

Dynamic analysis of hoisting viscous damping string with time-varying length

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Abstract. The nonlinear dynamic analysis of a hoisting viscous damping string with time-varying length is investigated. The hoisting string is modeled as a taut translating string with a rigid body attached at its low end. A systematic procedure for deriving the system model of hoisting viscoelastic string with time-varying is presented. The governing equations are developed employing the extended Hamilton's principle considering coupling of axial movement and flexural deformation of string. The Galerkin's method and the 4th Runge-Kutta method are employed to numerically analyze the resulting equations. The motions of elevator hoisting system are presented to illustrate the proposed mathematical models. The results of simulation show that the material viscous damping helps stabilize the transverse vibration. The modeling methods can represent the transverse vibration of hoisting viscous damping string with time-varying length.

1. Introduction

While rope is employed in hoisting industry such as mine hoists, elevators, cranes etc, it is subject to vibration due to its high flexibility and relatively low internal damping characteristics ^[1-3]. Most often these systems are modeled as either an axially moving tensioned beam or string with time-varying length and a load at its lower end ^[4, 5]. It was shown that the vibration energy of the rope changes in general during elongation and shortening. When the rope length is being shortened, vibration energy increases exponentially with time, causing dynamic instability ^[6]. So the vibration of the rope with time-varying length has been studied by some researchers.

Kotera and Kawai ^[7] analyzed free vibrations of a string with time-varying length and a weight at one end. New variables of position and time are introduced which allow the equation of motion to be solved by Laplace transformation. Almost exact solutions of free vibrations induced by a distributed initial displacement are also obtained. Kaczmarczyka and Ostachowicz ^[8] studied coupled vibration of deep mine hoisting cable and built a distributed-parameter model. They found that response of the catenary-vertical rope system may feature a number of resonance phenomena. Kimure and Iijima et al. ^[9-12] published a series of studies on vibrations of elevator rope. Finite difference analyses of the rope vibration were performed by considering the time-varying length of the rope, based on the assumption that the movement velocity is constant. The calculated results of the finite difference analyses are in



fairly good agreement with the calculated results of the exact solution. The above studies assume that the velocity is constant during the process of modeling. Some researches^[13-15] assumed that the transport speed is characterized as a small simple harmonic variation about the constant mean speed. The assumption has its physical meaning. For example, if the axially moving rope with time-varying length models a string on a rotating pulley, the rotation vibration of the pulley will result in a small fluctuation in the axial speed of the string. However, the study of vibrations of varying length rope with arbitrary varying axial velocity is a relatively less studied problem in literature. Zhu and Chen^[16] investigated a comprehensive, theoretical and experimental study of the uncontrolled and controlled lateral responses of a moving cable in a high-rise elevator. A novel experimental method was developed to validate the uncontrolled and controlled response and shown good agreement with the theoretical predictions. Zhang and Zhu et al.^[17] derived the governing equation and energy equation of longitudinal vibration of flexible hoisting system with arbitrarily varying length and velocity.

Extensive research efforts on the flexible hoisting rope with time-varying length have been done in the last few decades as aforementioned, however, most studies assumed the rope is linearly elastic, and damping was ignored since the primary focus was on the intrinsic stability. In the paper, the dynamic characteristics of the hoisting viscous damping rope with time-varying length are the subject of this investigation. The governing equations are developed employing the extended Hamilton's principle. The derived governing equations are shown to be partial differential equations (PDF) with variable coefficients. On choosing proper mode functions that satisfy the boundary conditions, the solution of the governing equations was obtained using the Galerkin's method. The motions of elevator hoisting rope were illustrated to evaluate the proposed mathematical models. According to the numerical simulations, the effects of the viscous damping of material on the dynamic characteristics are analyzed for the hoisting rope with time-varying length. Based on the proposed dynamic analysis, further vibration control will be adopted for such the hoisting systems in the near future.

2. Model of hoisting system

Hoisting system can be simplified as an axially moving string with time-varying length and a rigid body m at its lower end, as shown in Figure 1. The rail and the suspension of the rail are assumed to be rigid. The string has Young's modulus E , diameter d and the density per unit length ρ . The origin of coordinate is set at the top end of string and the instantaneous length of string is $l(t)$ at time t . The instantaneous translational velocity, acceleration and jerk of the string are $v(t)$, $a(t)$ and $j(t)$ respectively. At any instant t , the transverse displacements of string is described by $y(x,t)$, at a spatial position $x(t)$, where $0 \leq x(t) \leq l(t)$. The following assumptions constrain the analysis: 1. Young's modulus E , diameter d and density ρ of the string are always constants; 2. Only transverse vibration is considered here. The elastic distortion of string arousing from the transverse vibration is much less than the length of the string.

