Spin-Triplet Superconductivity Induced by Longitudinal Ferromagnetic Fluctuations in UCoGe: Theoretical Aspect

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Abstract. Identification of pairing mechanisms leading to the unconventional superconductivity realized in copper-oxide, heavy-fermions, and organic compounds is one of the most challenging issues in condensed-matter physics. Clear evidence for an electron-phonon mechanism in conventional superconductors is seen by the isotope effect on the superconducting transition temperatures T_{SC}, since isotopic substitution varies the phonon frequency without affecting the electronic states. In unconventional superconductors, magnetic fluctuations have been proposed to mediate superconductivity, and considerable efforts have been made to unravel relationships between normal-state magnetic fluctuations and superconductivity. Here, we show that characteristic experimental results on the ferromagnetic (FM) superconductor UCoGe ($T_{\mathrm{Curie}} \sim 2.5$ K and $T_{\mathrm{SC}} \sim 0.6$ K) can be understood consistently within a scenario of the spin-triplet superconductivity induced by FM spin fluctuations. Temperature and angle dependencies of the upper critical magnetic field of the superconductivity (H_{c2}) are calculated on the basis of the above scenario by solving the Eliashberg equation. Calculated H_{c2} well agrees with the characteristic experimental results observed in UCoGe. This is a first example that FM fluctuations are shown to be a pairing glue of superconductivity.

1. Introduction

The discovery of superconductivity in ferromagnetic (FM) UGe_2 under pressure gave great impact for condensed-matter community [1], since ferromagnetism and superconductivity have been considered to be mutually exclusive phenomena. After the discovery, superconductivity has been found in several U-based FM compounds [2, 3]. UCoGe is one of the compounds among the FM superconductors discovered so far which can be readily explored in experiments, because of its high superconducting (SC) transition temperature (T_{SC}) and low Curie temperature (T_{Curie}) at ambient pressure. Actually, important and intriguing experimental results have been reported for UCoGe [4, 5], and some of them are summarized as follows.

(i) Microscopic measurements of μSR [Fig.1(a)] [6] and Co-NQR [Fig.1(b)] [7] have shown that the superconductivity occurs within the FM region, resulting in homogeneous microscopic coexistence of the ferromagnetism and superconductivity. The superconductivity is also found in the paramagnetic state and T_{SC} shows a broad maximum around a critical pressure of ferromagnetism [Fig.1(c)] [3], which is in sharp contrast with UGe_2 where the SC is seen only in the FM phase [1].

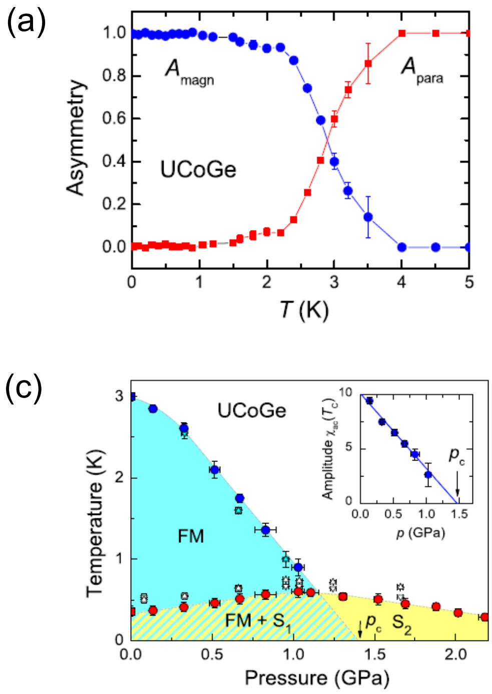

Figure 1. (a)Temperature variation of the normalized asymmtries ($A_{\mathrm{magn}} + A_{\mathrm{para}} = 1$) of the FM and paramagnetic phases measured with zero-field muon-spin rotation on UCoGe [6]. Below ~ 1.5 K, magnetism is observed in the whole sample volume although superconductivity occurs at ~ 0.8 K, indicative of coexistence of ferromagnetism and superconductivity on a microscopic scale. (b) Temperature dependence of ^{57}Co $1/T_1$ in the single-crystal sample [7]. $1/T_1$ above 2.3 K, shown by the blue dots, was measured at the paramagnetic (PM)-frequency (8.3 MHz) peak arising from the $\pm 5/2 \leftrightarrow \pm 7/2$ transitions, and $1/T_1$ below 2.3 K, shown by the red dots, was measured at the FM-frequency (8.1 MHz) peak shifted from the PM peak due to the appearance of the internal field at the Co site. The inset shows the NQR signal arising from the $\pm 5/2 \leftrightarrow \pm 7/2$ transitions obtained at 4.2 K (PM) and 0.9 K (FM). Two $1/T_1$ components were observed in the SC state, the faster (slower) component denoted by red open (closed) squares; the red broken curve below T_{SC} represents the temperature dependence calculated assuming a line-node gap with $\Delta_0/k_{\mathrm{B}}T_{\mathrm{SC}} = 2.3$. The SC anomaly observed from the FM signal is clear evidence that superconductivity occurs in the FM region. (c) Pressure-temperature phase diagram of UCoGe [11]. Ferromagnetism is denoted as blue area, superconductivity is as yellow area. $T_{\mathrm{SC}}(p)$ extrapolates to a FM quantum critical point at the critical pressure p_{c}. Superconductivity coexists with ferromagnetism below p_{c} as blue-yellow hatched area. Inset: Amplitude of $\chi_{\mathrm{ac}}(T)$ at T_{C} as a function of pressure. The data follow a linear p dependence and extrapolate to $p_{\mathrm{c}} = 1.46 \pm 0.10$ GPa.

