

High-order Vibration Characteristics of Rotating Thin Shells and Hard-coating Damping Effects

X Y Song ¹, H J Ren ¹, X P Wang ², X J Li ³ and Q K Han ^{1,3*}

¹ School of Mechanical Engineering, Dalian University of Technology, Dalian, PR China 116024

² Shenyang Engine Design and Research Institute, Shenyang, PR China, 110015

³ Hunan Provincial Key Laboratory of Health Maintenance for Mechanical Equipment, Hunan University of Science and Technology, Xiangtan, PR China, 411201

E-mail: hanqingkai@dlut.edu.cn

Abstract. The high-order modes and the moving wave characteristics of rotating thin shell are investigated in this paper based on transfer matrix method when the damping effects of the hard-coating layers are applied on the shell. Firstly, the dynamic model of the thin shell is obtained based on the Love theory, with introducing centrifugal force and Coriolis force. Then, the transfer matrix formulas of the shell with partially coated layers at different segments are obtained under the clamped-free boundary condition, in which the mechanical property of the hard-coating material is simplified to be linear and isotropic. The modal characteristics of the shell are calculated numerically. The obtained higher order modal frequencies of the partially coated shells are compared with those of the bare shell. The effects of the material parameters and locations of the hard-coating layers are also illustrated based on the different modal characteristics., great care should be taken in constructing both.

1. Introduction

Rotating thin shells are widely used in many industry areas including the shafts of turbine machines, machining tools, combined disc-drum, etc. Thin shells often possess advantages of good stiffness and low mass. However, they are also prone to suffer from serious vibration and high cycle fatigue (HCF) due to multi-harmonic resonance, rubbing-impact and highly rotating speed. In practical design, the lower order resonances of a rotating shell is not permitted and must be guaranteed to avoid firstly. Besides, the higher-order vibrations of the thin shell also need to be paid attention because of their direct contribution to HCF. Sometimes, special damping treatments are urgently developed to compensate the lifetime of the shell under certain operating condition.

Nowadays, there are many classic theories and methods to analyze the vibrations of rotating thin shells. The natural characteristics of a rotating ring were investigated and moving waves were observed at the first time by Bryan ^[1]. The Coriolis effect on rotating shell was discussed by Taranto and Lessen ^[2]. The natural characteristics and the forward/backward moving waves of a rotating cylindrical shell with infinite length were investigated for different Coriolis and centrifugal forces by

^{1,3} hanqingkai@dlut.edu.cn



Srinivasan and Lauterbach^[3]. While the rotating shell with finite length were discussed by Zohar and Aboudi^[4], Wang and Chen^[5].

The vibrations of shells with different boundary conditions and corresponding solving methods were also explored extensively. The natural frequencies of the clamp-clamp supported by rotating shell were solved based on triangle-Galerkin combined method by Saito and Endo^[6]. The natural frequencies of rotating cylindrical shell with different boundary conditions were solved using complex exponent method by Penzes and Kraus^[7]. The forward and backward moving waves were compared by Lam and Loy^[8], for both different boundary conditions (simple-simple, clamped-clamped, clamped-free, respectively) and different shell theories (Donnell, Flügge, Love and Sander's theory). The effects of the rotating speed and thickness were also reported by them^[9].

Among the solving methods of shell dynamics, the transfer matrix method was introduced to solve the free vibration of clamp-free boundary condition shell by Tottenham and Shimizu^[10]. The dynamic characteristics of conical and cylindrical shells were solved using transfer matrix method by Irie^[11]. The forward and backward moving waves of the rotating shell were compared by Hong, Guo and Zhu^[12-13], for different thicknesses and Coriolis effects. The transfer matrix method can solve complex shell, the high order vibration character of the rotating thin shell were solved by Han^[14], considering the effect of sealing teeth.

Besides, constrained layer damping (CLD) is widely used in the aerospace and automotive industries as a novel technique of surface treatment to attenuate resonant structural vibrations of shells. The deformation energy of the structure is dissipated through the compliant layer made of viscoelastic material, sandwiched between the base structure and a stiff constraining layer. The performances of such treatment have been studied by many researchers. The overall damping achieved has been shown to be dependent not only on the inherent damping in the viscoelastic material but also on the thickness and elastic modulus of each layer^[15].

Hard-coating treatment is well known to play an important role in improving the structure lifetime and performance^[16]. In the last two decades, a so-called hard-coating damping technology for structure vibration reduction is interesting for many people. It can be achieved by using special coating materials with porous micro-structures that couple mechanical behaviour with electrical or magnetic fields applied to them^[17].

Most of the previous research, however, for a rotating thin shell, the effect of hard-coating damping and different coating parts, have no analytical, numerical and experiment results at all. This paper presents a strategy to solve the high-order vibrations of rotating thin shell based on transfer matrix method, and the damping effects of hard-coating treatment on its natural characteristics are compared. The method described in this paper is believed to include several novelties: First, the vibration equation of the rotating shell is established, considering the effect of centrifugal and Coriolis forces. Furthermore, based on Love shell model the governing differential equations for cylindrical shell segments are derived. Then, the state space vectors, propagation matrices and precise integrations are formed for different segments with or without hard-coating layers along axial directions. The mechanical property of the hard-coating material is simplified to be linear and isotropic. The obtained natural frequencies, especially higher-order modes, of the shell with different hard-coating segments are compared finally.

2. Analytic model of the rotating cylindrical shell and transfer matrix equations

Figure 1 shows a partly cylinder treated with hard-coating damping material which rotates in angular velocity speed Ω . The shell is modelled as a cylindrical shell consisting of sealing teeth effects. The figure 1(a) is a model of the partly coating damping material shell and the figure 1(b) is the cutaway view of the shell. The circular cylindrical shell is with a radius of R , thickness H and length L . The curve coordinate system is $o-x\theta z$. The variables u , v and w denotes the displacement of the shell in x -axis, y -axis and z -axis. The shell is divided into N parts and the lengths of every parts are $L_1, L_2, \dots, L_N$. The damping marital length is $b = x_2 - x_1$, where x_2 and x_1 denote the two ends of the damping parts in x direction.