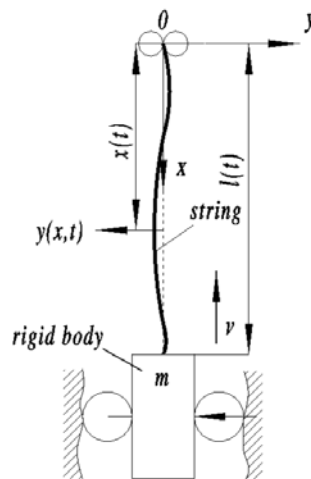


Figure 1. Schematic of hoisting viscoelastic string with time-varying length

The kinetic energy of flexible hoisting system is computed by

$$E_k(t) = 0.5(m + \rho l)v^2 + 0.5my_t^2(l(t), t) + 0.5\rho \int_0^{l(t)} (y_t(x, t) + vy_x(x, t))^2 dx \quad (1)$$

The elastic strain energy of the string is

$$E_e(t) = \int_0^{l(t)} \left(P\varepsilon + \frac{1}{2} EA\varepsilon^2 \right) dx + \frac{1}{2} \int_0^{l(t)} EIy_{xx}^2 dx \quad (2)$$

The first and the second terms on the right of Equation (2) represent axial strain energy of the string, the third term represents bending strain energy of the string. $P(x, t)$ is the quasi-static tension in the string and is given by

$$P = [m + \rho(l(t) - x)]g \quad (3)$$

And ε represents the strain measure at spatial position x of the string and can be expressed as

$$\varepsilon = (ds - dx) / dx \quad (4)$$

As shown in Figure 2, ds can be expressed as

$$ds \approx \sqrt{1 + (dy/dx)^2} dx \approx \left[1 + \frac{1}{2} \left(\frac{\partial y}{\partial x} \right)^2 - \frac{1}{8} \left(\frac{\partial y}{\partial x} \right)^4 + \dots \right] dx \approx \left[1 + \frac{1}{2} \left(\frac{\partial y}{\partial x} \right)^2 \right] dx \quad (5)$$

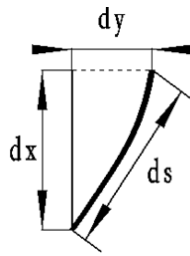


Figure 2. A small element of the string in a deformed position

Substituting Equation (5) into Equation (4) yields

$$\varepsilon = \frac{1}{2} y_x^2 \quad (6)$$

The virtual work done by the transverse damping forces of the string is given by

$$\delta W = \int_0^{l(t)} F_c \delta y dx \quad (7)$$

where F_c is the viscous damping distributed force^[2] in the string, and

$$F_c = -c_1 P y_{xxt} + c_2 \rho y_t \quad (8)$$

where c_1 and c_2 are constant coefficients of transverse damping. According to the characteristics of top restriction of the string, the boundary conditions at $x(t)=0$ are

$$y(0,t) = 0, \quad y_t(0,t) = 0 \quad (9)$$

On substitution of Eqs.(1), (2) and (7) in the extended Hamilton's Principle,

$$\delta \int_{t_1}^{t_2} (E_k(t) - E_e(t)) dt + \int_{t_1}^{t_2} \delta W dt = 0 \quad (10)$$

and apply the variational operation and the procedure for integration by parts to obtain,

$$\begin{aligned} & \int_{t_1}^{t_2} \left[m \frac{\partial}{\partial t} y_t(l,t) + \left(P + \frac{1}{2} E A y_x^2 \right) y_x - E I y_{xxx} \right] \delta y \Big|_l dt + \int_{t_1}^{t_2} (E I y_{xx}) \delta y_x \Big|_l dt + \\ & \int_{t_1}^{t_2} \int_0^l \left[\rho \frac{\partial}{\partial t} (y_t + v y_x) + \rho v \frac{\partial}{\partial x} (y_t + v y_x) - \frac{\partial}{\partial x} \left(\left(P + \frac{1}{2} E A y_x^2 \right) y_x \right) + F_c + E I y_{xxxx} \right] \delta y dx dt = 0 \end{aligned} \quad (11)$$

