Effects of the lateral propagating disturbance on wave propagation in periodic beams

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Abstract. The elastic wave propagation in periodic beams with the propagating disturbance is investigated. The effects of the disturbance on the wave propagation in periodic beams are studied. A propagating wave (disturbance) is incident upon the discontinuity and gives rise to transmitted and reflected waves. All of the transmitted and reflected waves of given flexural wave incident upon the beam at some location are found and superposed using the multiple reflections approach. The relation between the wave-field of incident wave (disturbance) and the wave-field of resulting waves on any segment is expressed. Much attention is devoted to the response in the frequency ranges with gaps in the band structure for the corresponding periodic beams with the disturbance. The numerical results of the frequency response function of finite periodic beams with propagating disturbance are presented. With the increase disturbance amplitude, the attenuation of the band gaps gradually decreases. The effects of the number of cells and material parameters on the band gap are also considered.

1. Introduction

In the last decade much attention is focused on periodic flexural systems that may be idealized as consisting of identical elements connected end to end before [1-5]. The work on phononic band gaps has led to a renewed interest in elastic wave propagation in periodic materials or structures, called phononic crystals, which are made of two or more elastic materials with large contrast between their mechanical properties in recent years [6-8]. The properties of frequency band gaps in the phononic crystals have been studied in both theoretical predictions and experiments [9-14]. Most of the previous studies are focused on the band structure in periodic materials with infinite dimensions. Many potential applications of periodic structures may be expected such as vibration isolation technology and acoustic filters.

As we know, many practical engineering structures are conceived as assemblies of relatively slender elements that can be models as one-dimensional mechanical waveguides. Periodic structures are important both as fundamental structural elements and as simple global models for some slender structures, and the length of such periodic materials or structures in practical engineering structures is finite. Jensen [15] investigated phononic band gaps of finite one-dimensional mass-spring periodic structures subjected to axial loading, and indicated that the frequency response function may be calculated to describe the band gap characteristics of finite periodic structures. Kobayashi et al. [16] studied wave transmission characteristics in one-dimensional multilayered media with a finite length using the transform matrix method. Xiang and Shi [17] determined the band gaps in finite periodic beams using differential quadrature method. Yu et al. [18] investigated the propagation properties of flexural wave in periodic beams using the transform matrix method.

Although many methods may be applied to investigate the band gap of periodic structures, it is very difficult to study effects of lateral disturbances on wave propagation in the periodic beam using existing methods, such as transfer matrix method. These methods may be applied to analyze the band gap characteristic of periodic structure, however, the relation between the wave-field of incident wave (disturbance) and the wave-field of resulting waves on any segment can not be expressed. It is not convenient for their promising applications in vibration isolation and other engineering application. Further investigation on the wave propagation in finite dimensional structures with one or more lateral disturbance is of great importance. Particularly, Ungar [5] first proposed the multiple reflections approach. The approach can set up the relation between the wave-field of incident wave and the wave-field of resulting waves on any segment. However, the approach does not be explained in detail, and the validity of the multi-reflection approach is also not proved numerically.

To our knowledge, no work on effects of lateral disturbances on wave propagation in periodic beams structures has been reported. The present paper aims to analyze effects of lateral disturbances on wave propagation in periodic beams from theoretical as well as numerical points of view. In Section 2, periodic binary beams are adopted, and the results of injecting a propagating wave are concerned. All of the transmitted and reflected waves of flexural wave incident upon the beam at some specified location are found and superposed using the multi-reflection approach. In Section 3, the relation of the wave-field on adjacent identical elements is given, comparing the propagating constant calculated by the relation with calculated by the transfer matrix method, iteration times of wave-field can be determined. In Section 4, the influences of the number of cells and material parameters on the band gap of periodic beams are analyzed. Frequency response of periodic beams can be changed by tuning the disturbance. The conclusions from this study are listed in Section 5. From the result, we can observe some interesting and valuable physical phenomena.Another section of your paper

2. Wave reflection and transmission at discontinuities

As shown in Fig. 1, a periodic binary beam consists of an infinite repetition of alternating segments A with length a_1 and segments B with length a_2. So the periodic beam's lattice constant is $a = a_1 + a_2$. In other word, the whole beam is split into successive unit cells. Each cell contains two segments with length a_1 and a_2. The uniform Euler-Bernoulli beam model with constant cross-section is used.