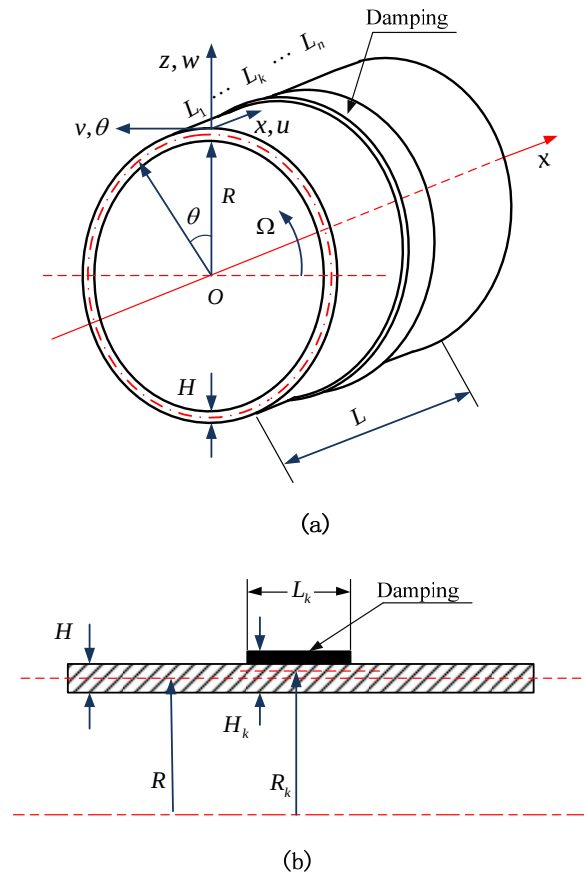


Figure 1. Rotating cylindrical shell partly treated with hard-coating damping material and its cutaway view.

The strains of the neutral surface of the shell, $\varepsilon_x^{(0)}$, $\varepsilon_\theta^{(0)}$ and shearing strain $\gamma_{x\theta}^{(0)}$, can be written as follows

$$\varepsilon_x^{(0)} = \frac{\partial u}{\partial x}, \varepsilon_\theta^{(0)} = \frac{\partial v}{R \partial \theta}, \gamma_{x\theta}^{(0)} = \frac{\partial v}{\partial x} + \frac{\partial u}{R \partial \theta} \quad (1)$$

Based on the Love thin shell theory, the intersection angle of the shell θ_x and θ_θ are described as

$$\theta_x = -\frac{\partial w}{\partial x}, \theta_\theta = \frac{v}{R} - \frac{\partial w}{R \partial \theta} \quad (2)$$

The curvature change of the neutral surface κ_x , κ_θ and torsion $\chi_{x\theta}$ are

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \kappa_\theta = \frac{1}{R^2} \left[\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right], \chi_{x\theta} = \frac{1}{R} \left[\frac{\partial v}{\partial \theta} - 2 \frac{\partial^2 w}{\partial x \partial \theta} \right] \quad (3)$$

For a thin shell, the displacements in x and θ directions are assumed to change linearly with the thickness, and the displacements in the transverse direction are independent on z . Thus, the strain-displacement relations based on Love theory are, therefore, with the following forms

$$\varepsilon_x = \varepsilon_x^{(0)} + z \kappa_x, \varepsilon_\theta = \varepsilon_\theta^{(0)} + z \kappa_\theta, \gamma_{x\theta} = \gamma_{x\theta}^{(0)} + z \chi_{x\theta} \quad (4)$$

where, z is the displacement of a point in transverse direction.

Based on the Hooke's law

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_\theta), \sigma_\theta = \frac{E}{1-\mu^2}(\varepsilon_\theta + \mu\varepsilon_x), \tau_{x\theta} = \frac{E}{2(1+\mu)}\gamma_{x\theta} \quad (5)$$

where, σ_x is the stress in x -axis, σ_θ is the stress in θ -axis and $\tau_{x\theta}$ is the shear force in x - θ plane.

The internal force is expressed as follow

$$N_x = \int_{-H/2}^H \sigma_x dz, N_\theta = \int_{-H/2}^H \sigma_\theta dz, N_{x\theta} = \int_{-H/2}^H \sigma_{x\theta} dz \quad (6a)$$

$$M_x = \int_{-H/2}^{H/2} \sigma_x z dz, M_\theta = \int_{-H/2}^{H/2} \sigma_\theta z dz, M_{x\theta} = \int_{-H/2}^{H/2} \sigma_{x\theta} z dz \quad (6b)$$

where, N_x, N_θ and $N_{x\theta}$ are the shear forces of the neutral surface per unit length, M_x, M_θ and $M_{x\theta}$ are the bending moment and torsion moment of the neutral surface per unit length.

Substituting equation (4) and (5) into equation (6) yields

$$N_x = K \left[\frac{\partial u}{\partial x} + \frac{\mu}{R} \left(\frac{\partial v}{\partial \theta} + w \right) \right] \quad (7a)$$

$$N_\theta = K \left[\frac{1}{R} \left(\frac{\partial v}{\partial \theta} + w \right) + \mu \frac{\partial u}{\partial x} \right] \quad (7b)$$

$$N_{x\theta} = K \frac{1-\mu}{2} \left(\frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \theta} \right) \quad (7c)$$

$$M_x = D \left[-\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{R^2} \left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right) \right] \quad (7d)$$

$$M_\theta = D \left[\frac{1}{R^2} \left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2} \right) - \mu \frac{\partial^2 w}{\partial x^2} \right] \quad (7e)$$

$$M_{x\theta} = D \frac{1-\mu}{2R} \left(\frac{\partial v}{\partial x} - 2 \frac{\partial^2 w}{\partial x \partial \theta} \right) \quad (7f)$$

where, K is film stiffness and D is bending rigidity

$$K = \frac{EH}{1-\mu^2}, D = \frac{EH^3}{12(1-\mu^2)}$$

The linear differential equation of the rotating thin shell can be written as follow. Equation (8a) is the oscillatory differential equation of the axial vibration. Equation (8b) is the oscillatory differential equation of the circumferential vibration and equation (8c) is the oscillatory differential equation of the radial vibration.