(ii) The temperature variation of the ^{59}Co-NQR spectra shown in Fig. 2 indicates that the FM region grows continuously around $T_{\rm Curie}$ coexisting with the PM region, and that the FM moment in the FM region appears discontinuously, whereas the hysteresis behavior was not observed in the temperature variation of the spectra and $1/T_1$ in the both states below $T_{\rm Curie}$ is almost identical [8]. Since the divergence of $1/T_1$ and the clear anomaly in the specific heat were observed at $T_{\rm Curie}$ [7, 5] and the FM ordered moment is small ($\mu_s \sim 0.05\mu_B$) at low temperatures [5] , the FM phase transition in UCoGe is considered to possess weak first-order character with critical FM fluctuations, which is close to second-order one. Although we need further investigation of sample and pressure dependence on the FM transition to understand the transition character thoroughly, the presence of the critical fluctuations implies the validity of the scenarios of spin fluctuation-induced superconductivity.

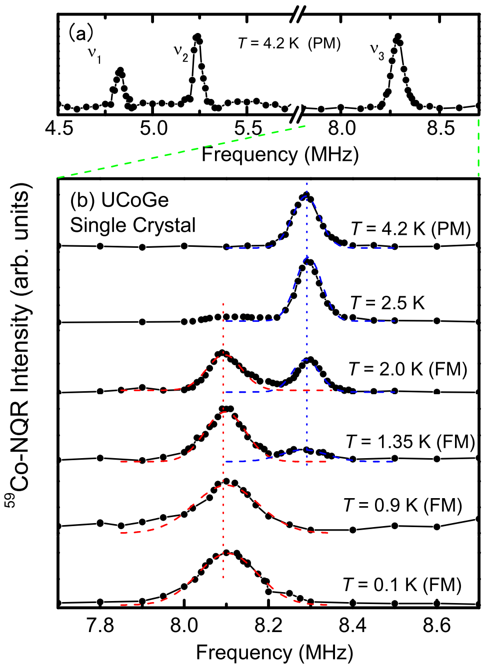

Figure 2. (a) ^{59}Co-NQR spectra (ν_1, ν_2, ν_3) at 4.2 K in the PM state[8]. (b) Temperature dependence of the NQR spectrum from the $\pm 5/2 \leftrightarrow \pm 7/2$ transitions (ν_3)[8]. The PM and FM signals coexist between 1 and 2.7 K. The blue (red) broken lines represent Gaussian fits to the PM (FM) peaks; the solid lines are guides to the eye.

(iii) Although transport properties show rather three dimensional isotropic characters [9], studies of the SC upper critical magnetic field (H_{c2}) [Fig. 2 (a)] and angle dependence of H_{c2} [Fig. 2 (b)] have shown that SC properties are highly anisotropic [10, 11, 12]; the superconductivity survives in fields exceeding 15 T along the a and b axes, whereas H_{c2} for fields along the c direction (H^c_{c2}) is as small as 0.8 T.

(iv) The static magnetic susceptibility is strongly anisotropic [10]; the magnetic anisotropy is Ising like with the c axis as a magnetic easy axis [Fig. 4 (a)]. In addition, direction-dependent nuclear-spin lattice relaxation rate ($1/T_1$) measurements on a single crystalline sample have revealed the magnetic fluctuations in UCoGe to be Ising-type FM ones along the c axis (longitudinal-mode fluctuations) as shown in Fig. 4 (b) [13]. The Ising-type fluctuations are also suggested from the anisotropic correlation length measured by the inelastic neutron measurements [23].

(v) From detailed angle-resolved NMR and Meissner measurements [9], it was found that magnetic fields along the c axis (H^c) is a tuning parameter of the longitudinal FM fluctuations and strongly suppresses the FM fluctuations, as shown in Fig. 5 (a) and 5 (b). This sharp angle dependence of $1/T_1$ was also confirmed at 600 mK just above $T_{\rm SC}$ [14]. It

