Application of Higher-Order Cumulant in Fault Diagnosis of Rolling Bearing

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Abstract. In this paper a new method of pattern recognition based on higher-order cumulant and envelope analysis is presented. The core of this new method is to construct analytical signals from the given signals and obtain the envelope signals firstly, then compute and compare the higher-order cumulants of the envelope signals. The higher-order cumulants could be used as a characteristic quantity to distinguish these given signals. As an example, this method is applied in fault diagnosis for 197726 rolling bearing of freight locomotive. The comparisons of the second-order, third-order and fourth-order cumulants of the envelope signals from different vibration signals of rolling bearing show this new method could discriminate the normal and two fault signals distinctly.

1. Introduction

In recent years, Higher-Order Statistics (HOS) has received increasing interest in signal processing [1]. It has also been used in many other fields, such as communication, sonar, biomedicine, speech signal, image processing, et al [2-5]. As we all know, there are some shortcomings of second-order statistics used in signal processing, such as inability to analyze the phase relation between different harmonic signals, and invalidity for nonlinear and non-Gaussian signals. On the other hand, application of HOS in signal processing could avoid these problems, due to its abilities including suppressing additive colored noise, identifying non-minimum phase systems, extracting information due to deviations from Gaussianity, detecting and characterizing nonlinear properties in signals or systems, analyzing the cyclostationary time series, separating unknown sources or Blind Source Separation, et al.

In this paper a new method of pattern recognition is presented, which is based on the envelope analysis of the given signals and the corresponding higher-order cumulant, one kind of HOS. The procedure of this method is to construct analytical signals by Hilbert Transform from the given signals, and obtain the envelope signals at first, then compute and compare the higher-order cumulants of these envelope signals. The higher-order cumulants could be used as the characteristic quantity to discriminate these given signals. In order to illustrate the validity of this method, it is used to discriminate the measured signals from 197726 rolling bearings of freight locomotive in different conditions. The comparison of the second-order, third-order and fourth-order cumulants of different vibration signals of rolling bearing show this new method could discriminate the normal and two fault signals distinctly.

2. Definition and Properties of Higher-Order Cumulant

In this part we would review the definition and properties of HOS curtly, which is the basis of the new method of pattern recognition.

2.1. Definition of Higher-Order Cumulant

Letting x denote a random variable and $f(x)$ its probability density function, its moment generating function could be defined as

$$\phi_x(\omega) = \mathrm{E}[\exp(\mathrm{j}\omega x)] = \int_{-\infty}^{\infty} f(x)\mathrm{e}^{\mathrm{j}\omega x}\,\mathrm{d}x, \tag{1}$$

where E denotes the expectation operator.

The definition of cumulant generating function of x is

$$C(\omega) = \ln\{\mathrm{E}[\exp(\mathrm{j}\omega x)]\}. \tag{2}$$

Considering the random vector $\mathbf{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ composed of n random variables, and supposing $\boldsymbol{\omega} = [\omega_1, \omega_2, ..., \omega_n]^{\mathrm{T}}$, one could obtain the definition of joint characteristic function for $\mathbf{x}$ as follow

$$\phi_{\mathbf{x}}(\omega_1, \omega_2, \cdots, \omega_n) = \mathrm{E}[\exp\mathrm{j}(\omega_1 x_1 + \omega_2 x_2 + \cdots + \omega_n x_n)], \tag{3}$$

and it could also be written as

$$\phi_{\mathbf{x}}(\boldsymbol{\omega}) = \mathrm{E}[\exp(\mathrm{j}\boldsymbol{\omega}^{\mathrm{T}}\mathbf{x})]. \tag{4}$$

And the cumulant generating function of $\mathbf{x}$ is defined as

$$C(\boldsymbol{\omega}) = \ln\{\mathrm{E}[\exp\mathrm{j}(\boldsymbol{\omega}^{\mathrm{T}}\mathbf{x})]\}. \tag{5}$$

Then the joint cumulant of $\mathbf{x}$ is

$$\mathrm{cum}(\mathbf{x}) = (-\mathrm{j})^k \frac{\partial^k \ln\phi_{\mathbf{x}}(\boldsymbol{\omega})}{\partial\omega_1^{k_1}\partial\omega_2^{k_2}\cdots\partial\omega_n^{k_n}}\Big|_{\omega_1=\omega_2=\cdots\omega_n=0}, \tag{6}$$

where $k = (k_1 + k_2 + \cdots + k_n)$.

Supposing $\{x(t)\}$ as k-order stationary random process and meet the condition of ergodicity, the k-order cumulant of $\{x(t)\}$ could be defined as the k-order joint cumulant of $[x(t), x(t+\tau_1), \cdots, x(t+\tau_{k-1})]^{\mathrm{T}}$, that is to say,

$$C_{k,x}(\tau_1, \tau_2, \cdots, \tau_{k-1}) = \mathrm{cum}[x(t), x(t+\tau_1), \cdots, x(t+\tau_{k-1})], \tag{7}$$

where $k_1 = k_2 = \cdots k_n = 1$.