$$\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} + N_\theta^0 \left(\frac{1}{R^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{1}{R} \frac{\partial w}{\partial x} \right) = \rho H \frac{\partial^2 u}{\partial t^2} \quad (8a)$$

$$\frac{1}{R} \frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{Q_\theta}{R} + \frac{N_\theta^0}{R} \frac{\partial^2 u}{\partial x \partial \theta} = \rho H \frac{\partial^2 v}{\partial t^2} + 2\rho H \Omega \frac{\partial w}{\partial t} - \rho H \Omega^2 v \quad (8b)$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_\theta}{R \partial \theta} - \frac{N_\theta}{R} + \frac{N_\theta^0}{R^2} \left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right) = \rho H \frac{\partial^2 w}{\partial t^2} - 2\rho H \Omega \frac{\partial v}{\partial t} - \rho H \Omega^2 w \quad (8c)$$

where, N_θ^0 is the strain due to the centrifugal force; $N_\theta^0 = \rho H \Omega^2 R^2$; $\rho H \frac{\partial^2 u}{\partial t^2}$, $\rho H \frac{\partial^2 v}{\partial t^2}$, $\rho H \frac{\partial^2 w}{\partial t^2}$ are inertia force; $2\rho H \Omega \frac{\partial w}{\partial t}$, $2\rho H \Omega \frac{\partial v}{\partial t}$ are Coriolis force and $\rho H \Omega^2 v$, $\rho H \Omega^2 w$ are centrifugal force.

Q_x and Q_θ are shear force per unit length in x and θ direction, and can be written as follows

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{x\theta}}{R \partial \theta} \quad (9a)$$

$$Q_\theta = \frac{\partial M_{x\theta}}{\partial x} + \frac{\partial M_\theta}{R \partial \theta} \quad (9b)$$

The general solution of equation (8) is expressed as follows

$$u(x, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \tilde{u}(x) \cos(n\theta \pm \omega t) \quad (10a)$$

$$v(x, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \tilde{v}(x) \sin(n\theta \pm \omega t) \quad (10b)$$

$$w(x, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \tilde{w}(x) \cos(n\theta \pm \omega t) \quad (10c)$$

where, ω is the natural frequency of the shell; m, n are longitudinal half-wave number and the circumferential wave number; '+' and '-' is the forward traveling wave and backward wave of the rotating shell.

Substituting Equation (10) into Equation (2) yields

$$\theta_x = \tilde{\theta}_x \cos(n\theta \pm \omega t) \quad (11a)$$

$$\theta_\theta = \tilde{\theta}_\theta \sin(n\theta \pm \omega t) \quad (11b)$$

where, $\tilde{\theta}_x = -d\tilde{w}/dx$ and $\tilde{\theta}_\theta = \tilde{v}/R + n\tilde{w}/R$.

Substituting Equation (10) into Equation (9a) and Equation (9b), one obtains that that

$$\begin{bmatrix} N_x & N_\theta & M_x & M_\theta & Q_x & V_x \end{bmatrix}^T = \begin{bmatrix} \tilde{N}_x & \tilde{N}_\theta & \tilde{M}_x & \tilde{M}_\theta & \tilde{Q}_x & \tilde{V}_x \end{bmatrix}^T \cos(n\theta \pm \omega t) \quad (12a)$$

$$\begin{bmatrix} N_{x\theta} & M_{x\theta} & Q_\theta & S_x \end{bmatrix}^T = \begin{bmatrix} \tilde{N}_{x\theta} & \tilde{M}_{x\theta} & \tilde{Q}_\theta & \tilde{S}_x \end{bmatrix}^T \sin(n\theta \pm \omega t) \quad (12b)$$

where,

$$\tilde{N}_x = K \left[\frac{d\tilde{u}}{dx} + \mu \left(\frac{n}{R} \tilde{v} + \frac{1}{R} \tilde{w} \right) \right],$$

$$\tilde{M}_x = D \left(\frac{d\tilde{\theta}_x}{dx} + \mu \frac{n}{R} \tilde{\theta}_\theta \right), \quad \tilde{N}_\theta = K \left(\frac{n}{R} \tilde{v} + \frac{1}{R} \tilde{w} + \mu \frac{d\tilde{u}}{dx} \right), \quad \tilde{M}_\theta = D \left(\frac{n}{R} \tilde{\theta}_\theta + \mu \frac{d\tilde{\theta}_x}{dx} \right),$$

$$\tilde{N}_{x\theta} = K \frac{1-\mu}{2} \left(\frac{d\tilde{v}}{dx} - \frac{n}{R} \tilde{u} \right), \quad \tilde{M}_{x\theta} = D \frac{1-\mu}{2} \left(\frac{d\tilde{\theta}_\theta}{dx} - \frac{n}{R} \tilde{\theta}_x \right), \quad \tilde{Q}_x = \frac{d\tilde{M}_x}{dx} + \frac{n}{R} \tilde{M}_{x\theta}, \quad \tilde{Q}_\theta = \frac{d\tilde{M}_{x\theta}}{dx} - \frac{n}{R} \tilde{M}_\theta,$$

$$\tilde{V}_x = \tilde{Q}_x + \frac{n}{R} \tilde{M}_{x\theta}, \quad \tilde{S}_x = \tilde{N}_{x\theta} + \frac{1}{R} \tilde{M}_{x\theta}.$$

Substituting equation (10) and equation (11) into vibration equation (8) yields

$$\frac{d\tilde{N}_x}{dx} + \frac{n}{R} \tilde{N}_{x\theta} - \rho H \Omega^2 R \left(\frac{n^2}{R} \tilde{u} + \frac{d\tilde{w}}{dx} \right) + \rho H \omega^2 \tilde{u} = 0 \quad (13a)$$

$$\frac{d\tilde{N}_{x\theta}}{dx} - \frac{n}{R} \tilde{N}_\theta + \frac{1}{R} \tilde{Q}_\theta - n \rho H \Omega^2 R \frac{d\tilde{u}}{dx} + \rho H (\omega^2 \tilde{v} \pm 2\omega \Omega \tilde{w} + \Omega^2 \tilde{v}) = 0 \quad (13b)$$

$$\frac{d\tilde{Q}_x}{dx} + \frac{n}{R} \tilde{Q}_\theta - \frac{\tilde{N}_\theta}{R} - \rho H \Omega^2 (n^2 \tilde{w} + n \tilde{v}) + \rho H (\omega^2 \tilde{w} \pm 2\omega \Omega \tilde{v} + \Omega^2 \tilde{w}) = 0 \quad (13c)$$

From the second factor and third factor of equation (13), the control equation contain the terms of $\pm 2\omega \Omega$. It is why the rotating shell has two different frequencies which are defined as the forward traveling wave and backward wave.

