Combining Flavour and CP Symmetries

Ferruccio Feruglio

Dipartimento di Fisica e Astronomia 'G. Galilei', Università di Padova,
and INFN, Sezione di Padova,
Via Marzolo 8, I-35131 Padua, Italy

E-mail: `feruglio@pd.infn.it`

Abstract. I shortly review the impact of the most recent neutrino oscillation data on our attempts to construct a realistic model for neutrino masses and mixing angles. Models based on anarchy and its variants remain an open possibility, reinforced by the latest experimental findings. Many models based on discrete symmetries no longer work in their simplest realizations. I illustrate several proposals that can rescue discrete symmetries. In particular I discuss the possibility of combining discrete flavour symmetries and CP, and I describe a recently proposed symmetry breaking pattern that allows to predict all mixing parameters, angles and phases, in terms of a single real unknown. I analyze several explicit examples of this construction, providing new realistic mixing patterns.

1. Introduction

The discovery of neutrino oscillations brought lot of excitement in particle physics. It still represents the first and, in some respect, unique evidence of new physics. The smallness of neutrino masses evokes the breaking of the total lepton number, thus making neutrinos a privileged observatory to investigate energy scales that would otherwise be unaccessible. Moreover lepton mixing properties together with those of the quark sector can shed a new light on the flavour mystery. Perhaps form this information we can identify some principle allowing us to describe in a more economic and rational way the multitude of Yukawa interactions needed to account for fermion masses and mixing angles.

The early data on neutrino oscillations were compatible with a maximal atmospheric mixing angle and a vanishing reactor angle. The whole mixing pattern was consistent with the very simple ansatz [1]:

$$\sin^2\theta_{23} = \frac{1}{2} \quad , \qquad \sin^2\theta_{12} = \frac{1}{3} \quad , \qquad \sin^2\theta_{13} = 0 \quad . \tag{1}$$

Despite the large experimental errors affecting, until recently, both the atmospheric and the reactor mixing angles, this circumstance was taken by many of us as evidence for a symmetry principle beyond the data. Discrete symmetries based on small groups such as S_3, A_4 [2] and S_4 [3] were soon recognized at the basis of efficient mechanisms able to reproduce the tribimaximal (TB) pattern in eq. (1). There was also a certain confidence that deviations from the TB pattern had to be small, since the solar mixing angle was soon measured to a very good precision, about a couple of degrees, in impressive agreement with eq. (1). On this basis we could hope that the TB ansatz were correct to few degrees. We have now evidence at the 10σ level that θ_{13} is non-vanishing [4]. Its size is comparable to that of the Cabibbo angle. We also have a first hint for a non-maximal atmospheric mixing angle. Results from a recent global fit to neutrino oscillations are reported in table 1.

$\sin^2\theta_{12}$	0.30 ± 0.013
$\sin^2\theta_{23}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$
$\sin^2\theta_{13}$	0.023 ± 0.0023
Δm^2_{21}	$(7.50 \pm 0.185) \times 10^{-5}\ eV^2$
$\Delta m^2_{31}(N)$	$(2.47^{+0.069}_{-0.067}) \times 10^{-3}\ eV^2$
$\Delta m^2_{31}(I)$	$-(2.43^{+0.042}_{-0.065}) \times 10^{-3}\ eV^2$

Table 1. Results of a global fit to neutrino oscillations, from ref. [5] There are two best fit values for the atmospheric mixing angle θ_{23} due to the presence of two minima in the χ^2 function. The labels N and I refer to normal and inverted ordering, respectively.

There are still several open questions concerning neutrino properties. We do not know whether the total lepton number L is violated or not, whether the neutrino mass ordering is normal or inverted. The Dirac phase δ_{CP} is essentially unconstrained and the absolute scale of neutrino masses is still undetermined. Despite the remaining unknowns, our knowledge of neutrinos has greatly improved thanks to the most recent experiments and many models have been ruled out. We might expect that some coherent theoretical description had already become apparent from the data. Unfortunately this is not the case. Present data can still be described by widely different approaches.