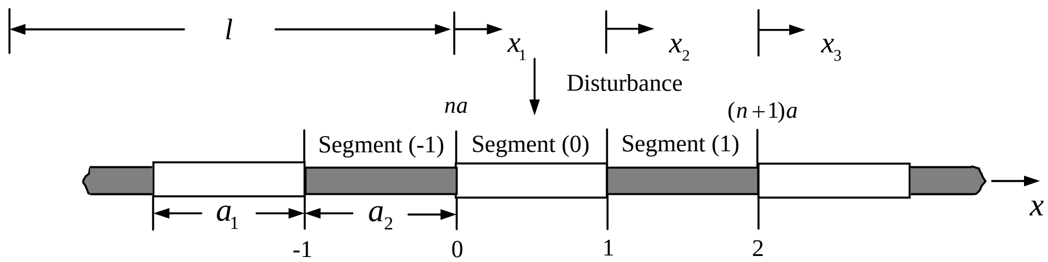

Fig. 1 Designation used for wave multiple-reflection and transmission analysis of a periodic binary beam with a propagating disturbance.

2.1 Multiple reflections between two discontinuities

As shown in Fig. 1, let an external disturbance generate a harmonic wave $w_{in}(x,t)$ on segment (0) and propagating in the positive x direction. The positive-going wave produced will give rise to transmitted and reflected waves when it impinges on discontinuity (1). As the beginning to dealing with more complicated case, one may first study the wave-field set up between two discontinuities due to multiple reflections. The incident wave is described by

$$w_{in}(x) = A_{in1}\exp(-k_1 x\mathrm{i}), \tag{1}$$

where $k_1 = \sqrt[4]{(\rho S)_1 \omega^2 /(EI)_1}$, E denotes Young's modulus, I the area moment of inertia, ρ is the density and S the cross-sectional area, A_{in1} is the wave amplitude.

For simplifying the analysis, the present section deals with the hypothetical case where near-field effects are neglected. Then at the discontinuity (1), a transmitted component $w_{tr1}(x)$ and a reflected component $w_{re1}(x)$ are generated

$$w_{tr1}(x) = A_{tr1}\exp(-k_2 x\mathrm{i}), \tag{2}$$

$$w_{re1}(x) = A_{re1}\exp(-k_1 x\mathrm{i})\exp(2k_1 x_2\mathrm{i}) = A_{re1}\tau\eta\exp(k_1 x\mathrm{i}), \tag{3}$$

where $k_2 = \sqrt[4]{(\rho S)_2 \omega^2 /(EI)_2}$, $x_2 = x - l - a_1$, $\tau = \exp(-2k_1 l\mathrm{i})$, $\eta = \exp(-2k_1 a_1\mathrm{i})$, $A_{re1} = RA_{in1}$, $A_{tr1} = TA_{in1}$, R and T are the transmission and reflection coefficients at the discontinuity (1) of the segment (0), l is the distance between the origin of the x system and the discontinuity (0).

The reflected portion $w_{re1}(x)$ will travel in the negative x direction until it is incident upon discontinuity (0), and there will be transmitted and reflected. The reflected portion

$$w_{re2}(x) = A_{re11}\exp(-k_1 x\mathrm{i})\exp(2k_1 x_2\mathrm{i})\exp(-2k_1 x_1\mathrm{i}) = A_{re11}\eta\exp(-k_1 x\mathrm{i}), \tag{4}$$

where $x_1 = x - l$, $A_{re11} = RA_{re1}$.