The primary variables of the rotating cylindrical shell are expressed in the vector form as

$$\tilde{Z}(x)=[\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\theta}_x, \tilde{M}_x, \tilde{V}_x, \tilde{S}_x, \tilde{N}_x]^T \quad (14)$$

The general solution of the state variables is expressed as

$$Z(x, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \begin{bmatrix} \tilde{u}(x) \cos(n\theta \pm \omega t) \\ \tilde{v}(x) \sin(n\theta \pm \omega t) \\ \tilde{w}(x) \cos(n\theta \pm \omega t) \\ \tilde{\theta}_x(x) \cos(n\theta \pm \omega t) \\ \tilde{M}_x(x) \cos(n\theta \pm \omega t) \\ \tilde{V}_x(x) \cos(n\theta \pm \omega t) \\ \tilde{S}_x(x) \sin(n\theta \pm \omega t) \\ \tilde{N}_x(x) \cos(n\theta \pm \omega t) \end{bmatrix} \quad (15)$$

where ω is the natural frequency of the rotating shell.

Based on equation (11)- equation (13), the only state equation for the primary variables is considered for simplification

$$\frac{d\tilde{Z}(x)}{dx} = \tilde{U} \cdot \tilde{Z}(x) \quad (16)$$

where, $\tilde{U}$ is a constant coefficient matrix of 8×8 and $\tilde{U}$ is expressed as follows

$$\tilde{U} = \begin{bmatrix} 0 & U_{12} & U_{13} & 0 & 0 & 0 & 0 & U_{18} \\ U_{21} & 0 & 0 & U_{24} & 0 & 0 & U_{27} & 0 \\ 0 & 0 & 0 & U_{34} & 0 & 0 & 0 & 0 \\ 0 & U_{42} & U_{43} & 0 & U_{45} & 0 & 0 & 0 \\ U_{51} & 0 & 0 & U_{54} & 0 & U_{56} & U_{57} & 0 \\ 0 & U_{62} & U_{63} & 0 & U_{65} & 0 & 0 & U_{68} \\ 0 & U_{72} & U_{73} & 0 & U_{75} & 0 & 0 & U_{78} \\ U_{81} & 0 & 0 & U_{84} & 0 & 0 & U_{87} & 0 \end{bmatrix} \quad (17)$$

The elements which is not equal zero in matrix $U_{ij} (i, j=1, \dots, 8)$ are relate to the natural frequency ω of the shell, rotating speed Ω , geometrical parameter and material parameter. The expressions of the matrix $\tilde{U}$ are given in Appendix A.

The rotating shell in figure1 is divided into n_0 parts. The propagating relationship given in equation (16) can be used repeatedly so that we can propagate the physical quantities from the one edge to the other edge of the rotating shell. Thus, we have

$$\tilde{Z}_{n_0}(\xi_{n_0}) = \tilde{T}(\omega) \cdot \tilde{Z}_1(\xi_0) \quad (i=1, \dots, n_0) \quad (18)$$

Where, $\tilde{T}(\omega)$ is the propagator matrix, which can be written as $\tilde{T}(\omega) = \prod_{i=1}^{n_0} \tilde{T}_i(\omega)$.

According to the differential equations theory, state vector equations can be expressed as

$$\tilde{Z}(\xi_i) = \tilde{T}_i(\omega) \tilde{Z}(\xi_{i-1}) \quad (i=1, \dots, n_0) \quad (19)$$

where, for the i^{th} rotating cylindrical thin shell, the propagator matrix can be written as $\tilde{T}_i(\omega) = \exp(\tilde{U}_i L_i)$.

In order to get a more exact result, the i^{th} shell can be divided into k parts, whose size is $L_1, L_2, \dots, L_{k-1}, L_k$ in x -direction, and the transfer matrix is defined as follow

$$\tilde{Z}(L_k) = \tilde{G}(L_k) \tilde{Z}(L_{k-1}) \quad (20)$$

where, $\tilde{G}(L_k)$ is the transfer matrix of the k^{th} shell and $\tilde{G}(L_k) = \exp(\tilde{U}_k L_k)$.

When the character of a rotating thin shell is solved, the characteristic equation is defined by boundary condition.

For a clamp-free supported rotating cylindrical thin shell, the boundary conditions are as follows:

$$u = v = w = \varphi_x = 0 \quad \text{where } x = 0,$$

$$\tilde{M}_x = \tilde{V}_x = \tilde{S}_x = \tilde{N}_x = 0 \quad \text{where } x = L.$$

Substituting the boundary conditions into equation (14), then yields

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{T}_{11} & \tilde{T}_{12} & \tilde{T}_{13} & \tilde{T}_{14} \\ \tilde{T}_{21} & \tilde{T}_{22} & \tilde{T}_{23} & \tilde{T}_{24} \\ \tilde{T}_{31} & \tilde{T}_{32} & \tilde{T}_{33} & \tilde{T}_{34} \\ \tilde{T}_{41} & \tilde{T}_{42} & \tilde{T}_{43} & \tilde{T}_{44} \end{bmatrix} \begin{bmatrix} \tilde{u}_x \\ \tilde{v}_x \\ \tilde{w}_x \\ \tilde{\varphi}_x \end{bmatrix} \quad (21)$$

The hard-coating damping material is anisotropic. In this paper, the Voigt approach method is introduced to solve the hard-coating damping shell.

Here E_d is Young's modulus of the hard damping material; E_m is Young's modulus of the thin shell; V_d is the volume fraction of the hard damping material; V_m is the volume fraction of thin shell; μ_d is Poisson's ratio of damping material and μ_m is Poisson's ratio of shell, based on the Voigt approach method then yields

$$E = E_d V_d + E_m V_m \quad (22)$$

$$\mu = \mu_d V_d + \mu_m V_m \quad (23)$$

$$G = \frac{G_d G_m}{G_d V_d + G_m V_m} \quad (24)$$

$$\rho = \rho_d V_d + \rho_m V_m \quad (25)$$

3. Numerical results

The geometrical parameter and material parameter of the shell is given in table1.