2. Anarchy

One possible interpretation of the current results is in term of anarchy [6], which does not recognise any special pattern in the neutrino data. Lepton mixing angles and neutrino mass ratios are generic order one parameters, the smallness of θ_{13} and $\Delta m^2_{sol}/\Delta m^2_{atm}$ being accidental features with no special meaning. The actual size of θ_{13} and the indications in favour of a non maximal θ_{23} have strengthened this point of view.

It is worth saying that the idea of anarchy has several good aspects. Anarchy can be easily incorporated in valuable theoretical frameworks. For instance it can be realized in SU(5) Grand Unified Theories (GUTs), by the inclusion of a Froggatt-Nielsen $\mathrm{U}(1)_{FN}$ group [7] under which the three generations of pentaplets $\bar{5}$ are assumed to have the same charge. It can also be realized in models with extra dimensions (ED). For example, standard model fermions in the bulk of one extra dimension, spanning an interval of finite length, develop zero modes whose profiles are controlled by bulk mass parameters. If the Higgs doublet is localized at one of the two edges of the five dimensional interval, the Yukawa couplings mimic those of a Froggatt-Nielsen setup, with the role of the Froggatt-Nielsen charges played by the fermion bulk masses. Anarchy is also compatible with the known solutions to the hierarchy problem, such as supersymmetry (SUSY) and warped ED. Finally, models based on a Froggatt-Nielsen $\mathrm{U}(1)_{FN}$ symmetry are flexible enough to allow for the implementation of several variants of anarchy, where neutrino $\mathrm{U}(1)_{FN}$ charges are not necessarily equal. Such variants proved successful in reproducing, at the level of order of magnitudes, all fermion masses and mixing angles [8].

The main drawback of anarchy and its variants is the difficulty to identify a quantitative test of the idea. Models based on a Froggatt-Nielsen $\mathrm{U}(1)_{FN}$ symmetry contain a large number of order-one independent parameters, thus preventing predictions beyond the order-of-magnitude accuracy. By accepting the principle of anarchy, the best we can do is to estimate probability distributions for the physical observables and evaluate the likelihood of our universe. We loose all the potentiality offered by the excellent experimental precision with which fermion masses and mixing angles are known today. Moreover, if new degrees of freedom carrying flavour charges are present at the TeV scale, as expected in the known solutions to the hierarchy problem, new sources of flavour change and/or CP violation appear and additional mechanisms should be invoked to avoid conflict with the present data.

3. Discrete Symmetries

An alternative description of the data is based on flavour symmetries. There are many types of flavour symmetries, global or local, continuous or discrete. There is no evidence for exact flavour symmetries and one of the most important aspects in model building is represented by the symmetry breaking. There is a large freedom related to the choice of the symmetry breaking sector and to the characteristic symmetry breaking scale. It is impossible to give here even a short account of all types of models. A special class of models is the one based on discrete flavour symmetries [9], adopted to reproduce some simple pattern U^0_{PMNS}, which provides a first approximation to the observed lepton mixing matrix U_{PMNS}. We have

$$U_{PMNS} = U^0_{PMNS} + O(u) \quad , \tag{2}$$

where $O(u)$ denotes a set of small corrections, proportional to some adimensional parameter u. Such an approach was well motivated before 2012. Several examples of leading order (LO) patterns U^0_{PMNS} have been suggested. A well-known example is the TB one, given in eq. (1) and described by a mixing matrix U_{TB} of the type

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0.82 & 0.58 & 0 \\ -0.41 & 0.58 & -0.71 \\ -0.41 & 0.58 & 0.71 \end{pmatrix} \quad . \tag{3}$$

This pattern can be easily derived from small discrete groups such as A_4 or S_4.