The reflected waves $w_{re2}(x)$ will again travel in the positive x direction, and will suffer the same situation as the initial incident waves. So two set of waves will be result in the region between the two continuities. One consists of an infinite number of portions traveling in the positive x direction; the other consists of a same number traveling in the negative x direction. Multiple reflections produce a total wave $w_{na+}^{(0)}(x)$ and a total wave $w_{na-}^{(0)}(x)$ on segment (0) by superposition, where

$$\begin{aligned} w_{na+}^{(0)}(x) &= A_{in1}\exp(-k_1 x\mathrm{i}) + R^2\eta A_{in1}\exp(-k_1 x\mathrm{i}) + R^4\eta^2 A_{in1}\exp(-k_1 x\mathrm{i}) + \cdots \\ &= \frac{1}{1-R^2\eta} A_{in1}\exp(-k_1 x\mathrm{i}) = A_{n1}^{(0)}\exp(-k_1 x\mathrm{i}), \end{aligned} \tag{5}$$

$$\begin{aligned} w_{na-}^{(0)}(x) &= R\tau\eta A_{in1}\exp(k_1 x\mathrm{i}) + R^3\tau\eta^2 A_{in1}\exp(k_1 x\mathrm{i}) + R^5\tau\eta^3 A_{in1}\exp(k_1 x\mathrm{i}) + \cdots \\ &= \frac{R\tau\eta}{1-R^2\eta} A_{in1}\exp(k_1 x\mathrm{i}) = A_{n2}^{(0)}\exp(k_1 x\mathrm{i}), \end{aligned} \tag{6}$$

where the superscript (0) is added to denote the initial approximations. Periodic beam structures is considered, so the modulus of $R^2\eta$ is less than 1.

2.2 Unidirectional propagation

Above Section neglected transmitted waves across the boundaries of segment (0) in Fig. 1. It is difficult to account for all of the transmitted waves of two directions using the method of multiple reflections at the same time, so wave-field will be first set up when wave transmission takes place in only one coordinate direction.

If waves are transmitted in only rightward direction, then wave-field established on segment (0) $w_{na+}^{(0)}(x)$ will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (1) and (2). Multiple reflections produce a total wave $w_{nb+}^{(0)}(x)$ and a total wave $w_{nb-}^{(0)}(x)$ on segment (1) by superposition, where

$$w_{nb+}^{(0)}(x)=\frac{T}{1-R^2\eta}A_{in1}\exp(-k_2x\mathrm{i})+R_1^2\gamma\frac{T}{1-R^2\eta}A_{in1}\exp(-k_2x\mathrm{i})+R_1^4\gamma^2\frac{T}{1-R^2\eta}A_{in1}\exp(-k_2x\mathrm{i})+\cdots$$
$$=\frac{T}{(1-R^2\eta)(1-R_1^2\gamma)}A_{in1}\exp(-k_2x\mathrm{i})=B_{n1}^{(0)}\exp(-k_2x\mathrm{i}),\tag{7}$$
$$w_{nb-}^{(0)}(x)=R_1\tau_1\eta_1\frac{T}{1-R^2\eta}A_{in}\exp(k_2x\mathrm{i})+R_1^3\gamma\tau_1\eta_1\frac{T}{1-R^2\eta}A_{in}\exp(k_2x\mathrm{i})+\cdots$$
$$=R_1\tau_1\eta_1\frac{T}{(1-R^2\eta)(1-R_1^2\gamma)}A_{in}\exp(k_2x\mathrm{i})=B_{n2}^{(0)}\exp(k_2x\mathrm{i}),\tag{8}$$
where $\tau_1=\exp(-2k_2l\mathrm{i})$, $\eta_1=\exp(-2k_2a\mathrm{i})$, $\gamma=\exp(-2k_2a_2\mathrm{i})$, R_1 is the reflection coefficient of the segment (1).