Setting the coefficients of δy in Equation (11) to zero yields the governing equation for the string,

$$\rho(y_{tt} + 2v y_{xt} + a y_x + v^2 y_{xx}) - P_x y_x - P y_{xx} - \frac{3}{2} E A y_x^2 y_{xx} - c_1 P y_{xxt} + c_2 \rho y_t + E I y_{xxxx} = 0, 0 < x < l(t) \quad (12)$$

The first four terms in Equation (12) correspond to the local, Coriolis, tangential, and centripetal acceleration, respectively. Equation (12) is a partial differential equation which describes the dynamics of the flexible hoisting string. The equation defined over time-dependent spatial domain rendering the problem non-stationary. Hence, the exact solution to this problem is not available, and recourse must be made to an approximate analysis. In what follows, numerical techniques are employed to obtain approximate solution for the governing equation.

3. Discretization of the governing equation

Equation (12) is a partial differential equation with infinite dimensions and many parameters are time-variant. It is impossible to obtain an exact analytical solution from Equation (12). In this section, Galerkin's method is applied to truncate the infinite-dimensional partial differential equation into a linear finite-dimensional ordinary differential equation with time-variant coefficients. Then, solve them with numerical methods. In order to map Equation (12) onto the fixed domain, a new independent variable $\zeta = x/[l(t)]$ is introduced and the time-variant domain $[0, l(t)]$ for x is converted to a fixed domain $[0, 1]$ for ζ . According to the characteristic of taut translating string, the solution of $y(x, t)$ is assumed in the forms^[5,13]

$$y(x, t) = \sum_{i=1}^n \varphi_i(\zeta) q_i(t) = \sum_{i=1}^n \varphi_i\left(\frac{x}{l}\right) q_i(t) \quad (13)$$

where $q_i(t)$ ($i=1,2,3,\dots,n$) is the generalized coordinate respect to $y(x,t)$, n is the number of included mode. $\phi_i(\zeta)$ is trial function^[5,13],

$$\phi_i(\zeta) = \sqrt{2} \sin i\pi\zeta \quad (14)$$

Consequently, expansion Equation (13) results in the expressions for partial derivatives of transverse displacement functions,

$$\begin{aligned} y_x(x,t) &= \frac{1}{l(t)} \sum_{i=1}^n \phi'_i(\zeta) q_i(t), \quad y_{xx}(x,t) = \frac{1}{l(t)^2} \sum_{i=1}^n \phi''_i(\zeta) q_i(t) \\ y_{xxx}(x,t) &= \frac{1}{l(t)^4} \sum_{i=1}^n \phi^{(4)}_i(\zeta) q_i(t), \\ y_{xt}(x,t) &= \sum_{i=1}^n \frac{1}{l(t)} \phi'_i(\zeta) \dot{q}_i(t) - \sum_{i=1}^n \left(\frac{\zeta v(t)}{l(t)^2} \phi''_i(\zeta) + \frac{v(t)}{l(t)^2} \phi'_i(\zeta) \right) q_i(t), \\ y_{tt}(x,t) &= \sum_{i=1}^n \phi_i(\zeta) \ddot{q}_i(t) - \frac{2\zeta v(t)}{l(t)} \sum_{i=1}^n \phi'_i(\zeta) \dot{q}_i(t) + \sum_{i=1}^n \left(\frac{2\zeta v(t)^2}{l(t)^2} \phi'_i(\zeta) - \frac{\zeta a(t)}{l(t)} \phi'_i(\zeta) + \frac{\zeta^2 v(t)^2}{l(t)^2} \phi''_i(\zeta) \right) q_i(t) \end{aligned} \quad (15)$$

where the overdot denotes time differentiation. Substituting Equation (13)-(15) into Equation (12), multiplying the governing equation by $\phi_k(\zeta)$ ($k=1,2,3,\dots,n$), integrating it from $\zeta=0$ to 1, and using the boundary conditions and the orthonormality relation for $\phi_i(\zeta)$ yield the discretized equation of transverse vibration for the flexible hoisting string with time-variant coefficients