The general mechanism allowing to constrain U^0_{PMNS} by a discrete symmetry is very simple. The underlying model is assumed to be invariant under a discrete flavour symmetry G_f, broken down in such a way that neutrino and charged lepton sectors have different residual symmetries, at least in a LO approximation where small effects are neglected. The combination $m_l^\dagger m_l$ of the charged lepton mass matrix m_l is invariant under the subgroup G_e of G_f, while the neutrino mass matrix m_ν is invariant under the subgroup G_ν. It is interesting to note that, if neutrinos are of Majorana type, as assumed here, the most general group leaving m_ν invariant (and the individual masses m_i unconstrained) is $Z_2 \times Z_2$, a finite group. The subgroup G_e can be continuous, but G_e discrete remains the simplest option. We require a sufficiently large G_e to distinguish the three charged leptons. For instance we can choose $G_e = Z_n$ ($n \geq 3$) or $G_e = Z_2 \times Z_2$. Once G_e and G_ν have been chosen inside G_f, the embedding automatically fixes the relative alignment of $m_l^\dagger m_l$ and m_ν in flavour space. Lepton masses are unconstrained and U^0_{PMNS} is determined up to Majorana phases and up to permutations of rows and columns. This freedom apart, we can predict the three mixing angles θ^0_{ij} and the Dirac phase δ^0_{CP}. In most concrete models, where symmetry breaking is achieved via vacuum expectation values (VEVs) of a set of flavons, the LO results are modified by small corrections, as in eq. (2). In the specific case $U^0_{PMNS} = U_{TB}$ these corrections were expected to be very small, of the order of few percent, not to spoil the good agreement in the predicted value of the solar mixing angle. On this basis many models reproducing U_{TB} at the leading order predicted θ_{13} not larger than few degrees, later proved wrong by experiments. Discrete flavour symmetries can also be extended to quarks and even incorporated in GUTs, but in the existing constructions the symmetry has to be badly broken in the quark sector, leaving very few hints at the level of physical quantities.

Due to the sizable deviation of the latest data from the TB ansatz, many people have contemplated several modifications of the simplest models based on discrete symmetries. If we keep adopting $U^0_{PMNS} = U_{TB}$ as LO approximation, perhaps the most economic way to reproduce the actual value of θ_{13} is to introduce large correction terms, $O(u) \approx 0.2$. This is also viable in some scheme where U^0_{PMNS} differs substantially from U_{TB}, such as the so-called bimaximal (BM) mixing. Introducing large corrections has the disadvantage that beyond the LO the number of independent contributions is generally quite large. If their typical size is about 0.2, all mixing angles tend to be affected by generic corrections of this type and predictability is lost [10]. Moreover large correction terms are dangerous if new sources of flavour changing and/or CP violation are present at the TeV scale.

Another possibility is to look for alternative LO approximations where θ_{13} is closer to the measured value. Remarkably, several groups G_f giving rise to more realistic LO approximations have been found. Of particular interest are the groups leading to special form of trimaximal (TM) mixing:

$$U^0_{PMNS} = U_{TB}\; U_{13}(\alpha) \tag{4}$$

where $U_{13}(\alpha)$ describes a rotation in the 13 plane by an angle α. Early examples are the groups of the series $\Delta(6n^2)$ [11]. The angle α is fixed by n. For $n = 4(8)$ we have $\alpha = \pm 1/12(\pm 1/24)$ and $\sin^2\theta^0_{13} = 0.045(0.011)$. The Dirac phase is zero (modulo π).