Similarly, if waves are transmitted in only leftward direction, then wave-field established on segment (0) $w_{na-}^{(0)}(x)$ will be transmitted past discontinuity (0), and the total transmission will suffer multiple reflections between discontinuity (-1) and (0). Multiple reflections produce a total wave $w_{(n-1)b-}^{(0)}(x)$ and a total wave $w_{(n-1)b+}^{(0)}(x)$ on segment (-1) by superposition, where
$$w_{(n-1)b-}^{(0)}(x)=\frac{TR\tau\eta}{(1-R^2\eta)(1-R_1^2\gamma)}A_{\mathrm{in}1}\exp(k_2x\mathrm{i})=B_{(n-1)2}^{(0)}\exp(k_2x\mathrm{i}),\tag{9}$$
$$w_{(n-1)b+}^{(0)}(x)=R_1\tau_1^{-1}\gamma\frac{TR\tau\eta}{(1-R^2\eta)(1-R_1^2\gamma)}A_{\mathrm{in}1}\exp(-k_2x\mathrm{i})=B_{(n-1)1}^{(0)}\exp(-k_2x\mathrm{i}).\tag{10}$$
So wave-field on any segment (n) can also be established.

2.3 Bidirectional propagation

Here wave propagation in both directions is considered by means of an iteration procedure. In this procedure, each step deals with propagation in only one direction and gives a correction term that, in essence, accounts for the propagation direction not considered in the proceeding step [5]. Above Section give initial approximations that neglect the effects of propagation in the leftward direction on each segment when waves are transmitted in only rightward direction. The problem is taken into account subsequently.

The first correction terms on segment (0) are due to the leftward propagating components on all segments to the right of the segment (0) when only leftward propagation is permitted. $w_{nb-}^{(0)}(x)$ on segment (1) will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (0) and (1). Multiple reflections produce a total wave on segment (0); $w_{(n+1)a-}^{(0)}(x)$ on segment (2) will be transmitted past discontinuity (2), and the total transmission will suffer multiple reflections between discontinuity (1) and (2), then will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (0) and (1). Multiple reflections produce a total wave on segment (0). By the same token, the first correction terms $w_{na-}^{(1)}(x)$ and $w_{na+}^{(1)}(x)$ can be obtained by superposing the waves resulting on segment (0)
$$w_{na-}^{(1)}(x)=\left(PB_{n2}^{(0)}+PP_1A_{(n+1)2}^{(0)}+P^2P_1B_{(n+1)2}^{(0)}+\cdots\right)\exp(k_1x\mathrm{i}),\tag{11}$$
$$w_{na+}^{(1)}(x)=\tau^{-1}R\left(PB_{n2}^{(0)}+PP_1A_{(n+1)2}^{(0)}+P^2P_1B_{(n+1)2}^{(0)}+\cdots\right)\exp(-k_1x\mathrm{i}),\tag{12}$$

where $T_1/(1-R^2\eta)=P$, $T/(1-R_1^2\gamma)=P_1$, number of term in series equals to the number of segment to the right of the segment (0). The first correction terms on any segment can also be obtained in the same way.

Similar to the above considered case, the second correction terms on a given segment can be obtained. The second correction terms on segment (0) are due to the rightward propagating components on all segments to the left of the segment (0). $w_{na+}^{(2)}(x)$ and $w_{na-}^{(2)}(x)$ can be obtained by superposing the waves resulting on segment (0).

$$w_{na+}^{(2)}(x)=\left(PB_{(n-1)1}^{(1)}+PP_1A_{(n-1)1}^{(1)}+P^2P_1B_{(n-2)1}^{(1)}+\cdots\right)\exp(-k_1x\mathrm{i}),\tag{13}$$

$$w_{na-}^{(2)}(x)=\tau\eta R\left(PB_{(n-1)1}^{(1)}+PP_1A_{(n-1)1}^{(1)}+P^2P_1B_{(n-2)1}^{(1)}+\cdots\right)\exp(k_1x\mathrm{i}),\tag{14}$$

where number of term in series equals to the number of segment to the left of the segment (0). The second correction terms on any segment can also be obtained in the same way.

The third and the fourth correction terms can also be obtained by means of this iteration procedure. The third correction terms on a given segment are due to the leftward propagating components of the second correction terms on all segments to the right of the segment when only leftward propagation is permitted. The fourth correction terms on a given segment are due to the rightward propagating components of the third correction terms on all segments to the left of the segment. The wave-field on a given segment can be obtained by superposing the initial approximation and the various corrections. For example, the wave-field on segment (0) can be obtained

$$w_{na+}(x)=\sum_{i=0}^{\infty}w_{na+}^{(i)}(x)=W_{na+}\exp(-k_1x\mathrm{i}),\qquad w_{na-}(x)=\sum_{i=0}^{\infty}w_{na-}^{(i)}(x),\tag{15a}$$

where number of term in series is determined by iteration times.