Table 1. Properties and geometrical parameter of the thin cylindrical shell

Properties	Symbol	Value	Units
Young's modulus	E	206×10^{11}	Pa
Poisson's ratio	μ	0.3	
Density	ρ	7850	kg/m ³
Length	L	0.256	m
Radius	R	0.141	m
Thickness	H	0.002	m

The hard-coating thin shell composed of an isotropic metal substrate and a hard-coating damping layer is analyzed. The hard-coating damping layer is made of Al₂O₃+MgO, which is an anisotropic material. The material parameters of this kind of hard-coating materials of Al₂O₃+MgO are listed in table 2.

Table 2. Material parameters of hard-coating damping material

Elastic coefficients C_{ik}					Unit Gpa			
C11	C12	C13	C22	C23	C33	C44	C55	C66
183.1	173.2	170.5	183.1	170.5	183.1	45.3	45.3	56.5

In this paper we set the Young's modules $E_d = 170.5 \text{ GPa}$, material density $\rho_d = 5300 \text{ kgm}^{-3}$ and Poisson's ratio $\mu_d = 0.3$. Based on equation (22) and equation (25), the linear Young's modules are calculated as $E = 204 \times 10^9 \text{ Pa}$, density $\rho = 7660 \text{ kgm}^{-3}$. Substituting the linear material parameters into equation (18), the natural frequency of the partially hard-coating damping material shell can be calculated.

3.1 The high-order vibration characteristics of the bare shell when rotating speed $\Omega = 0$

First of all, the transfer matrix method and finite element method are introduced to solve the static shell with the clamp-free boundary condition. For boundary conditions, the natural frequencies can be solved from equation (21). It is assumed that the top and bottom edges are clamp and free boundaries.

In order to validate the transfer matrix method results, the finite element method (FEM) is used to calculate the natural frequency values of the corresponding mode orders for the non-coated shell under the same boundary condition. The finite element model of one of the thin shell here composes of 920 elements (SHELL181 of ANSYS) of the substrate thin shell. The natural frequencies and mode shapes are solved by Block Lanczos method.

Figure 2 shows the natural frequency of the static shell with different half-wave number m and the circumferential wave number n . The curve $m=1$ shows that the transfer matrix method and FEM have the same precision. The frequencies based on the transfer matrix corresponding to modes 1–5 are exactly the same as that by FEM. The natural frequency of the thin shell decreases firstly, then enlarge from 4389.4 Hz to 4864.4 Hz, at the (1,6) step the natural is the lowest 1355.5 Hz. Compare with $m=1$ and $m=2$ the natural frequency is different with the same circumferential wave number n .

When $n < 6$, the difference between the transfer matrix method and the FEM is slight, but the difference increases with the circumferential wave number changes from 1 to 20.

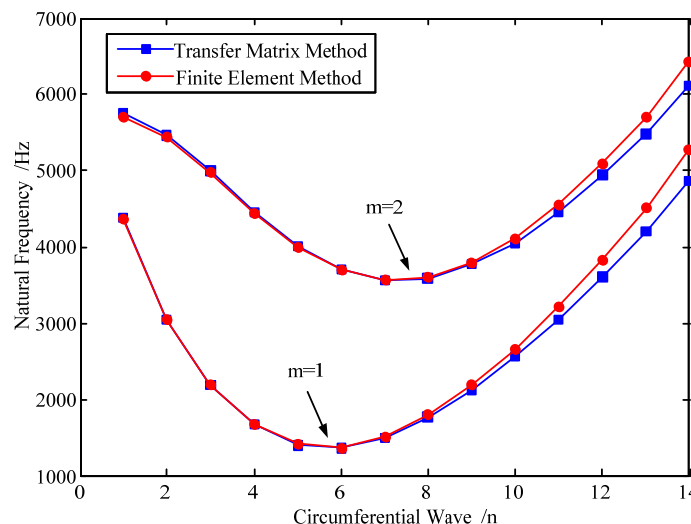


Figure 2. The high-order frequencies of the static shell.

3.2 The Forward and backward waves of the high-order modes of the bare shell under different rotating speeds

Figure 3 and figure 4 show the relationship between the rotating speed Ω and the natural frequency. Based on equation (12)- (18) the transfer matrix of a clamp-free boundary condition shell with rotating speed Ω . When the rotating speed Ω increases, the natural frequency of the rotating shell are divided

into two parts, the top curve is the forward traveling wave frequency and the other one is the backward traveling wave. Different from the low-order condition, the high-order vibration's backward frequency curve increase with the rotating speed changed from 0 rad/sec to 2000 rad/sec. In figure 8 and figure 10 the difference between FEM and TRM are enlarged as the rotating speed increases.

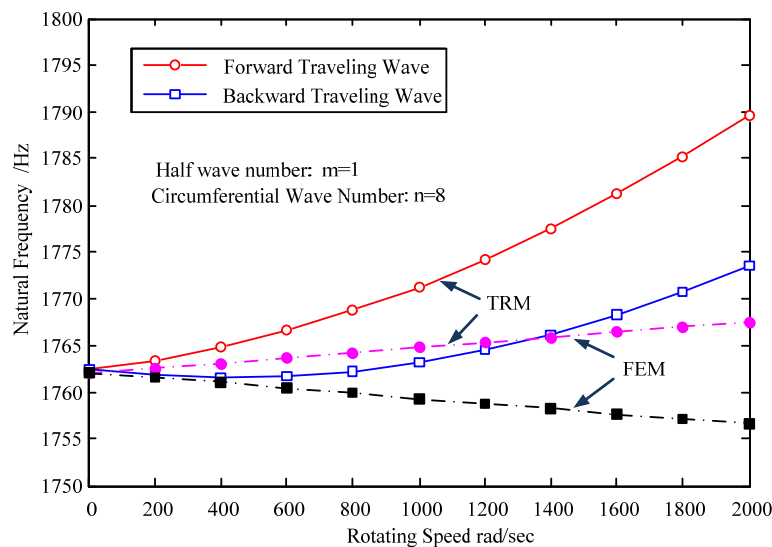


Figure 3. Forward and backward waves of the rotating cylindrical shells with clamped-free supported boundary conditions at both edges for high nodal diameters $n=8$.