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{C}\dot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} + \mathbf{S}(\mathbf{Q}) = 0 \quad (16)$$

where $\mathbf{Q} = [q_1(t), q_2(t), \dots, q_n(t)]^T$ is vector of generalized coordinate, \mathbf{M} , \mathbf{C} and \mathbf{K} are matrices of mass, damp and stiffness respect to \mathbf{Q} , respectively. $\mathbf{S}(\mathbf{Q})$ is higher order item of generalized coordinate. The overdot denotes time differentiation. Where entries of the matrices are expressed as follows,

$$\begin{aligned} M_{ik} &= \rho \delta_{ik} \\ C_{ik}(t) &= \frac{2\rho v}{l} \int_0^1 (1-\zeta) \phi'_i(\zeta) \phi'_k(\zeta) d\zeta - \frac{c_1 \rho g}{l} \int_0^1 (1-\zeta) \phi''_i(\zeta) \phi'_k(\zeta) d\zeta - \frac{c_1 mg}{l^2} \int_0^1 \phi''_i(\zeta) \phi'_k(\zeta) d\zeta + c_2 \rho \delta_{ik} \\ K_{ik}(t) &= \frac{\rho a}{l} \int_0^1 (1-\zeta) \phi'_i(\zeta) \phi'_k(\zeta) d\zeta - \frac{\rho v^2}{l^2} \int_0^1 (1-\zeta)^2 \phi'_i(\zeta) \phi'_k(\zeta) d\zeta + \frac{\rho g}{l} \int_0^1 (1-\zeta) \phi'_i(\zeta) \phi'_k(\zeta) d\zeta - \\ &\quad \frac{mg}{l^2} \int_0^1 \phi''_i(\zeta) \phi'_k(\zeta) d\zeta + \frac{EI}{l^4} \int_0^1 \phi^{(4)}_i(\zeta) \phi'_k(\zeta) d\zeta + \\ &\quad \frac{2vc_1 \rho g}{l^2} \int_0^1 (1-\zeta) \phi''_i(\zeta) \phi'_k(\zeta) d\zeta + \frac{vc_1 \rho g}{l^2} \int_0^1 \zeta (1-\zeta) \phi''_i(\zeta) \phi'_k(\zeta) d\zeta + \frac{2vc_1 mg}{l^3} \int_0^1 \phi''_i(\zeta) \phi'_k(\zeta) d\zeta + \\ &\quad \frac{vc_1 mg}{l^3} \int_0^1 \zeta \phi''_i(\zeta) \phi'_k(\zeta) d\zeta - \frac{c_2 \rho v}{l} \int_0^1 \zeta \phi''_i(\zeta) \phi'_k(\zeta) d\zeta \\ S_k(\mathbf{Q}) &= -\frac{3EA}{2l^4} \int_0^1 \left(\sum_{i=1}^n \phi'_i(\zeta) q_i(t) \right)^2 \sum_{i=1}^n \phi''_i(\zeta) q_i(t) \phi'_k(\zeta) d\zeta \end{aligned} \quad (17)$$

where the superscript “” denotes partial differentiation for normalized variable ζ , δ_{ij} is the Kronecker delta defined by $\delta_{ik} = 1$ if $i=k$ and $\delta_{ik} = 0$ if $i \neq k$ ($i=1,2,3,\dots,n$, $k=1,2,3,\dots,n$). If the initial displacement and velocity of the string are given by $y(x,0)$ and $y_t(x,0)$, respectively, where $0 < x < l(0)$, the initial conditions for the generalized coordinate can be obtained from Equation (13) and (15),

$$q_i(0) = \int_0^1 y(\zeta l(0), 0) \phi_i(\zeta) d\zeta \quad (18)$$

$$\dot{q}_i(0) = \int_0^1 y_t(\zeta l(0), 0) \phi_i(\zeta) d\zeta + \frac{v(0)}{l(t)} \sum_{i=1}^n q_i(0) \int_0^1 \zeta \phi_i'(\zeta) \phi_k(\zeta) d\zeta \quad (19)$$

Solving the ordinary differential Equation (16) with numerical methods may yield the instantaneous values of Q . Substituting these values into Equation (13) may yield the instantaneous values of transverse vibration of the string $y(x, t)$.

4. Numerical simulation and discussions

Equation (16) is a second-order ordinary differential equations with variable coefficients. They will be used to investigate the dynamic responses of hoisting viscous damping string with time-varying length. The solutions of the equation are obtained by the Runge-Kutta method. Kimura^[9-12] published a series of studied on vibrations of a string with time-varying length. However, these studies considered the string with uniform motions only. In the study, we will consider the speed changes with time. In what follows, the motions of elevator hoisting system were illustrated to evaluate the proposed mathematical model. Elevator hoisting system is simplified as an axially translating viscous damping string with a rigid body attached at its lower end. The simulation parameters for the elevator are given in Table 1. The flight time for a travel distance of 135m (45 stories) is 33 second. Figure 3 gives the prescribed displacement, velocity, acceleration and jerk curves of elevator hoisting system. Utilizing the curves as the input of Equation (16) with aid of MatLab may obtain dynamic responses of elevator hoisting system. In this work, all numerical analyses were implemented with aid of MatLab.