Similarly, we have

$$w_{(n-1)b+}(x)=\sum_{i=0}^{\infty}w_{(n-1)b+}^{(i)}(x)=W_{(n-1)b+}\exp(-k_2x\mathrm{i}),\qquad w_{(n-1)b-}(x)=\sum_{i=0}^{\infty}w_{(n-1)b-}^{(i)}(x),\tag{15b}$$

$$w_{nb+}(x)=\sum_{i=0}^{\infty}w_{nb+}^{(i)}(x)=W_{nb+}\exp(-k_2x\mathrm{i}),\qquad w_{nb-}(x)=\sum_{i=0}^{\infty}w_{nb-}^{(i)}(x).\tag{15c}$$

The wave-field on any segment can also be obtained. So the amplitudes of the wave-field resulting from the incident propagating wave can be determined.

3. The determination of Iteration times

Iteration times may be determined by calculating the ratio of the wave-field on adjacent cells in the next step. Considering Eq. (15), we have

$$\overline{W}_{nb+}=\exp(-k_2(l+a_1)\mathrm{i})W_{nb+},\qquad \overline{W}_{(n-1)b+}=\exp(-k_2(l-a+a_1)\mathrm{i})W_{(n-1)b+},\tag{16}$$

where the symbols with the bar denote components referred the local coordinate systems originating at the left ends of the segments. The Bloch theorem states that $\overline{W}_{nb+}=\exp(\mathrm{i}\mu a)\overline{W}_{(n-1)b+}$. So we have the following relation of the wave-field on adjacent cells

$$W_{nb+}=\exp(\mathrm{i}\mu a)\exp(k_2a\mathrm{i})W_{(n-1)b+}.\tag{17a}$$

Similarly, we can get

$$W_{na+}=\exp(\mathrm{i}\mu a)\exp(k_1a\mathrm{i})W_{(n-1)a+}.\tag{17b}$$

Comparing the propagating constant μ calculated by Eq. (17) with μ calculated by the transfer matrix method, iteration times of Eq. (15) can be determined within the desired accuracy [5].

4. Numerical examples and discussions

From this section, numerical calculations of band structures for finite periodic beams with epoxy as material A and aluminum as material B are performed. Material constants used in the calculation are mainly listed in Table 1. The lattice constant is chosen to be $a = 2\mathrm{m}$, and $a_1 = a_2 = 1\mathrm{m}$. The beam cross-sectional area is $A = 5.969\times10^{-3}\mathrm{m}^2$ and the second area moment of inertia is $I = 2.701\times10^{-5}\mathrm{m}^4$ for segments A and B. The distance between the origin of the x system and the discontinuity (0) is $l = 3a$.

Table 1
Material constants

Materials	Yong's modulus E (Gpa)	Density ρ ($\mathrm{kg/m^3}$)
Epoxy (A)	4.35	1180
Aluminum (B)	77.56	2730

4.1 Band gap of periodic beam

The band structure of an infinite periodic beam is depicted in Fig. 2. The shadow areas are the first two band gaps.. It is repeated here for the sake of completeness and comparison.