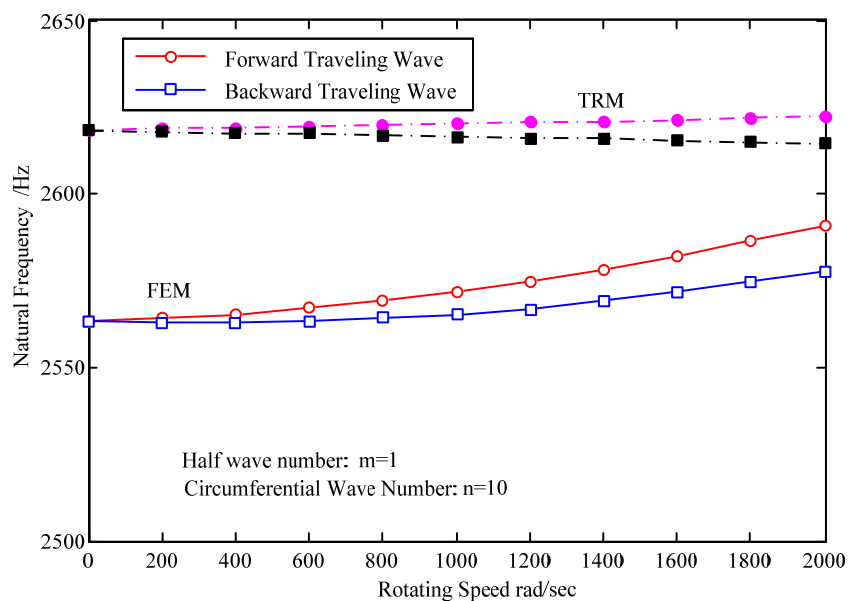


Figure 4. Forward and backward waves of the rotating cylindrical shells with clamped-free supported boundary conditions at both edges for high nodal diameters $n=10$.

3.3 The high-order vibration characteristics of the hard-coating shell when rotating speed $\Omega = 0$

The hard-coating layers are applied on the total out surface of the shell by APS. Figure 5 compares the natural frequency of hard-coating shell and thin shell. By checking the obtained modes of the bare plate and the coated shell with hard-coating damping material, it is observed that the natural frequency changes. When the half-wave number $m=1$, the first 6 natural frequencies of the two shells values have a very little difference. When the mode changes from $n=7$ to $n=20$, the different between the shell and hard-coating damping shell increases at the same, especially at the high-order vibration.

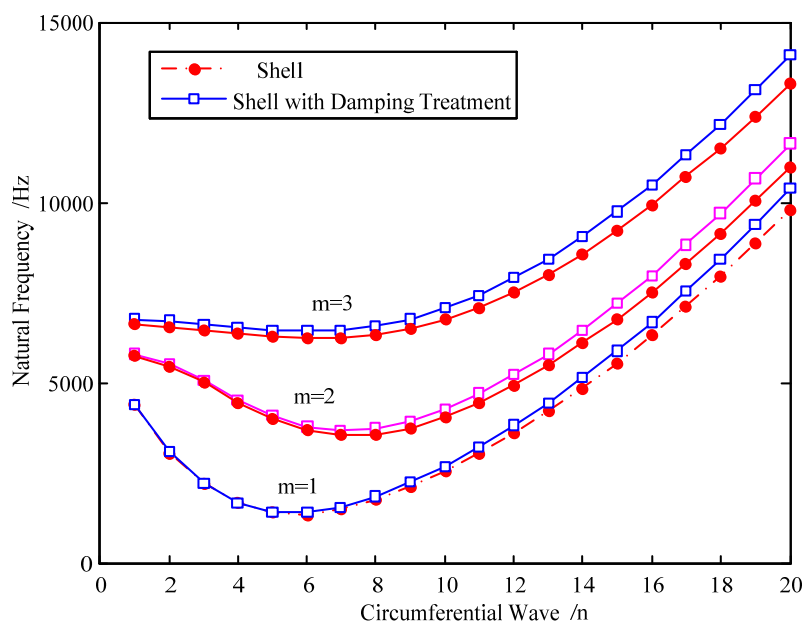


Figure 5. The natural frequencies of the shell with hard-coating damping material treatment totally.

3.4 The high-order vibration characteristics of the partly coated shell on the middle position when rotating speed

The transfer matrix method is introduced to solve the partially hard-coating damping material shell which is 30% area of the total shell, shown in figure 1, with the clamp-free boundary condition. In this paper, the coating position is in the middle of the x -axis and the results are shown in figure 6. By checking the obtained modes of the bare shell and the coated shell with 30% hard-coating damping material, it is obtained that the natural frequency has the same changing trend. In the case $m=1, n \leq 13$, the coating damping material of 30% area has little effect on the thin shell; at the high vibration order $m=1, n > 13$, the damping material has an obviously effect on the natural frequency. In the case $m > 1$, the damping material works better than that when $m=1$.

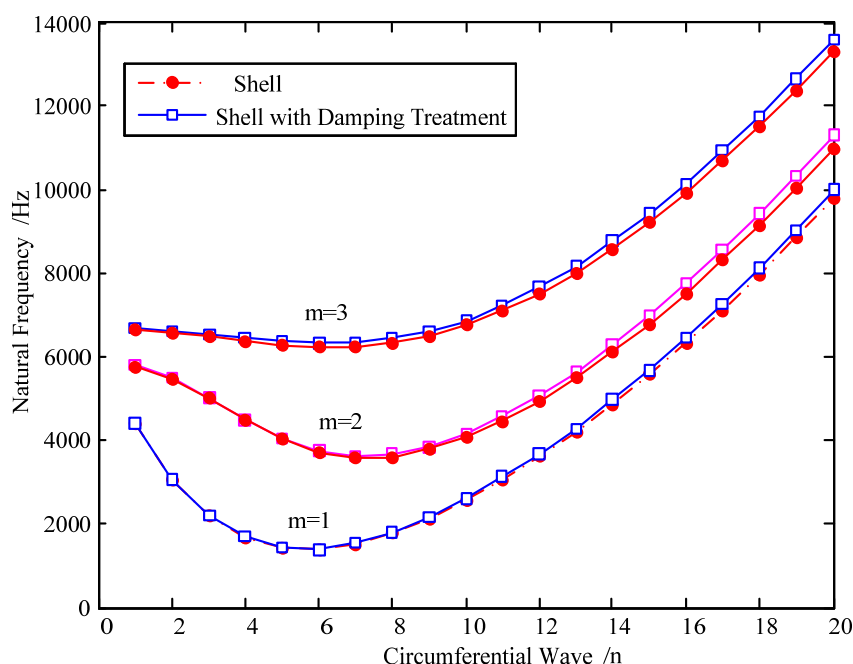


Figure 6. The natural frequencies of the shell with hard-coating damping material treatment of 30% area.