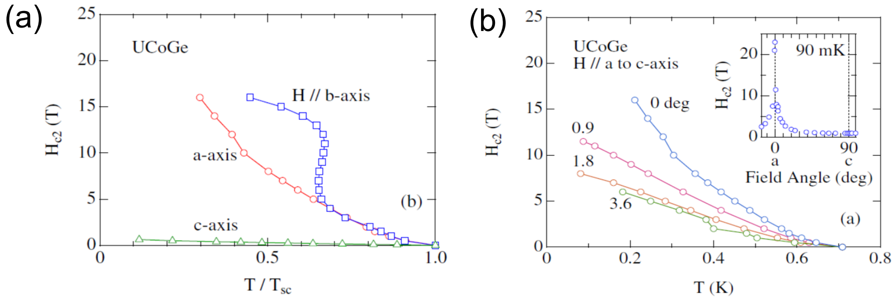

Figure 3. (a) Temperature dependence of the upper critical fields for $H \parallel a, b$ and c-axis [12]. The temperature is normalized by the SC critical temperature $T_{\rm SC}$ at zero field. (b) Temperature dependence of the upper critical field $H_{\rm c2}$ close to $H \parallel a$-axis [12]. The field direction is tilted from the a to c-axis. The inset shows the angular dependence of $H_{\rm c2}$ at 90 mK. The values of $H_{\rm c2}$ greater than 16 T are the results of linear extrapolations to 90 mK.

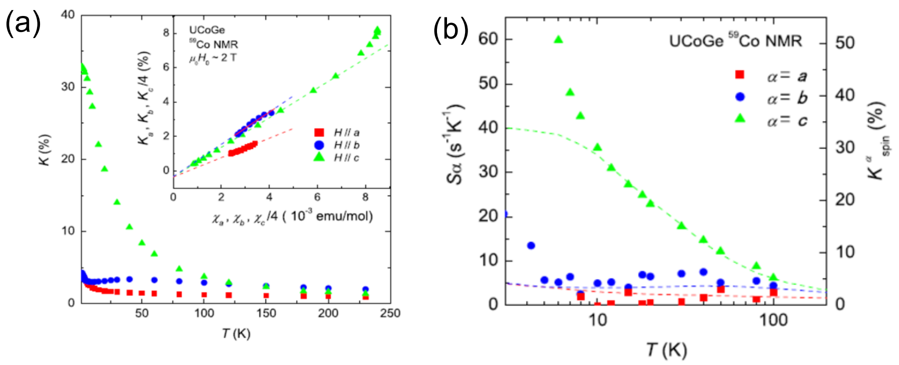

Figure 4. NMR shift K measured in field along the a, b and c directions [13]. Remarkable anisotropy was observed. In the inset, K is plotted against the bulk susceptibility χ measured in 2 T. The nearly identified slopes for all the directions indicate that the hyperfine coupling constants are isotropic. (b) Direction-decomposed dynamic susceptibility S_α and static susceptibility $K_{\rm spin}$ along each direction [13]. Identical Ising anisotropy for both quantities found above 8 K suggests that the longitudinal mode dominates these fluctuations. It is noteworthy that in this T range, the Knight shift scales with S_α, which is predicted by the self-consistent renormalization theory for a 3-dimensional nearly FM metals [15].

was also shown that the superconductivity is observed in the limited magnetic-field region where the longitudinal FM spin fluctuations are active [Fig. 4 (c)]. These results suggest that the superconductivity in UCoGe is tightly coupled with the longitudinal FM spin fluctuations along the c-axis.

From a theoretical point of view, when an itinerant FM superconductor possesses the large energy splitting between the majority and minority spin Fermi surfaces, exotic spin-triplet superconductivity is anticipated, in which pairing is between parallel spins within each spin Fermi surface. In addition, it has been argued that critical FM fluctuations near a quantum phase transition could mediate spin-triplet superconductivity [16, 17, 18, 19, 20, 21], and actually, (iv) in the above indicates that the superconductivity in UCoGe is intimately coupled with the Ising FM spin fluctuations. However, as far as we know, there have been no reports identifying