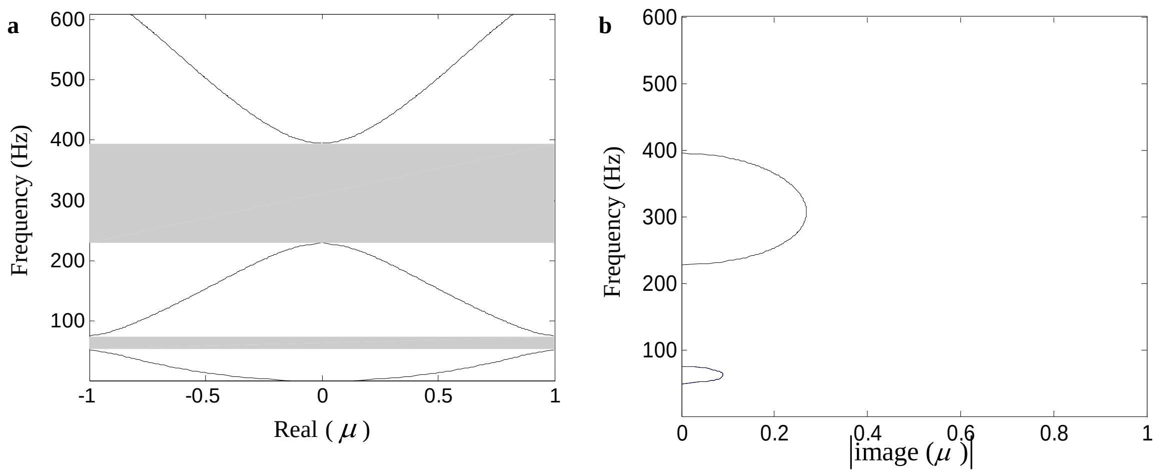

Fig. 2 Band structure (dispersion relationship) for an infinite periodic beam. (a) Real wave vector. (b) The absolute value of the imaginary part of complex wave vector (propagating constant).

4.2 Multi-reflection approach

4.2.1 The number of cells In this section, the frequency response on different segment of the beam is given to describe the band gap characteristics of finite periodic beams. In Fig. 3, Frequency response functions curves of finite periodic beams considering the propagating disturbance are given. Curves are shown for *M*=6, 10 and 15 (the number of cell).