A further possibility is to relax the symmetry requirements. It is worth mentioning that the smallest group reproducing TB mixing through the breaking down to $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$ is S_4. In the basis where charged leptons are diagonal, we can identify one of the two parities in G_ν with that generated by the so-called $\mu\tau$ exchange symmetry, directly responsible for the vanishing of θ_{13} and for θ_{23} being maximal. If the residual symmetry G_ν is reduced from $Z_2 \times Z_2$ down to Z_2 by eliminating the $\mu\tau$ exchange symmetry, the transformations belonging to G_e and G_ν only generate A_4, not the whole S_4. Assuming such a breaking pattern we find that the predicted mixing is again of TM type, as in eq. (4), but $U_{13}(\alpha)$ generalises to a unitary matrix, parametrized by a rotation angle α and a phase, both unconstrained [12]. We obtain a testable sum rule, which for small θ_{13} reads

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\theta_{13}\cos\delta_{CP} + O(\sin^2\theta_{13}) \quad . \tag{5}$$

Explicit models based on A_4 realizing such a breaking pattern were indeed proposed before the measurement of θ_{13} [13]. The possibility of reducing the residual symmetry G_ν to Z_2 can be systematically investigated [14]. As in the previous example, this framework only predicts two combinations of the three mixing angles.

4. Combining Discrete Symmetries and CP

A more recent possibility, which I would like to illustrate more in detail in this talk, is to combine discrete and CP symmetries and explore the symmetry breaking patterns such a combination can give rise to. This idea is not new and relies on many existing examples and suggestions. A well-known example is that of the so-called $\mu\tau$ reflection symmetry [15, 16] (not to be confused with the $\mu\tau$ exchange symmetry), which exchanges a muon (tau) neutrino with a tau (muon) antineutrino in the charged lepton mass basis. If such a symmetry is imposed, the atmospheric mixing angle is predicted to be maximal, while θ_{13} is in general non-vanishing for a maximal Dirac phase δ. Models combining S_4 and CP can be found in the recent literature [17,18]. Other approaches dealing with discrete symmetries and CP are illustrated in [19–21].

In ref. [22] we have proposed a general formalism which combines CP with a discrete flavour symmetry. Consider a flavour symmetry group G_f and a set of fields ϕ transforming in some representation ρ of G_f:

$$\phi'(x) = \rho(g)\;\phi(x) \qquad\qquad g \in G_f \quad , \tag{6}$$

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where $\rho(g)$ is a unitary matrix. We can define a CP transformation on ϕ as follows [23]

$$\phi'(x) = X\ \phi^*(x_{CP}) \qquad\qquad x_{CP} \equiv (x^0, -\vec{x}) \quad , \tag{7}$$

where X is a matrix in flavour space. If the fields ϕ are fermions, the well-known action of CP on spinor indices is understood. The choice of X is not arbitrary, but has to fulfill certain consistency conditions. First, we require the matrix X to be unitary and symmetric,

$$XX^\dagger = XX^* = \mathbb{1}\ . \tag{8}$$

In the presence of a flavour symmetry, requiring X symmetric is not the most general option, but if we do so then $CP^2 = 1$ automatically holds. A second condition arises by considering the action of CP combined with some transformation of G_f [22, 24]. For any element g of the group G_f, an element g' belonging to G_f should exist such that

$$\left(X^{-1}\rho(g)\ X\right)^* = \rho(g') \quad . \tag{9}$$

Notice that in general g and g' are distinct. As has been shown in [22], the mathematical structure of the group G_{CP} comprising G_f and CP is of the form $G_{CP} = G_f \rtimes H_{CP}$ with H_{CP} being the parity group generated by CP.

In our proposal we consider a theory invariant under G_{CP} with the three generations of lepton doublets l in some representation ρ of G_f

$$l'(x) = \rho(g)\ l(x) \quad . \tag{10}$$

Under CP we have

$$l'(x) = X\ l^*(x_{CP}) \quad , \tag{11}$$

where X satisfies both eq. (8) and eq. (9). We assume that in some limit of the theory G_{CP} is broken to the subgroups G_e and G_ν in the charged lepton and neutrino sectors, respectively. G_e is a subgroup of G_f generated by a set of elements Q_i, while $G_\nu = Z_2 \times CP$ is the direct product of a parity contained in G_f, generated by the element Z, and CP, generated by X. Notice that while Q_i, Z and X are defined in the representation ρ, for simplicity we make no distinction between (Q_i, Z, X) and the corresponding abstract elements of G_{CP} they represent. The subgroup G_ν involves the direct product between Z_2 and CP, and thus the transformations described by Z and X should commute. This gives rise to the condition

