

2.1. Problem formulation

The finite element model was developed using commercial FE software ABAQUS. Figure 1 shows the scheme of the beam used for simulations. The beam's dimensions are 300 mm × 25 mm × 10 mm. Simulation parameters are given in Table 1. The crack was placed in the middle of the beam, its depth was 10 mm. The beam was meshed with 2D plane-stress elements CPS4R (4-node bilinear, reduced-integration with hourglass control) and CPS3 (3-node linear). A basic element size applied was 1mm; smaller elements were used to mesh the crack's vicinity. The meshed beam is shown in Figure 2a and details of crack-tip meshing are shown in Figure 2b. One end of the beam was fixed; the other end was excited by a longitudinal sinusoidal force with frequency of the first longitudinal mode. The response was measured at eight different locations from the crack, marked with number 1 – 8, as shown in Figure1.

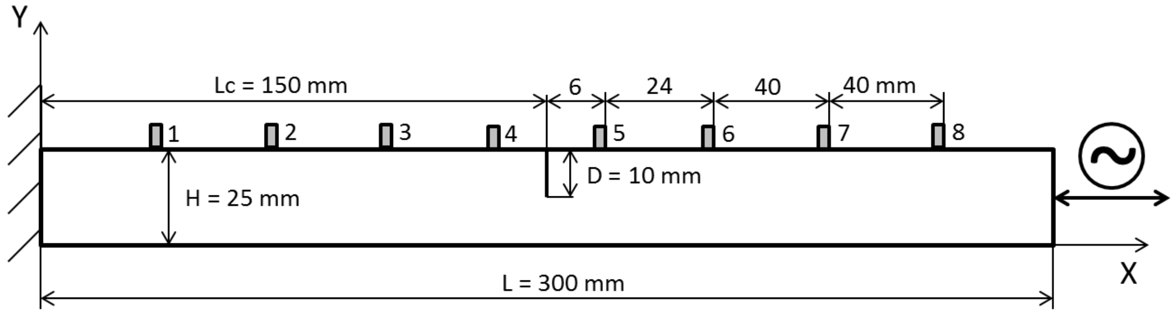

Figure 1. Scheme of cracked beam

Table 1. Simulation parameters

Material	Aluminium (Grade 6082T6)
Modulus of elasticity ($kg/mm\ s^2$)	7×10^7
Density (kg/mm^3)	2.70×10^{-6}
Poisson's ratio	0.33
Longitudinal stiffness of beam (N/mm)	5.84×10^7
Normal contact stiffness (N/mm)	7×10^9

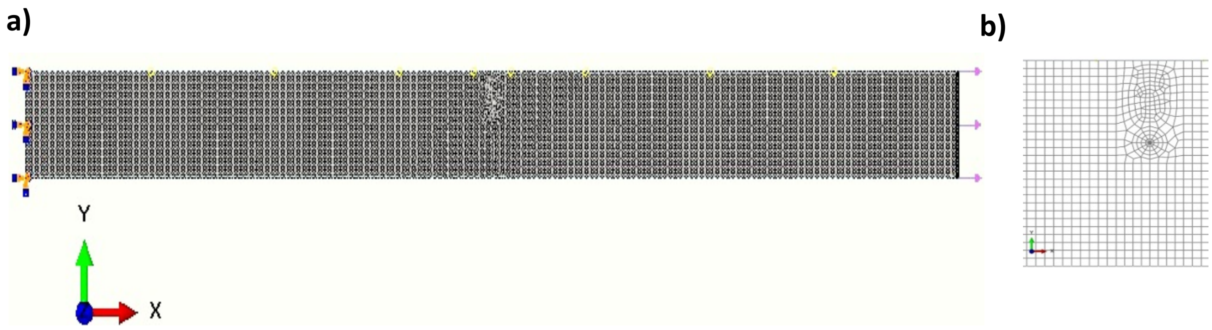

Figure 2.(a) Meshed beam, (b) mesh near crack

2.2.Results

Longitudinal natural frequencies of the un-cracked beam were extracted by modal analysis in Abaqus. It was found that the first longitudinal natural frequency of un-cracked beam was 4248 Hz (Figure 3; notation *acc* and *ampl* for axis titles in Figures 3-6 is used for *acceleration* and *amplitude*, respectively). In the figure 3, the time and frequency responses of intact beam are shown for sensors 1-4. The results obtained for sensors 5-8 are similar and thus are not shown. The cracked beam was excited at the first longitudinal natural frequency of the un-cracked beam, i.e. at 4248 Hz. The results obtained with Abaqus were post processed with Matlab software. The time and frequency responses were measured at different locations along the beam and the obtained results were compared. It was found that presence of the crack led to distortion of the time response (Figures. 4a and 5a) that is stronger near the crack. The frequency response (Figures. 4b and 5b) of the distorted time response (Figures. 4a and 5a) shows the generation of higher harmonics, which are the multiples of the frequency of excitation. Figures 6a and 6b show the magnified view of frequency response of the cracked bar at the excitation frequency measured at different location from sensors 1-8. The results demonstrate that crack-induced nonlinearity has the localised effect on dynamics of the beam as the amplitude of measured response decreases with the distance from the crack.