Table 1. Simulation parameters

Items	Data values
Density per unit length $\rho(\text{kg/m})$	0.707
Young's modulus $E(\text{N/m}^2)$	2.01×10^{11}
String diameter $d(\text{m})$	14×10^{-3}
Hoisting mass $m(\text{kg})$	300
Minimum length of the string $l_{\min}(\text{m})$	5
Maximum length of the string $l_{\max}(\text{m})$	140
Maximum velocity $v_{\max}(\text{m/s})$	5
Maximum acceleration $a_{\max}(\text{m/s}^2)$	1
Maximum jerk $j_{\max}(\text{m/s}^3)$	1
Total travel time $t(\text{s})$	33
Number of transverse modes n	4

Consider the free vibration caused by a distributed initial displacement and released from rest. The initial displacement and velocity are respectively

$$y(x,0) = y_0 \sin \frac{x\pi}{l_0}, \quad y_t(x,0) = 0 \quad (20)$$

where $y_0=0.005\text{m}$ is the initial amplitude. Transverse vibration responses of hoisting viscous damping string at 2m above the car during movement of elevator are illustrated in Figure 4 and Figure 5.

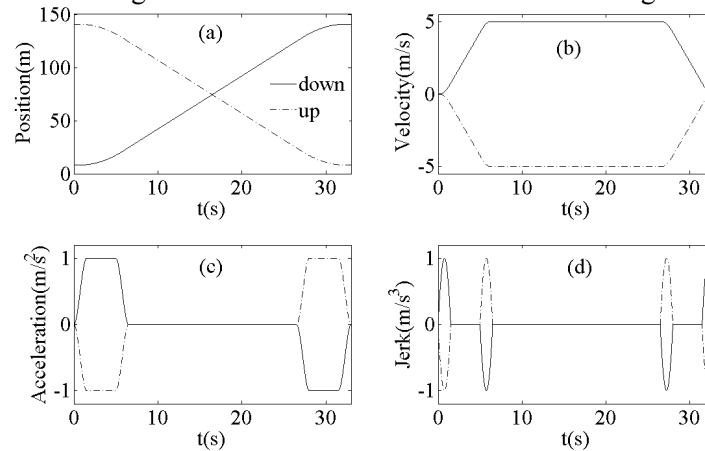


Figure 3. Movement profile of the elevator: (a) $l(t)$; (b) $v(t)$; (c) $a(t)$; (d) $j(t)$.

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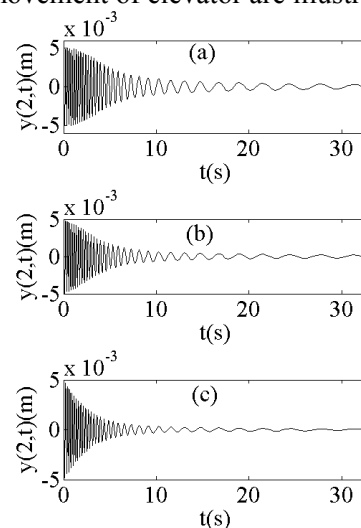


Figure 4. Transverse vibration responses of hoisting viscous damping string at 2m above the car during downward movement of the elevator: (a) $c_1=0$; (b) $c_1=0.0003$; (c) $c_1=0.0008$.

Figure 4 displays reducing vibration amplitudes with increasing length of the rope during downward movement. This is due to the energy of flexible hoisting system transfers from the transverse vibration to the axial motion by bringing some mass into the domain of effective length, i.e., the axially hoisting rope is dissipative during downward movement, thus leading to a stabilized transverse dynamic response. A possible physical interpretation of the result is as follows: during downward movement negative external work is required to maintain the prescribed axial motion which, in turn, brings about a convection of mass in the domain of effective length. At the same time,

frequencies of the transverse vibration reduce with increasing length of the rope. This is due to the fact that the mass of the rope increase and the stiffness of the rope decrease, i.e., the rope becomes somewhat “softer”.

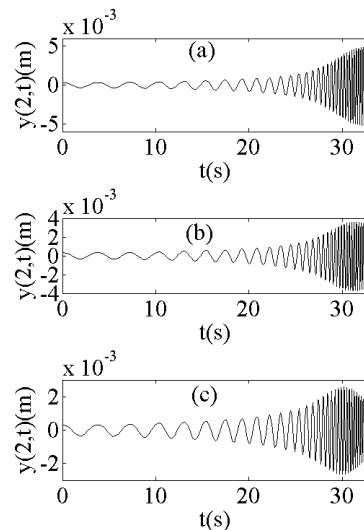


Figure 5. Transverse vibration responses of hoisting viscous damping string at 2m above the car during upward movement of the elevator: (a) $c_1=0$; (b) $c_1=0.0003$; (c) $c_1=0.0008$.