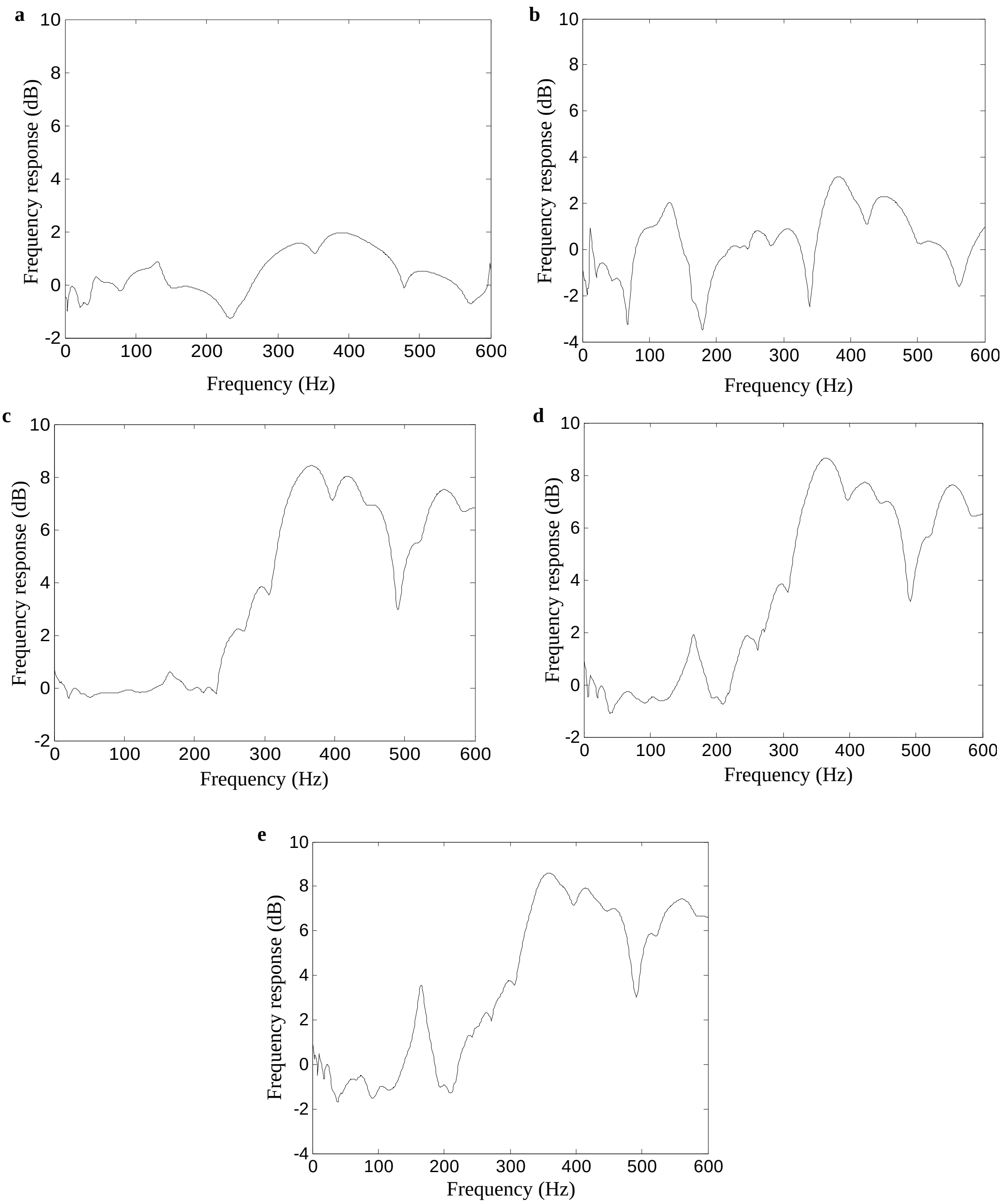

Fig. 3 Frequency response of periodic beams. (a) M=6, segment (4), Iteration times N=5, (b) M=10, segment (12), N=5, (c) M=15, segment (4), N=9, (d) M=15, segment (12), N=9, (e) M=15, segment (22), N=9.

In Fig. 3a, 3b and 3e, we can see obviously that the band gaps of a finite periodic beam is more agreement with the band gaps of infinite periodic beam for larger M . However, the first band gap is not obvious. We can also observe from Fig. 3c, 3d and 3e that the longer the distance from the segment (0) (propagating disturbance), the more obvious the band gaps become. It is shown from Fig. 3a and 3c that the band gap characteristics can not be found because the distance from the segment (0) (propagating disturbance) is too short. So the longer the distance from disturbance, the more obvious the band gaps become for periodic structures with a lateral disturbance force.

4.2.2 Effect of the material parameters In what follows, parametric studies are undertaken to investigate the effect of the ratio ρ_A / ρ_B and E_A / E_B on the band gaps in periodic beams with disturbance. Frequency response on segment (22) of periodic beams is considered. M=15, N=9. The material A is assumed to be epoxy, however, the density of material B varies with the change of ratio ρ_A / ρ_B. All other parameters are taken the same as above. The effect of ρ_A / ρ_B on the second band gap is plotted in Fig. 4. The bandwidth of the first band gap is too small to consider. The beginning frequency of the second band gap increases and the cutoff frequency decreases with the increasing of the ratio ρ_A / ρ_B. Now the Young's modulus of the beam is considered. We consider the case in which the material A is epoxy. The density of material B varies with the change of ratio E_A / E_B. All other parameters are taken the same as above. The effect of E_A / E_B on the second band gap is plotted in Fig. 5. It can be observed from Fig. 5 that both the beginning frequency and cutoff frequency of the second band gap are decrease with the increasing of the ratio E_A / E_B.

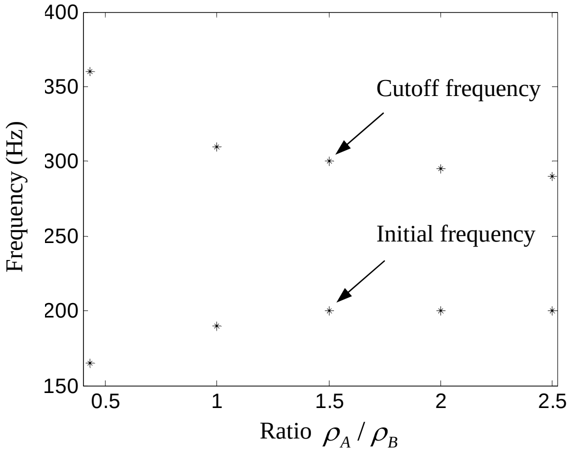

Fig. 4 Effect of the ratio ρ_A / ρ_B on the second band gap.