According to the hypothesis of stationarity, it could be concluded that the k-order cumulant of $\{x(t)\}$ is only the function of $k-1$ time intervals $\tau_1, \tau_2, \cdots, \tau_{k-1}$. Then one could obtain the equations

$$C_{2,x}(\tau_1) = \mathrm{E}[x(t)x(t+\tau_1)], \tag{8}$$

$$C_{3,x}(\tau_1, \tau_2) = \mathrm{E}[x(t)x(t+\tau_1)x(t+\tau_2)], \tag{9}$$

$$C_{4,x}(\tau_1, \tau_2, \tau_3) = \mathrm{E}[x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)]$$

$$-C_{2,x}(\tau_1)C_{2,x}(\tau_2-\tau_3)-C_{2,x}(\tau_2)C_{2,x}(\tau_3-\tau_1)$$

$$-C_{2,x}(\tau_3)C_{2,x}(\tau_1-\tau_2)\,. \tag{10}$$

2.2. Common Properties of Higher-order Cumulant

(1) The higher-order cumulant meets the symmetry between all the random variables, i.e.

$$\mathrm{cum}(\mathrm{x}_1,\mathrm{x}_2,\cdots,\mathrm{x}_k)=\mathrm{cum}(\mathrm{x}_i,\mathrm{x}_j,\cdots,\mathrm{x}_q) \tag{11}$$

where $(i,\ j,\cdots,q)$ is an arbitrary permutation form of $(1,2,\cdots,k)$.

(2) The higher-order cumulant is linear, namely,

$$\mathrm{cum}(\lambda_1 x_1,\lambda_2 x_2,\cdots,\lambda_k x_k)=\mathrm{cum}(x_1,x_2,\cdots,x_k)\prod_i \lambda_i\,, \tag{12}$$

where $\lambda_i, i=1,2,\cdots,k$ are complex constants.

(3) The cumulant of the sum of two random variables equals the sum of the cumulant of the two variables, that is to say,

$$\mathrm{cum}(x_0+y_0,z_1,z_2,\cdots,z_k)=\mathrm{cum}(x_0,z_1,\cdots,z_k)+\mathrm{cum}(y_0,z_1,\cdots,z_k)\,, \tag{13}$$

where x_0 and y_0 are two different random variables. And this property could be extended to the case of many random variables.

(4) If x_i is mutually independent to y_i, one could obtain

$$\mathrm{cum}(x_1+y_1,x_2+y_2,\cdots,x_k+y_k)=\mathrm{cum}(x_1,x_2,\cdots,x_k)+\mathrm{cum}(y_1,y_2,\cdots,y_k)\,. \tag{14}$$

Accordingly, application of higher-order cumulant in signal processing would restrain the noise when non-Gaussian signal added by Gaussian noise.

(5) If λ is a constant, it has no effect on the result of cumulant, i.e.

$$\mathrm{cum}(\lambda+x_1,x_2,\cdots,x_k)=\mathrm{cum}(x_1,x_2,\cdots,x_k) \tag{15}$$

3. Analysis of the Envelope Signal

In this section we review the Hilbert Transform and the analysis of complex envelope spectrum.

3.1 Hilbert Transform

Hilbert Transform is linear, which maps a function to another one in the same space. It reveals the mutual relation between the real and the imagery part of the system function. Based on Hilbert Transform, some new communication and digital signal systems have been developed and used in many cases.

Letting $x(t)$ ($t\in(-\infty,+\infty)$) be a real function, its Hilbert Transform could be defined as

$$\hat{x}(t)=\int_{-\infty}^{+\infty}\frac{x(\tau)}{\pi(t-\tau)}\,\mathrm{d}\,\tau \tag{16}$$

which is also denoted by $\hat{x}(t)=\mathrm{H}[x(t)]$. It could be found that $\hat{x}(t)$ is the convolution of $x(t)$ and πt, that means,

$$\hat{x}(t)=x(t)*\frac{1}{\pi t} \tag{17}$$

3.2 Analysis of complex envelope spectrum

Analysis of complex envelope spectrum is to recover the modulated signal from the complex analytical signal constructed by the measured signal based on Hilbert Transform. The procedure of this analysis could be illustrated as follow:

(1) obtaining the measured signal from the sensors which may be written as $x(t) = r(t)\cos(2\pi ft + \phi(t))$, where the amplitude modulation $r(t)$ and phase modulation $\phi(t)$ all exist.