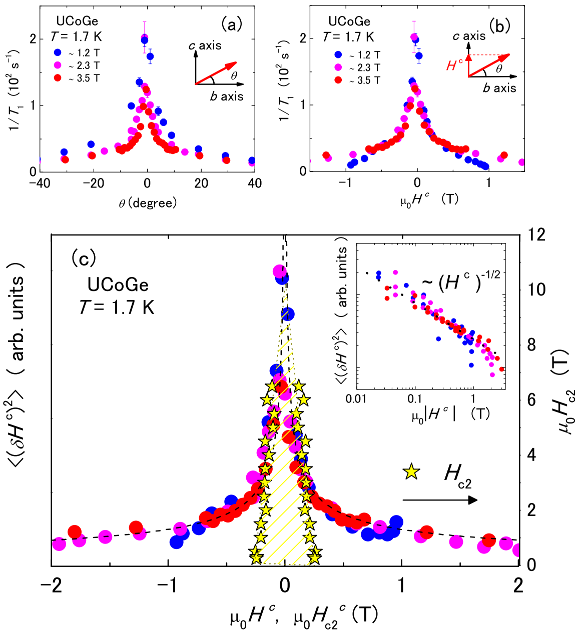

Figure 5. (a) Angle dependence of $1/T_1$ in the bc plane measured in three different magnetic fields at 1.7 K [9]. (b) Plot of the $1/T_1$ against H^c. The $1/T_1$ data collapse onto a single curve when plotted against H^c. (c) H^c dependence of magnetic fluctuations along the c axis $\langle(\delta H^c)^2\rangle$ at 1.7 K, extracted using $\langle(\delta H^c)^2\rangle \propto \frac{1}{\cos^2\theta}\left(\frac{1}{T_1}(\theta) - \frac{(1+\sin^2\theta)}{2}\frac{1}{T_1^c}\right)$. Angle dependence of H_{c2}, determined by the onset of the Meissner signal, is plotted against $H_{c2}^c \equiv H_{c2}\sin\theta$. It is found that the superconductivity is observed in the limited field region (yellow slashed area), where the longitudinal fluctuations are active. Inset: plot of $\langle(\delta H^c)^2\rangle$ against $|H^c|$. The relation of $\langle(\delta Hc)^2\rangle \propto 1/\sqrt{H^c}$ is shown by dashed lines in (c) and the inset.

magnetic fluctuations inducing unconventional superconductivity. Here, we show, from model calculations on the basis of the scenario of FM spin fluctuation-mediated superconductivity in UCoGe, that unconventional temperature and angle dependence of H_{c2} is consistently understood by adopting the characteristic longitudinal FM fluctuations revealed experimentally. Besides, by comparing calculations for possible pairing states, we also discuss candidates of SC gap symmetries in UCoGe.

2. Calculations and Results

2.1. Model

In this section, we introduce a simple model to study H_{c2} in UCoGe at ambient pressure where the ferromagnetism and superconductivity coexist. Although UCoGe has a complicated multi-band structure, since the spin fluctuations dominate the low energy physics in UCoGe, we can use a simplified model to study its superconductivity with appropriately taking into account the critical FM spin fluctuations observed in the experiments [9, 13, 23]. The model relevant to UCoGe in the FM phase with an applied magnetic field $\boldsymbol{H}$ is

$$S = S_0 + S_{\rm ex} + S_{\rm int}, \tag{1}$$

$$S_0 = \sum_k \int_0^{1/T} d\tau c_k^\dagger(\tau)[\partial_\tau + \varepsilon_k]c_k(\tau), \tag{2}$$

$$S_{\rm ex} = -(\boldsymbol{h}_{ex} + \mu_B \boldsymbol{H}) \cdot \sum_k \int_0^{1/T} d\tau c_k^\dagger(\tau)\boldsymbol{\sigma} c_k(\tau), \tag{3}$$

$$S_{\rm int} = -\frac{2g^2}{3}\sum_q \int_0^{1/T} d\tau \int_0^{1/T} d\tau' S_q^c(\tau)\chi^c(\boldsymbol{q}, \tau-\tau')S_{-q}^c(\tau'), \tag{4}$$

and effects of the vector potential on electron motions are taken into account in a semiclassical way. $c_k = (c_{k\uparrow}, c_{k\downarrow})^T$ is an annihilation operator of low energy quasiparticles which are formed through hybridizations between conduction electrons and f-electrons. A typical energy scale of the quasiparticles is $40 \sim 50$ (K) estimated from the specific heat [3] and the coherence temperature [13]. Since the anisotropy in the resistivity in UCoGe is small as discussed in the introduction, we use an isotropic dispersion $\varepsilon_k = -2t\sum_{i=a,b,c}\cos k_i a_0 - \mu$ where a_0 is the lattice constant and μ is the chemical potential. We fix the electron density per site as $n = 0.15$ which leads to a small but nearly spherical Fermi surface. As discussed in Ref.[9], the anisotropic mass model cannot explain the anomalous angle dependence of H_{c2}. In our model, the anisotropy in H_{c2} arises from the exchange term and the pairing interaction term in Eq.(1). We use an energy unit $t = 1$ and a length unit $a_0 = 1$.

The second term of Eq.(1) includes the exchange splitting of the Fermi surface in the FM phase which is assumed to be large compared to the SC transition temperature at zero-field $T_{\mathrm{SC}0}$. The exchange field is parallel to the c-axis and fixed to be $\boldsymbol{h}_{ex} = (0, 0, h_{ex}) = (0, 0, 0.5t)$, which would lead to suppression of the Pauli depairing effect against in-plane magnetic fields in the equal spin pairing states of the superconductivity [22]. This is a key assumption to understand the large anisotropy in H_{c2} in the present study, because the equal spin pairing states are not robust against the in-plane Zeeman field while the observed $H_{c2}^{a,b}$ are huge in UCoGe. However, we put a remark on this assumption. UCoGe has a multi-band structure with a strong spin-orbit interaction and $5f_{5/2}$ states are located near the Fermi level. In such a system, it is possible that, even if the magnetization in the FM state is small, the exchange splitting for each Fermi surface is large. Actually, in the band calculations, the orbital magnetizations and the spin polarizations cancel to result in a small total magnetic moment. The present assumption can be checked by NMR Knight shift measurements in the SC states.