$$\left(X^{-1}Z\ X\right)^* = Z \quad , \tag{12}$$

a special version of eq. (9) with $g = g'$. Given a discrete group G_f and a parity subgroup generated by Z, eqs. (8), (9) and (12) can be read as a set of constraints on X, i.e. on the possible CP definitions we can adopt to realize the desired symmetry breaking pattern. The residual symmetries G_e and G_ν imply the following conditions on $m_l^\dagger m_l$ and m_ν:

$$Q_i^\dagger (m_l^\dagger m_l) Q_i = (m_l^\dagger m_l) \quad , \qquad Z^T m_\nu Z = m_\nu \quad , \qquad X m_\nu X = m_\nu^* \quad . \tag{13}$$

From these conditions we can derive the mixing matrix U^0_{PMNS}, and we find that in general it can be parametrized by one real parameter θ, ranging from 0 to π:

$$U^0_{PMNS} = U^0_{PMNS}(Q_i, Z, X, \theta) \qquad\qquad 0 \le \theta \le \pi \quad . \tag{14}$$

Mixing angles and phases, both Dirac and Majorana, are then predicted as a function of θ, modulo the ambiguity related to the freedom of permuting rows and columns and to the intrinsic parity of neutrinos. The formalism is completely invariant under any change of basis in field space. The physical results only depend on G_{CP} and the residual symmetries specified by (Q_i, Z, X).

5. The case $G_f = S_4$

To exemplify our results, in ref. [22] we have perform an exhaustive analysis of the case $G_f = S_4$. The group S_4 can be defined in terms of three generators S, T and U [25] which fulfill the following relations

$$\begin{aligned} &S^2 = E\ ,\ \ T^3 = E\ ,\ \ U^2 = E\ , \\ &(ST)^3 = E\ ,\ \ (SU)^2 = E\ ,\ \ (TU)^2 = E\ ,\ \ (STU)^4 = E \end{aligned} \tag{15}$$

with E being the neutral element of S_4. The generators S and T alone give rise to the group A_4. The group S_4 has five irreducible representations: $\mathbf{1}$, $\mathbf{1'}$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3'}$. We assign the three generations of left-handed leptons to the faithful representation $\mathbf{3'}$ (equivalent results are obtained by choosing the representation $\mathbf{3}$) and for this representation we adopt a basis where the elements S, T and U are represented by the real matrices [25]

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ ,\quad T = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ -1 & \sqrt{2} & -1 \end{pmatrix}\ ,\quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ . \tag{16}$$

We have considered all possible choices of G_e and G_ν matching our symmetry breaking pattern. We have found that all the independent physical results are exhausted by considering the representative cases listed in table 2.

G_e	Q_i
Z_3	T
Z_4	STU
$Z_2 \times Z_2$	(TST^2S, UT^2)

G_ν	Z	X
$Z_2 \times CP$	S	$X_i\ (i = 1, ..., 6)$
$Z_2 \times CP$	SU	$X_i\ (i = 1, ..., 4)$
$Z_2 \times CP$	U	$X_i\ (i = 1, ..., 4)$

Table 2. Representative choice of generators Q_i for the subgroup G_e and (Z, X) for the subgroup G_ν. For any Z chosen in S_4 only a finite number of CP transformations X_i satisfying eqs. (8,9,12) are allowed. Their expressions in the representation $3'$ are given in eq. (17).

For any Z chosen in S_4 only a finite number of CP transformations X_i satisfying eqs. (8,9,12) are allowed. For Z chosen as in table 1, the admissible CP transformations X_i $(i = 1, ..., 6)$ are given by:

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\ ,\quad X_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ ,\quad X_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ ,$$

$$X_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\ ,\quad X_5 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\ ,\quad X_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\ . \tag{17}$$

These transformations are defined up to an irrelevant overall phase. X_1 is the canonical CP transformation.