3.5 The high-order vibration characteristics of the partly coated shells on different positions when rotating speed $\Omega=0$

In order to compare different cover position of the damping material effect on the shell, the results are shown in table 2 and figure 6. It is clearly that the natural frequencies of the shells are different due to the effect of the cover position of the hard-coating damping material. When the material is covered near the free edge, the natural frequency of the shell increases more obviously than the other two cases. The natural frequencies have the same trend due to the circumferential wave number n .

Table 3. Comparison of different coating position for the natural frequency of the shell
(the coating area is 30%)

Order	Coating position			Order	Coating position		
	Top	Middle	Bottom		Top	Middle	Bottom
1	4386.4	4405.8	4424.5	11	3185.6	3119.3	3085.4
2	3040.3	3063.1	3085.6	12	3759.3	3680.8	3639.6
3	2189.1	2209.3	2229.4	13	4386.5	4295.2	4246.4
4	1673.2	1688.6	1704.7	14	5066.2	4960.9	4904.3
5	1412.8	1420.2	1430.4	15	5797.1	5677.3	5612.7
6	1379.2	1374.8	1377.2	16	6579.2	6443.9	6371.1
7	1534.7	1516.8	1511.1	17	7412.2	7260.3	7179.1
8	1825.8	1794.9	1781.7	18	8295.7	8126.5	8036.7
9	2211.1	2168.1	2147.9	19	9229.6	9042.1	8943.8
10	2667.9	2613.3	2586.3	20	10213.6	10007.7	9900.3

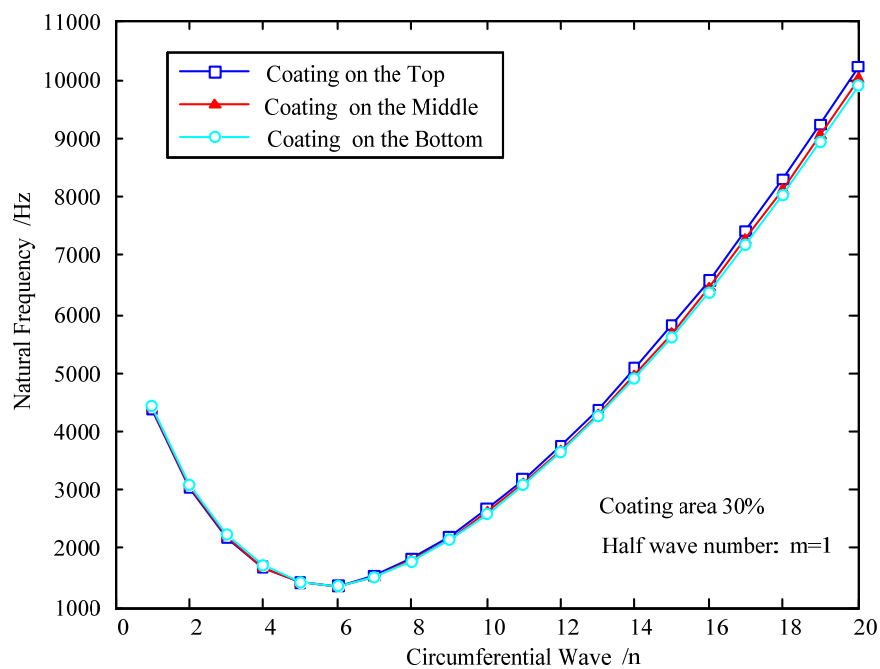


Figure 7. Comparison of different coating positions of the natural frequencies of the shell.

3.6 The high-order vibration characteristics of the partly coated shells on middle position with different rotating speeds

Figure 8 shows the relationship between the rotating speed Ω and the natural frequency of the hard-coating damping material. While the rotating speed Ω becomes fast, the natural frequency of the rotating shell are divided into two parts, the top curve is the forward traveling wave frequency and the other one is the backward traveling wave. In this figure, we consider the case that the half wave number $m=1$ and circumferential wave number $n=10$. At the high rotating speed, the backward traveling frequency is near to the forward traveling frequency in this case. The damping material has little effect on the rotating shell.

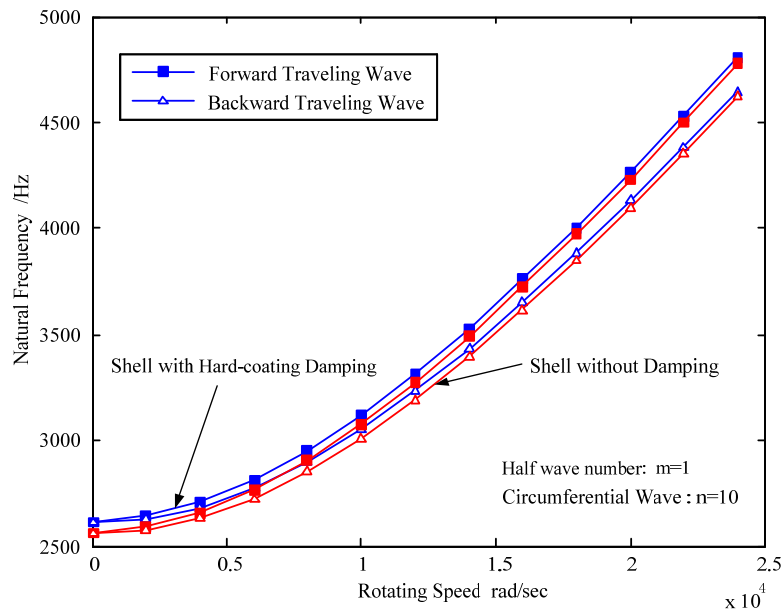


Figure 8. Bifurcations of the natural frequency of hard-rotating damping cylindrical shells with coating area of 30% for high nodal diameters.