Finally, the last term in Eq. (1) describes the interaction between the quasiparticles through the Ising spin fluctuations, and $S_q^c(\tau) = \sum_k c_{k-q}^\dagger(\tau)(\sigma^c/2)c_k(\tau)$. The dynamical susceptibility $\chi^c(\boldsymbol{q}, i\Omega_n)$ strongly depends on the c-axis magnetic field as revealed in the NMR experiments [9].

$$\chi^c(\boldsymbol{q}, i\Omega_n) = \frac{\chi_0}{\delta + q^2 + |\Omega_n|/\gamma_q}, \tag{5}$$

$$\delta(H^c) = h_{ex}^2 + c'\sqrt{H^c}, \tag{6}$$

where $\gamma_q = vq$ and $v = 4$ is approximately the Fermi velocity. Here, we have assumed a simple Landau damping in $\chi^c(\boldsymbol{q}, i\Omega_n)$, although existence of non-Landau damping was reported [23, 24]. The coefficient c' is determined by the NMR experiments, and a normalization factor $H_0^c \sim 1$ T is introduced as $c' \equiv ch_{ex}^2/\sqrt{H_0^c}$ so that the dimensionless parameter c is $c \sim \mathcal{O}(1)$. Although the origin of this magnetic field dependence in the susceptibility is not so clear, we have a possible explanation for it. According to the band calculation, the $5f_{5/2}$ bands in UCoGe have some semi-metallic like structures around several k-points [25]. For such structures, the density of states would be $\sim \sqrt{H^c}$, which results in $(\chi^c)^{-1} \sim \mathrm{const.} + \sqrt{H^c}$ near the criticality. In this scenario, detailed structures in the bands are important, and therefore, the spin fluctuations in UCoGe under a magnetic field can depend on sample qualities. This problem will be studied elsewhere. We note that the coupling constant g should be regarded as a renormalized one including the vertex corrections which enhance the SC transition temperatures. Although we have postulated a simple analytic q-dependence in χ^c, there might exist non-analytic corrections $\sim q^2\log q$ in 3-dimensional isotropic systems [26, 27, 28, 29]. The non-analytic corrections are important in connection with fluctuation-induced first order phase transitions. However, since the first order character of the FM transition is weak, and critical FM fluctuations are well developed in UCoGe as discussed in Sec.1, it would be legitimate to neglect the first order character, and assume that χ^c is analytic in q as long as calculations of the superconducting properties are concerned. In addition, if exist in the Ising-like system, the non-analytic corrections would not change the present study qualitatively as discussed in [30].

2.2. Eliashberg equation

In this study, we focus on the anisotropy between the a and c-axes upper critical fields (H_{c2}^a and H_{c2}^c). However, it was reported that the b-axis upper critical field H_{c2}^b shows a "S-shaped" curve at high fields, and relations to the reentrant superconductivity in URhGe were discussed [12]. In URhGe, the magnetization flips at a critical $H^b = H^{b*}$ and the spin fluctuations at H^{b*} are considered to be different from those of $H^b = 0$ [31]. Similar scenarios are expected to hold also in UCoGe where the S-shaped H_{c2}^b would indicate a crossover of the superconductivities of different mechanisms under magnetic field along the b axis. The problem of H_{c2}^b in UCoGe is an another issue and will be studied elsewhere.

To determine the upper critical field H_{c2} in the ac-plane, we solve the linearized Eliashberg equation in the presence of a vector potential [32]. The Eliashberg equation for an equal-spin pairing state in the FM phase is

$$\Delta_{\sigma\sigma}(k) = -\frac{T}{2N}\sum_{k'} V(k,k')[G_{\sigma\sigma'}(k'+\Pi)G_{\sigma\sigma'}(-k') + G_{\sigma\sigma'}(k')G_{\sigma\sigma'}(-k'+\Pi)]\Delta_{\sigma'\sigma'}(k'), \quad (7)$$

where $\Pi = (-i\nabla_R - 2e\boldsymbol{A}(\boldsymbol{R}), 0)$ and $\nabla \times \boldsymbol{A} = \boldsymbol{H} = H(\cos\theta, 0, \sin\theta)$. A short notation $k = (\boldsymbol{k}, i\omega_n)$ is used hereafter. Here we have neglected the effect of the magnetization in the orbital motion, because the observed moment is very small in UCoGe [33]. Note that the Green's function $\hat{G}(k)$ has off-diagonal elements when $\theta \neq 90°$. The gap function depends both on the momentum in the relative coordinate and the center-of-mass coordinate, and is expanded as $\Delta_{\sigma\sigma}(k, \boldsymbol{R}) = \Delta_{\sigma\sigma}(k) \cdot \phi_0(\boldsymbol{R})$, where ϕ_0 is the lowest level Landau function.