Several interesting cases arises when $G_e = Z_3$. Two of them are given in table 3, in terms of the generators (Q_i, Z, X).

case	Q_i	Z	X
I	T	S	X_1
IV	T	SU	X_1

Table 3. Choice of (Q_i, Z, X) defining cases I and IV. The subgroup G_e generated by T is Z_3.

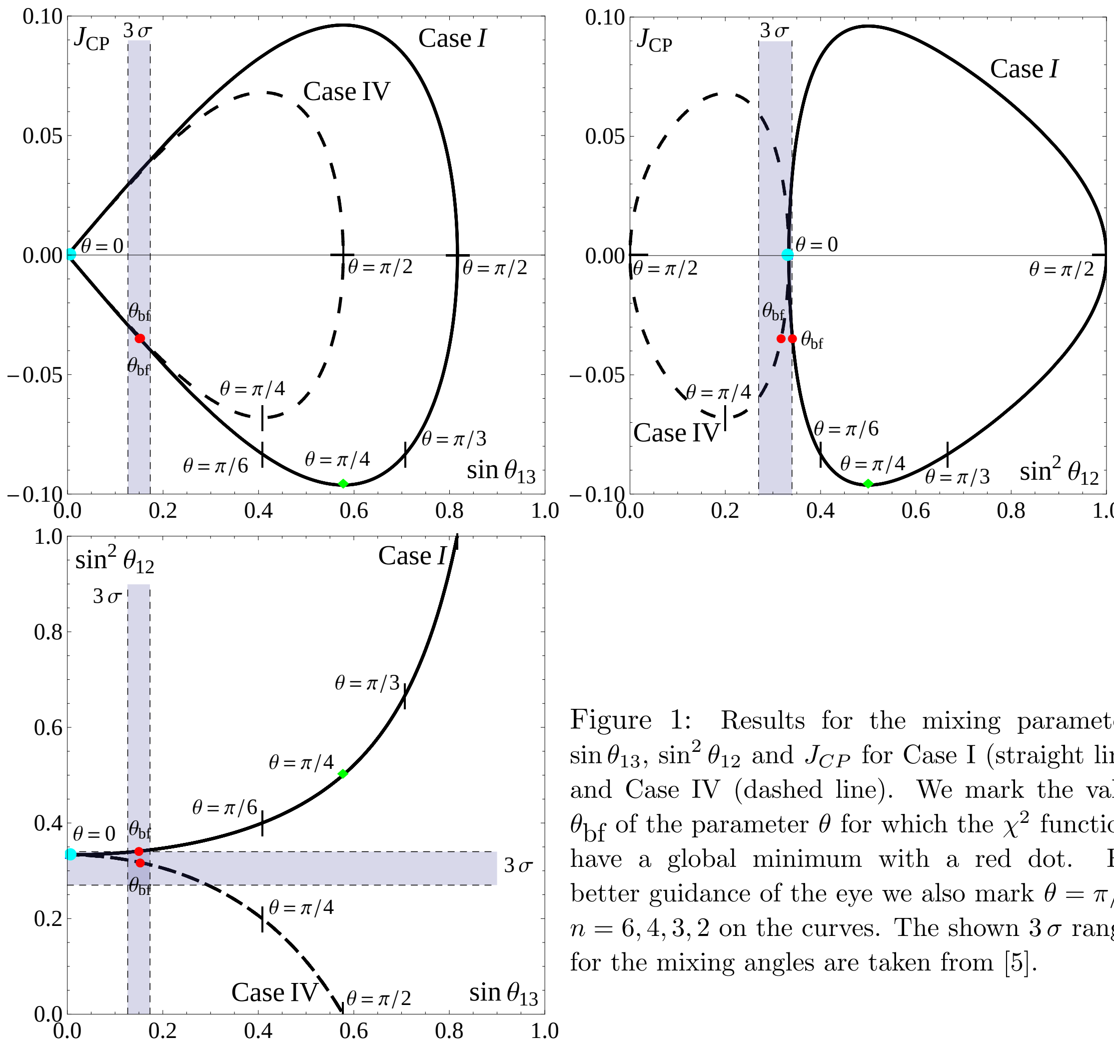


Figure 1: Results for the mixing parameters $\sin\theta_{13}$, $\sin^2\theta_{12}$ and J_{CP} for Case I (straight line) and Case IV (dashed line). We mark the value $\theta_{\rm bf}$ of the parameter θ for which the χ^2 functions have a global minimum with a red dot. For better guidance of the eye we also mark $\theta = \pi/n$, $n = 6, 4, 3, 2$ on the curves. The shown $3\,\sigma$ ranges for the mixing angles are taken from [5].