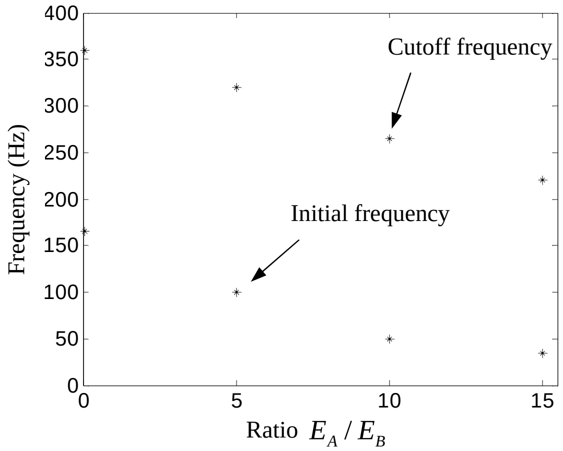

Fig. 5 Effect of the ratio E_A / E_B on the second band gap.

4.2.3 Effect of the disturbance Fig. 6 shows the frequency responses for the periodic beams with different the propagating disturbance. The material constants used in the calculation are the same as those in Table 1. Although the band gap characteristics can not be changed by tuning the disturbance, it can be seen that the attenuation of the band gaps gradually decreases with the increase disturbance amplitude.

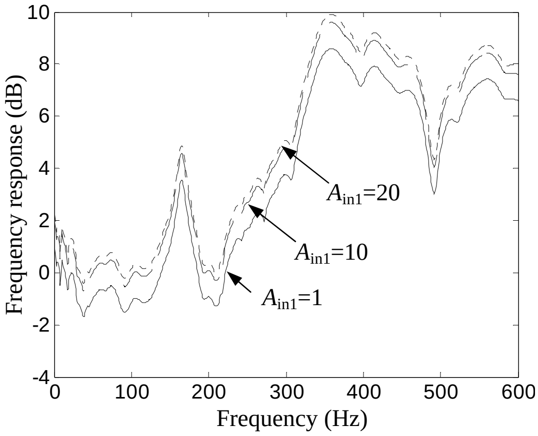

Fig. 6 Frequency response of periodic beams with different propagating disturbance. M=15, segment (22), N=9.

5. Conclusions
The elastic wave propagation in finite periodic beams considering the propagating disturbance is studied theoretically as well as numerically. The effects of the disturbance on the wave propagation in periodic beams are considered. Multi-reflection method is used to analyze the band gap characteristics of the periodic beam with one or more lateral disturbance. It is very difficult to study effects of the disturbance on the wave propagation in periodic beams using transfer matrix method. The validity of the multi-reflection is proved. The amplitudes of the wave-field on any segment resulting from a given incident amplitude can be determined. It is also not given by transfer matrix method. Expression is presented from which one may evaluate the propagation constant, and then study wave propagation characteristics of a finite periodic beam considering a propagating disturbance. Much attention is devoted to the response in the frequency ranges with gaps in the band structure for the corresponding periodic beams. From the results, the following conclusions can be drawn:

(1) The effects of the disturbance on the wave propagation in periodic beams is analyzed, and a relation between the amplitude of the incident wave and the resulting amplitudes of any segment is established using multi-reflection method, and these can not be given using the transfer matrix method.

(2) For the finite periodic beam, band gap characteristic are not obvious when the distance between the segment and the disturbance is short. It is shown that the influence of disturbance on the band gaps of the periodic beam is focus on the areas in the vicinity. Band gap characteristic will disappear because of the influence of external disturbance. At same time, the longer the distance from the disturbance, the more obvious the band gaps become, and the band gaps of the finite periodic beam is more agreement with the band gaps of the infinite periodic beam for larger M . The validity of the multi-reflection is proved.

(3) Frequency response of finite periodic beams can be changed by tuning the disturbance. With the increase disturbance amplitude, the attenuation of the band gaps gradually decreases.

Acknowledgements
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