(2) applying the Hilbert Transform to $x(t)$ so that one could obtain the $\hat{x}(t)$ which could approximately be expressed as $\hat{x}(t) = r(t)\sin(2\pi ft + \phi(t))$.

(3) constructing the analytical signal $z(t) = x(t) + \mathrm{j}\hat{x}(t)$, where its real and imaginary part is $x(t)$ and $\hat{x}(t)$ respectively.

(4) extracting the modules of $z(t)$ as $|z(t)| = \sqrt{x^2(t) + \hat{x}^2(t)} = r(t)$ which is the amplitude modulated signal, and the instantaneous phase of $z(t)$ as $\theta(t) = \arctan\left(\frac{\hat{x}(t)}{x(t)}\right) = 2\pi ft + \phi(t)$ which is the original phase modulated signal after subtracting the ramp signal $2\pi\, ft$.

4. New method of pattern recognition based on higher-order cumulant and analysis of complex envelope spectrum

Based on the aforementioned description, one could establish the new method of pattern recognition based on higher-order cumulant and analysis of complex envelope spectrum. The first step of this method is to obtain the amplitude modulated signal from the measured signal. And the second step is to calculate the higher-order cumulant of and then compare them.

In order to illustrate the validity of this method, we apply it to recognize the measured signals from 197726 rolling bearings of freight locomotive. The results are shown in Fig 1 to Fig 3, where the line 1, 2, 3 denote the signal from normal bearing, rolling element defect bearing, and outer race defect bearing respectively. It could be found that, second-order and third-order cumulants of the measured signals could not distinguish the three operating conditions, whereas the fourth-order cumulants of the measured signals or the second-order, the third-order and the fourth-order cumulants of the envelope signals could distinguish these three conditions distinctly.

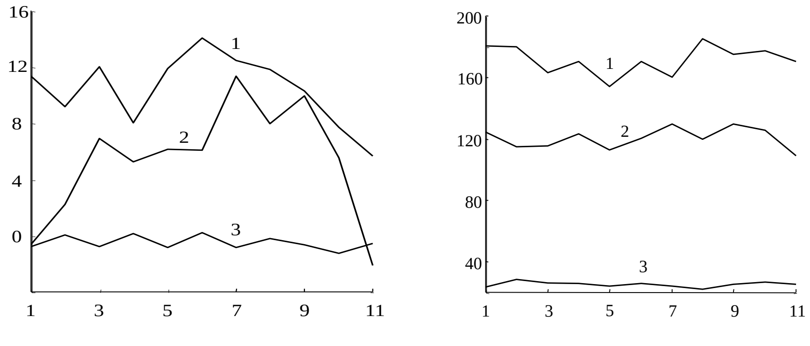
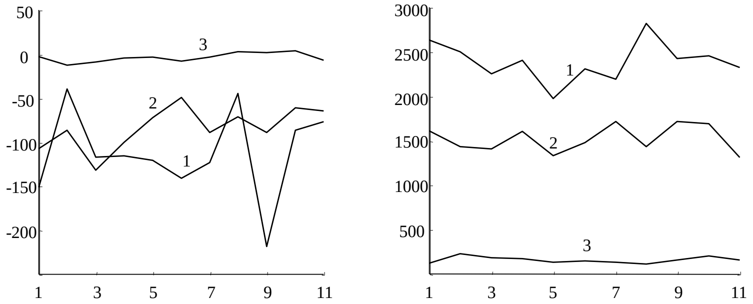
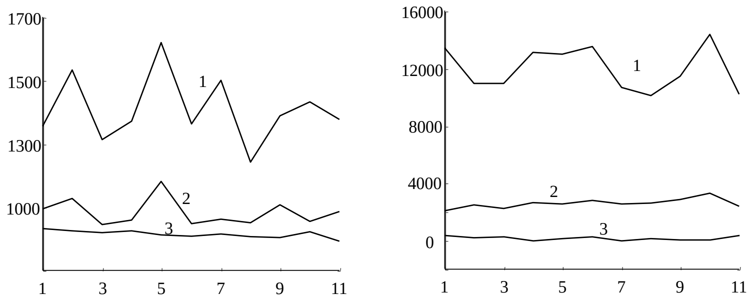


Figure 1. The Second-order cumulant ((a) for measured signals and (b) for envelope signals)



(a) (b)

Figure 2. The Third-order cumulant ((a) for measured signals and (b) for envelope signals)



(a) (b)

Figure 3. The Fourth-order cumulant ((a) for measured signals and (b) for envelope signals)

5. Conclusions

In this paper a new method for pattern recognition is investigated, which is based on higher-order cumulant and envelope analysis. And the efficiency of this method is illustrated by applying it in recognition of the operating conditions of 197726 rolling bearings of freight locomotive, which could distinguish the three conditions of rolling bearings distinctly.

6. References

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