We solve the linearized Eliashberg equation for two cases corresponding to the classification by the magnetic point group $D_2(C_2^z)$ for UCoGe which has an orthorhombic crystal structure [33, 34]. One is the so-called A state which corresponds to point node symmetry with a d-vector $\boldsymbol{d} \sim (a_1k_a + ia_2k_b, a_3k_b + ia_4k_a, 0)$ near the Γ point. The other one is the so-called B state corresponding to horizontal line node symmetry with $\boldsymbol{d} \sim (b_1k_c + ib_2k_ak_bk_c, ib_3k_c + b_4k_ak_bk_c, 0)$. Here, $\{a_i\}$ and $\{b_i\}$ are real coefficients. Both of the gap functions are determined by self consistent calculations, and the resulting SC states are non-unitary because of the presence of the exchange field $\boldsymbol{h}_{ex}$. We note that, under the applied magnetic field $\boldsymbol{H}$, the calculated gap functions are distorted and has lower symmetry than that of the cubic lattice in the present model.

The pairing interaction is evaluated at the first order in $g^2\chi_0$,

$$V(k,k') = -\frac{g^2}{6}\chi^c(k-k') + \frac{g^2}{6}\chi^c(k+k'). \quad (8)$$

For the selfenergy, we include only the term

$$\begin{aligned}\Sigma_0(k) &\equiv (\Sigma_{\uparrow\uparrow}(k) + \Sigma_{\downarrow\downarrow}(k))/2 \\ &= \frac{T}{6N}\sum_q g^2\chi^c(q)[G^0_{\uparrow\uparrow}(k+q) + G^0_{\downarrow\downarrow}(k+q)],\end{aligned} \quad (9)$$

where $G^0_{\sigma\sigma}$ is the non-interacting Green's function. Other neglected terms in the self energy are much smaller than Σ_0 in magnitude. As noted in the previous section, the coupling constant g is a renormalized one.

In the present study, the coupling constant is fixed as $g^2\chi_0 = 125t$ which is rather large to give a high transition temperature so that the numerical costs are reduced. However, qualitatively, our main results do not depend on the value of $g^2\chi_0$. For this value of $g^2\chi_0$, the mass enhancement factor $1/z$ is ~ 1.3, which is not so large because the density of states at the Fermi surface is small in the present parameters. The transition temperature for the A state without magnetic field is $T_{\rm SC0} = 0.020t$ which we regard as 1.0 K according to the experiments for UCoGe. Then, the hopping integral is fixed as $t = 50$ K throughout in our study, and the order of this value is consistent with the coefficient of the linear specific heat $\gamma \sim 50$ mJ/K^{2}mol [3] and the coherence temperature $T^* \sim 40$ K [13]. We also fix the lattice constant, $a_0 = 4.0$ Åwhich leads to an effective mass for the orbital motion of the Cooper pairs $m_{\rm eff} \equiv \hbar/ta_0^2 \sim 440m_0$ where m_0 is the bare electron mass. Correspondingly, we fix $H_0^c = 0.02t \sim 1$ T, which is consistent with the NMR experiments [9].

2.3. Results for A state

Before discussing numerical results of H_{c2} of the present model, we note that the orbital limit $H_{\rm orb}$ depends on positions of gap nodes in general [30]. A simple explanation is as follows. Let us consider a superconductor with isotropic Fermi surface which has nodes in a gap function. The effective velocity for the cyclotron motion of the orbital depairing effect can be of the form of $\tilde{\boldsymbol{v}}_k = \varphi_k \boldsymbol{v}_k$ for the basis function φ_k corresponding to the SC gap symmetry, where $\boldsymbol{v}_k$ is a velocity of electrons. For φ_k with horizontal line nodes, when the magnetic field is parallel to the c-axis, the cyclotron motion is suppressed at the nodes where in-plane $\tilde{\boldsymbol{v}}_k$ is zero, resulting in a large orbital limiting field. The cyclotron motion is not suppressed for $H \perp c$ because $\tilde{\boldsymbol{v}}_k \perp H$ is non-zero on the Fermi surface, which leads to the anisotropy of the orbital limiting field $H_{\rm orb}^{\parallel c} > H_{\rm orb}^{\perp c}$. On the other hand, for the point node gap function, $\tilde{\boldsymbol{v}}_k$ cannot be zero except for the poles of the Fermi surface $k_a = k_b = 0$ which are realized as cross points of two line nodes $k_a = 0$ and $k_b = 0$. Therefore, for point nodes at the poles of the Fermi surface, the order is turned over, $H_{\rm orb}^{\parallel c} < H_{\rm orb}^{\perp c}$. Following this consideration, if there is no suppression of the pairing interaction and the Pauli depairing effect is negligible, $H_{c2}^c < (>) H_{c2}^a$ is expected for the A (B) state.

We now turn to discussions of calculation results. Temperature dependence of $H_{c2}^{a,c}$ with $c = 1, 3$ for the A state is shown in Fig. 6. It is found that the qualitative behaviors of H_{c2}

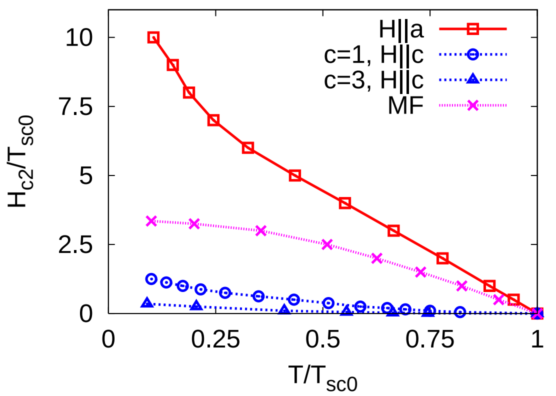

Figure 6. Temperature dependence of H_{c2} for the a and c axes in the A state. The purple curve "MF" is H_{c2}^c calculated with the mean field $\delta(H^c)$.