By contrast, in Figure 5, we observe that vibration amplitudes of the rope increase with decreasing length of the rope during upward movement. This is due to the energy of flexible hoisting system transfers from the axial motion to the transverse vibration by leaving some mass out of the domain of effective length, i.e., the axially hoisting rope gains energy during upward movement, thus leading to an unstabilized transverse dynamic response. A possible physical interpretation of the result is as follows: during upward movement positive external work is required to maintain the prescribed axial motion which, in turn, brings about a convection of mass out of the domain of effective length. In the mean time, frequencies of the transverse vibration increase with decreasing length of the rope. This is due to the fact that the mass of the rope decrease and the stiffness of the rope is increase, i.e., the rope becomes somewhat “stiffer”.

Further, Figure 4 and Figure 5 show that the viscous damping takes effect on the vibration amplitude. The response amplitude decreases as the viscous damping increases. Higher viscous damping leads to amplitude reduction faster, as shown Figure 4(c) and Figure 5(c). The reduced response amplitude indicates that the material viscous damping makes the dynamic system more stable.

5. Conclusion

The nonlinear vibration characteristics for a flexible hoist string with time-varying length considering coupling of axial movement and flexural deformation is analyzed in this paper. The flexible hoisting system is modeled as an axially moving string with time-varying length and a load m at its lower end. The governing equations are derived by using Leibnitz’s rule and the extended Hamilton’s principle. The Galerkin’s method is used to truncate the infinite-dimensional partial differential equations into a set of nonlinear finite-dimensional ordinary differential equations with time-variant coefficients. Based on the numerical simulations, the following conclusions can be obtained: The natural frequencies of flexible hoisting string with time-varying length are increasing because of the reducing mass and the increasing stiffness of the string, and the energy transforms from the axial movement into the flexible deformation. By contrast, the natural frequencies are decreasing because of the increasing mass and the reducing stiffness of the string, and the energy coverts from the flexible deformation into the axial movement. The material viscous damping always dissipates energy and

helps stabilize the transverse vibration during movement of the hoisting system. The proposed the theoretical model in this paper will be helpful for the researchers to comprehend its dynamic behavior and develop the proper method to suppress the vibration in practice.

References

- [1] Kaczmarczyk S 1997 *J. Sou. Vib.* **208** 243-69
- [2] Kaczmarczyk S and Ostachowicz W 2003 *J. Sou. Vib.* **262** 219-44
- [3] Kaczmarczyk S and Andrew P 2005 *Elev. Worl.* 126-29
- [4] Zhu W D and Xu G Y 2003 *J. Sou. Vib.* **263** 679-99
- [5] Zhu W D and Chen Y 2005 *Multi-body Dyna.* **219** 35-53
- [6] Lee S Y and Lee M 2002 *J. Appl. Phys.* **69** 83-87
- [7] Kotera T and Kawai R *JSME int. ser. III, Vib. Con. Eng. . Ind.* **31** 524-29
- [8] Kaczmarczyk S and Ostachowicz W 2003 *J. Sou. Vib.* **262** 245-89
- [9] Kimura H and Nakagawa T 2007 *J. Env. Eng.* **2** 76-86
- [10] Kimura H, Ito H and Nakagawa T 2007 *J. Env. Eng.* **2** 87-96
- [11] Kimura H, Ito H and Nakagawa T 2008 *J. Env. Eng.* **2** 540-49
- [12] Kimura H, Iijima T, Matsuo S and Fujita Y 2009 *J. Sys. Des. Dyn.* **3** 420-28
- [13] Fung R F, Wang P H and Lee M J 1998 *J. Chin. Ins. Eng.* **21** 109-17
- [14] Yao C M, Fung R F and Tseng C D 1999 *J. Sou. Vib.* **219** 323-37
- [15] Wang P H, Fung R F and Lee M J 1998 *J. Sou. Vib.* **209** 223-49
- [16] Zhu W D and Chen Y 2006 *J. Vib. Aco.* **128** 66-78
- [17] Zhang P, Zhu C M and Zhang L J 2008 *J. Shanghai JiaoTong Uni.* **42** 481-88