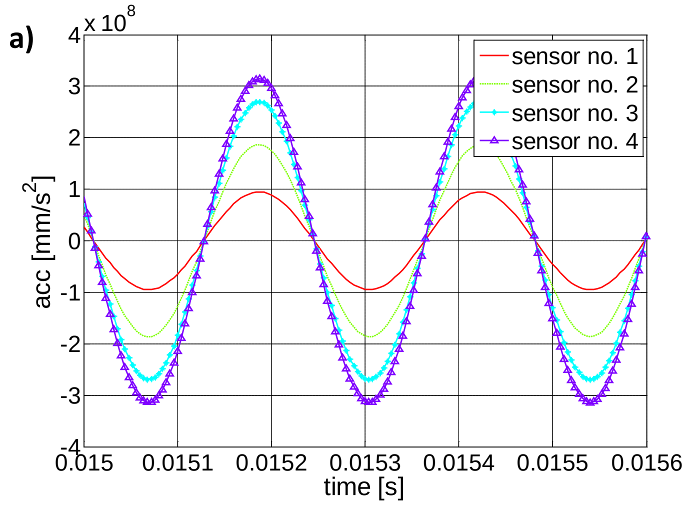

Figure 3. Measured time (a) and frequency (b) responses of un-cracked beam at sensors 1-4.

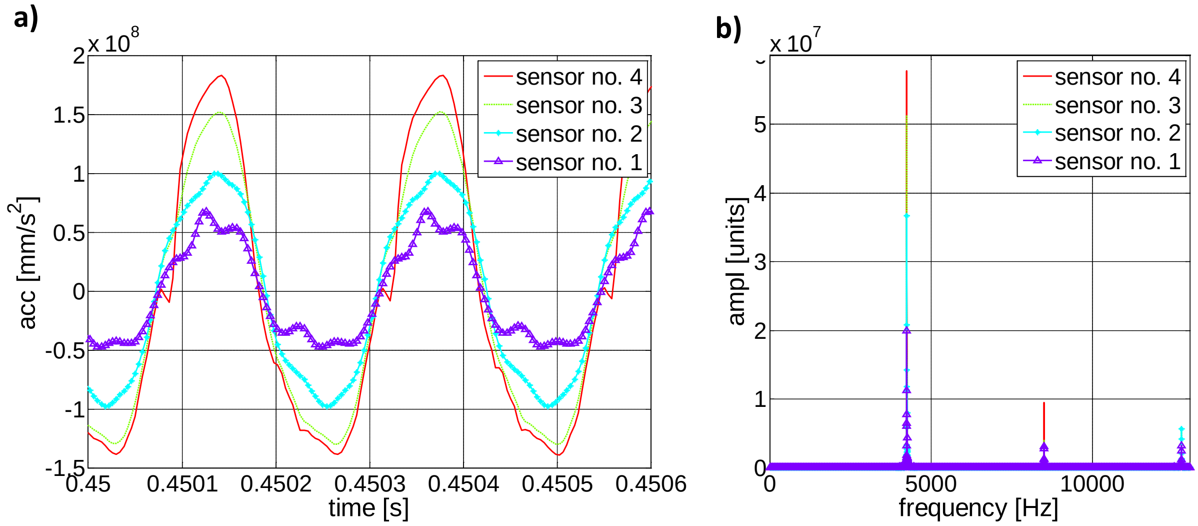

Figure 4. Measured time (a) and frequency (b) responses of cracked beam at sensors 1-4.

It was also observed that the most pronounced distortion was measured at the first harmonic. In order to compare the level of distortion (nonlinearity) of the first harmonic, the coefficient of distortion (D) in the form of

$$D = \frac{f_1}{T_1}, \tag{2}$$

where f_1 is the first harmonic amplitude and T_1 is the first longitudinal mode excitation amplitude, was introduced. Table 2 shows the ratios D for each measurement taken by sensor at 8 different locations on the beam. The ratios measured on the sensors placed between the crack and the fixed end are higher than on the sensor between the crack and free end of the beam.