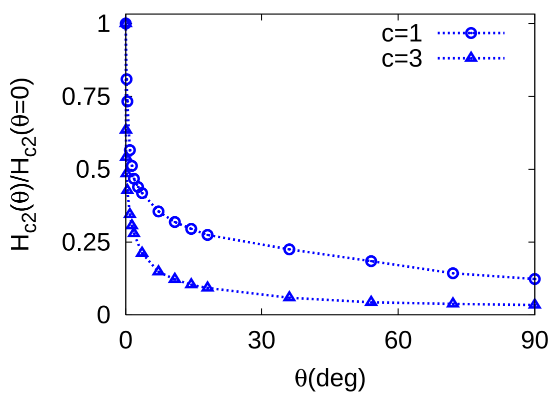

Figure 7. Field angle dependence of H_{c2} at $T = 0.1T_{\rm SC0}$ in the A state.

are independent of c. H_{c2}^a exhibits a strong coupling behavior and reaches $H_{c2}^a/T_{\rm SC0} \sim 10$. Although the precise value of $H_{c2}^a/T_{\rm SC0}$ depends on the choice of h_{ex} and $g^2\chi_0$, strong coupling calculations of H_{c2} naturally explain the upward curvature in H_{c2}^a and its large value observed in the experiments, which is a key to understand the anomalous anisotropic behavior in H_{c2}.

The calculated anisotropy ratio H_{c2}^a/H_{c2}^c becomes $10 \sim 20$ in consistent with the experiments, although the Fermi surface of our model is merely spherical. We emphasize that, for H_{c2} to have a large anisotropy between the a and c-axes, the experimentally observed $\sqrt{H^c}$-dependence in the susceptibility is crucially important. For a comparison, we calculate H_{c2}^c in the case that χ^c has weaker H-dependence, mean-field like dependence of $\delta(H^c) = (h_{ex} + \mu_B H^c)^2$, for which results are shown with purple symbols in Fig. 6. In this case, suppression of H_{c2} for the c-axis is only moderate, and we cannot obtain anomalously large anisotropy in H_{c2}. In Fig. 7, we show field angle dependence of $H_{c2}(\theta)$ at $T = 0.1T_{\rm SC0}$, where θ is the angle from the a-axis to the c-axis. It is seen that $H_{c2}(\theta)$ is strongly suppressed when the field direction is slightly tilted from the a-axis to the c axis. This strong angle dependence is again due to the $\sqrt{H^c}$ suppression of the pairing interaction. The calculation results well explain the unusual angle dependence of H_{c2} observed experimentally. This good agreement is strong evidence that the superconductivity in UCoGe is actually mediated by the critical Ising spin fluctuations.

We note that the relation between H^c and H_{c2} is considered to be an analog of the isotope effect in conventional superconductors where ion mass M controls transition temperature $T_{\rm SC}$. In both cases, suppression of the pairing interaction by increasing H^c or M leads to reduction of $T_{\rm SC}$. One important reason why we can establish such a relation between H^c and H_{c2} in the f-electron system UCoGe is that the critical spin fluctuations dominate the low energy physics. Details of the system and complexity of multi-band are sub-dominant factors for the superconductivity, and therefore, the SC properties are well understood once the spin fluctuations are appropriately taken into account.

2.4. Results for B state

The transition temperature at zero-field for the B state is $T_{\rm SC0} = 0.0213t$ which differs from that for the A state, because of the presence of the exchange field along the c-axis. However, the difference is not important in our study. Temperature dependence of $H_{c2}^{a,c}$ for the B state is shown in Fig. 8. For the B state, strong-coupling behaviors for the a-axis are not so significant,

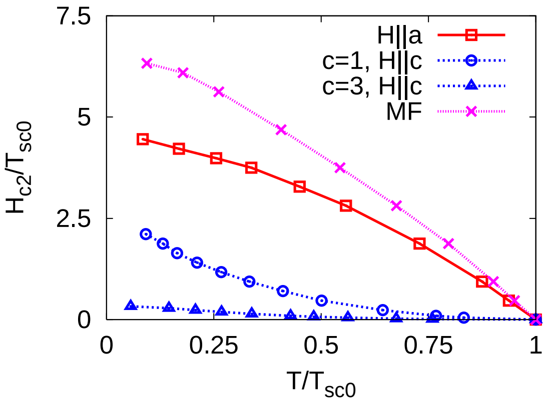

Figure 8. Temperature dependence of H_{c2} for the a and c axes in the B state. The purple curve "MF" is H_{c2}^c calculated with the mean field $\delta(H^c)$.