4. Conclusions

This paper presents a transfer matrix method to analyze the high-order vibrations and the damping effects of the hard-coating layers applying on different ring-like areas. The conclusions are conducted as follows:

(1) The transfer matrix method used for the bare shell under the same boundary condition has the same precision compare with the finite element method. And the transfer also calculates the natural frequency of the rotating shell.

(2) The Voigt approach method is introduced to solve the hard-coating damping shell and the anisotropic material can be approximated by linear material. The Voigt approach can calculate the hard-coating damping material shell.

(3) By checking the obtained modes of the bare shell and the coated shell with hard-coating damping material, it is observed that the natural frequency have the same change trend. At the high vibration order, the damping material has a more obviously effect on the natural frequency than at the low order.

(4) The covering position of the damping material has different effect on the natural frequency of the thin shell. The natural frequency values of the shell increase more obviously than the clamp and middle edges.

Reference

- [1] Bryan G H 1890 On the beats in the vibrations of a revolving cylinder or bell *Proceedings of the Cambridge Philosophical Society* **7** 100-101.
- [2] Lessen R A M 1964 Coriolis acceleration effect on the vibration of a rotating thin walled circular cylinder *Journal of Applied Mechanics* **31** 700-701.
- [3] Srinivasan A V, Lauterbach G F 1971 Traveling waves in rotating cylindrical shells *Journal of Engineering for Industry, ASME* **93** 1229-1232.
- [4] Zohar, Aboudi J 1973 The free vibrations of thin circular finite rotating cylinder *International Journal of Mechanical Sciences* **15** 269-278.

- [5] Wang S S, Chen Y 1974 Effects of rotation on vibrations of circular cylindrical shells *Journal of the Acoustical Society of America* **55** 1340–1342.
- [6] Saito T, Endo M. 1986 Vibration of finite length, rotating cylindrical shells *Journal of Sound and Vibration* **107** 17-28.
- [7] Penzes L E, Kraus H 1972 Free vibrations of prestressed cylindrical shells having arbitrary homogeneous boundary conditions *AIAA* **10** 1309-1313.
- [8] Lam K Y, Loy C T 1995 Analysis of rotating laminated cylindrical shells by different thin shell theories *Journal of Sound and Vibration*, **186** 23-35.
- [9] Lam K Y, Loy C T 1998 Influence of boundary conditions for a thin laminated rotating cylindrical shell *Composite Structures*, **41** 215-228.
- [10] Tottenham H, Shimizu K 1972 Analysis of the free vibration of cantilever cylindrical thin elastic shells by the matrix progression method *International Journal of Mechanical Sciences* **14** 293-310.
- [11] Irie T, Yamada G and Kaneko Y 1982 Free vibration of a conical shell with variable thickness *Journal of Sound and Vibration* **82** 83-94.
- [12] Hong J, Guo B T and Zhu Z G 1999 The research of rotating thin shell and the experiment *Aircraft Engine* **1** 40-44.
- [13] Hong J, Guo B T and Zhu Z G 1998 The experiment research of rotating thin shell *Journal of Aerospace Power* **13** 40-44.
- [14] Han Q K, Wang Y 2012 The high order vibration character of the rotating thin shell with the effect of sealing teeth *SCIENCE CHINA Physics, Mechanics & Astronomy*.
- [15] Rongong J A, Tomlinson G R 1996 Suppression of ring vibration modes of high nodal diameter using constrained layer damping methods *Smart Mater. Struct.* **5** 672-684.
- [16] Ding J N, Meng Y G and Wen S Z 2000 Mechanical properties and fracture toughness of multilayer hard coatings using nanoindentation *Thin Solid Films* **371** 178-182.
- [17] Buravalla V R, Remillat C and Rongong J A 2001 Advances in damping materials and technology *Smart Materials Bulletin* **8** 10-13.

Appendix

$$\begin{aligned}
 U_{12} &= -\frac{\mu n}{R}, U_{13} = -\frac{\mu}{R}, U_{18} = \frac{1-\mu^2}{EH}, U_{21} = \frac{12nR}{H^2-12R^2}, U_{24} = \frac{2H^2n}{H^2+12R^2}, \\
 U_{27} &= \frac{24R^2(1+\mu)}{EH(H^2+12R^2)}, U_{34} = -1, U_{42} = -\frac{\mu n}{R^2}, U_{43} = -\frac{\mu n^2}{R^2}, U_{45} = \frac{12(1+\mu^2)}{EH^3}, \\
 U_{51} &= -\frac{EH^3n^2}{R(1+\mu)(H^2+12R^2)}, U_{54} = \frac{2EH^3n^2}{(1+\mu)(H^2+12R^2)}, U_{56} = 1, U_{57} = -\frac{2H^2n}{H^2+12R^2}, \\
 U_{62} &= \rho H \left(n\Omega^2 \pm \omega\Omega \right) + n \frac{EH}{R^2} \left(1 + \frac{n^2H^2}{12R^2} \right), U_{63} = -\rho H \left[\omega^2 - (m^2-1)\Omega^2 \right] + \frac{EH}{R^2} \left(\frac{H^2n^4}{12R^2} + 1 \right), \\
 U_{65} &= \frac{\mu n^2}{R^2}, U_{68} = \frac{\mu}{R}, U_{72} = -\rho H \left[\omega^2 + (1+\mu n^2)\Omega^2 \right] + n^2 \frac{EH}{R^2} \left(1 + \frac{H^2}{12R^2} \right), U_{75} = \frac{\mu n}{R^2} \\
 U_{73} &= -\rho H \left(\pm 2\omega\Omega + \Omega^2 \right) + n \frac{EH}{R^2} \left(1 + \frac{n^2H^2}{12R^2} \right), U_{78} = \frac{\mu n}{R} + \frac{n(1-\mu^2)\rho R\Omega^2}{E}, \\
 U_{81} &= -\rho H \left(\omega^2 - n\Omega^2 \right) + \frac{n^2EH^3}{2R^2(1+\mu)(H^2+12R^2)}, \\
 U_{84} &= -\rho H\Omega^2 R - \frac{n^2EH^3}{R(1+\mu)(H^2+12R^2)}, U_{87} = -\frac{12nR}{H^2+12R^2}.
 \end{aligned}$$