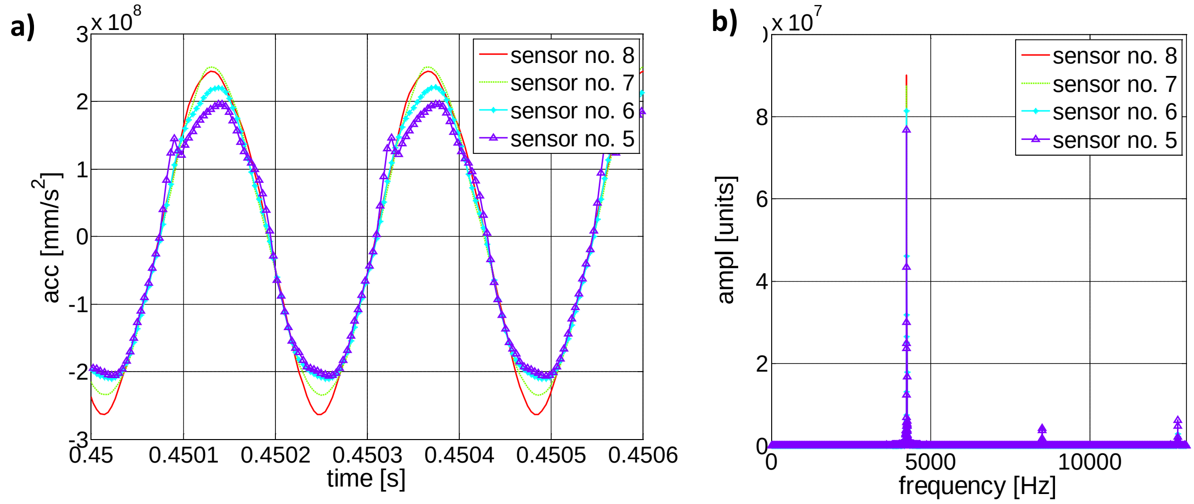

Figure 5. Measured time (a) and frequency (b) responses of cracked beam at sensors 5-8.

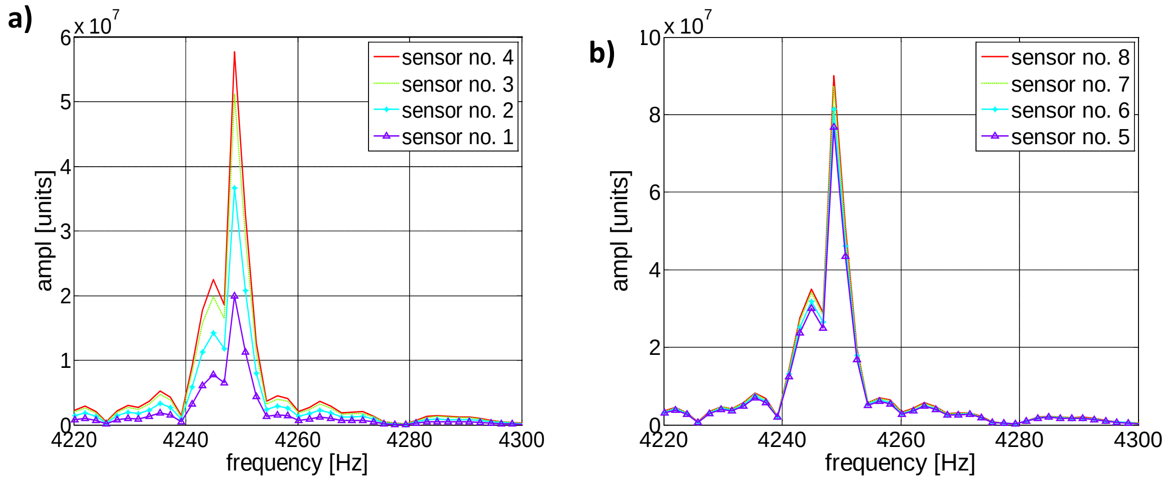

Figure 6. Magnified view of frequency response of cracked bar at excitation frequency: (a) sensors 1-4, (b) sensors 5-8.

The two highest values of distortion ratio were obtained at responses measured in the vicinity of the crack (sensors 4) and near the fixed end (sensor 1). High ratio was also measured on sensor 5, i.e. in the vicinity, but on the free end side, of the crack. From the values of distortion coefficient in Table 2 it is clear that the crack-induced nonlinearity in localized as the distortion coefficient decreases for responses measured far away from the crack with exception of the one measured at sensor 1 which needs further investigation.

3. Conclusions

From the results of two-dimensional finite-element simulations of longitudinal vibrations of the cracked beam it was established that crack-induced nonlinearity had localized effect on dynamics of the beam as the amplitude of measured response decreases when measured far away from the crack.

Further to this, it was also found that the distortion coefficient of the first harmonic decreased for the responses measured far away from the crack which can also be useful tool for monitoring the structural health.

Table 2. Coefficient of distortion D

Sensor	1st Harmonic
1	0.15660
2	0.01559
3	0.07765
4	0.16590
5	0.05603
6	0.01017
7	0.02500
8	0.02662

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