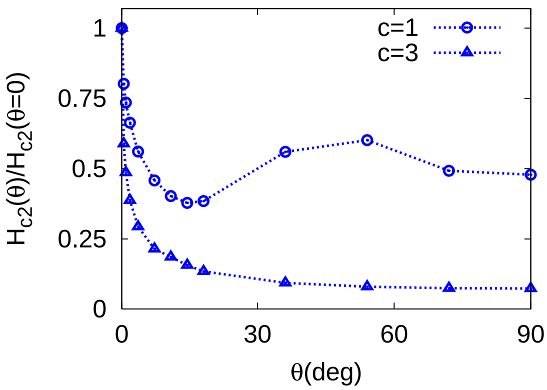

Figure 9. Field angle dependence of H_{c2} at $T = 0.1T_{\rm SC0}$ in the B state.

because the orbital limiting fields depend on the positions of gap nodes as discussed in the previous section, and the enhancement of H_{c2}^a for the B state is weaker than that for the A state. For the present value of $m_{\rm eff}$, the Pauli depairing effect cannot be negligibly small at high magnetic fields, which leads to the suppression of H_{c2}^a in that region. For the c-axis, detailed

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behaviors depend on the value of the parameter c. In the case of rather weak suppression of the Ising fluctuations, i.e. $c = 1$, H_{c2} is enhanced at low temperatures. If the suppression is sufficiently strong, for example $c = 3$, such enhancement cannot be seen. We also show H_{c2}^c calculated with the mean field $\delta(H^c)$ by the purple curve as in the A state. In the B state, H_{c2}^c is not strongly suppressed and H_{c2}^c is larger than H_{c2}^a in contrast to the experiments. The relation $H_{c2}^c > H_{c2}^a$ with the mean-field $\delta(H^c)$ is directly understood from the general relation $H_{\mathrm{orb}}^c > H_{\mathrm{orb}}^a$ for horizontal line node gap functions in simple systems.

Behaviors of H_{c2} as a function of the field angle also depend on the value of c as shown in Fig. 9. For $c = 1$, $H_{c2}(\theta)$ shows a non-monotonic behavior. This is because, for the B state, the relation $H_{c2}(\theta = 0°) < H_{c2}(\theta = 90°)$ holds in the orbital limit if there is no suppression of the Ising fluctuations, while in fact the suppression in $\chi^c(q)$ gets stronger as $\theta \to 90°$. On the other hand, for $c = 3$, H_{c2} monotonically decreases. Comparing these results with those for the A state which are robust against the parameter value, one may conclude that the A state is a promising candidate for the SC realized in the FM state in UCoGe, although the B state is not ruled out.

3. Summary

In this paper, we have studied the upper critical field H_{c2} for the FM superconductor UCoGe in the FM phase. In the introduction, we discussed several experimental observations which are key clues to understand relations between the FM and SC. To clarify the relationship, we introduced the simple model which includes the critical FM spin fluctuations with anomalous $\sqrt{H^c}$ dependence revealed by the NMR experiments. The linearized Eliashberg equations are solved within the first order in the spin fluctuations. In the A state with point nodes, temperature dependence of H_{c2}^a shows an upward curvature while H_{c2}^c is suppressed. When the field angle is tilted from the a to the c axis, H_{c2} is strongly suppressed. These behaviors are totally due to the characteristic suppression of the FM spin fluctuations by H^c, and in nice agreement with the experimentally observed H_{c2} in UCoGe. This is strong evidence of the scenario that the spin-triplet superconductivity is actually mediated by the critical Ising FM spin fluctuations in UCoGe. The relation between H^c and H_{c2} is an analog of the isotope effect in conventional superconductors. In the B state with a horizontal line node, both enhancement of H_{c2}^a and suppression of H_{c2}^c are moderate compared to those in the A state. Therefore, the A state is a promising candidate for the pairing symmetry in UCoGe, although the B state is not ruled out.

Finally, we put remarks on some remaining problems related to the present study. Firstly, in the present study, we have simply adopted the experimentally observed $\sqrt{H^c}$ dependence in $\chi^c(q)$. Although we suggested a possible explanation, the origin of this anomalous dependence is not clear.

Secondly, we have assumed that the exchange splitting is large enough to suppress the Pauli depairing effect. However, this assumption should be checked experimentally by e.g. NMR Knight shift measurements. There is also a related problem. We have focused on the coexistence region of the superconductivity and the ferromagnetism where the exchange splitting can be large. Under pressure [11, 12], T_{Curie} is suppressed and the exchange splitting becomes small. In this case and in the paramagnetic phase, the Pauli depairing effect would not be suppressed. Actually, it was reported that, around the critical pressure where T_{Curie} is extrapolated to zero, $H_{c2}^{a,b}(T \to 0)$ is suppressed while $H_{c2}^c(T \to 0)$ is increasing [35]. These behaviors could be understood as a result of two competing effects; although the Pauli depairing effect would not be suppressed by the exchange field near the criticality, the FM spin fluctuations are enhanced.

Investigations of these issues are needed for a comprehensive understanding of UCoGe, and they would be studied in the future.

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