These two cases correspond to two particular realizations of the $\mu\tau$ reflection symmetry. They predict a maximal atmospheric mixing angle, a maximal Dirac phase and vanishing Majorana phases. The solar and reactor angles are given by

$$\sin^2\theta_{12} = \begin{cases} \dfrac{1}{2+\cos 2\theta} & \text{(case I)} \\ \dfrac{\cos^2\theta}{2+\cos^2\theta} & \text{(case IV)} \end{cases} \qquad \sin^2\theta_{13} = \begin{cases} \dfrac{2}{3}\sin^2\theta & \text{(case I)} \\ \dfrac{1}{3}\sin^2\theta & \text{(case IV)} \end{cases} \quad . \tag{18}$$

The correlations between $\sin^2\theta_{12}$, $\sin\theta_{13}$ and the CP invariant J_{CP} are displayed in fig. 1. By optimizing the choice of θ through the minimization of the χ^2 function, we find a reasonable

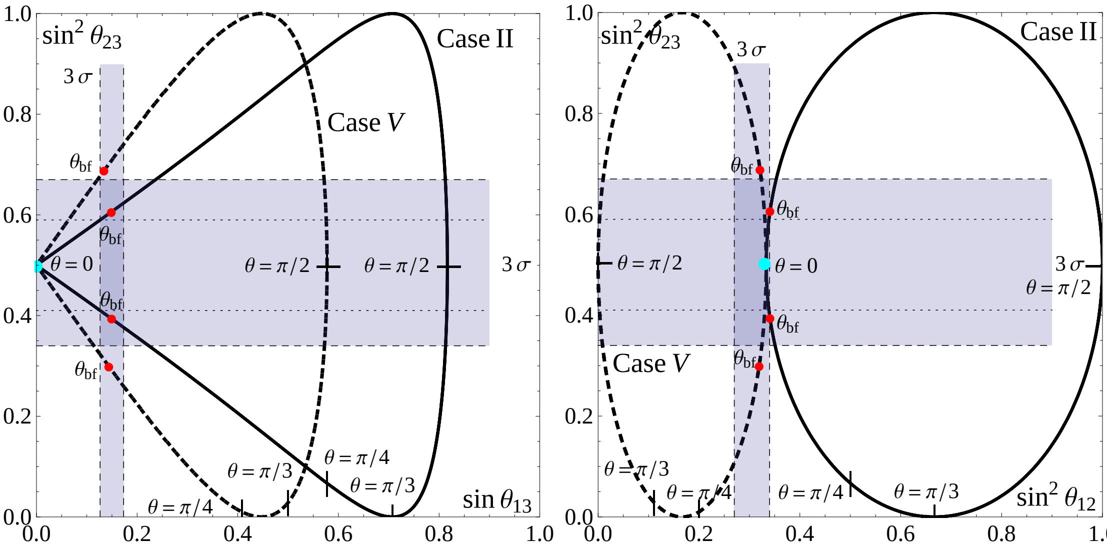


Figure 2: Results for the atmospheric, solar and reactor mixing angles for Case II (straight line) and Case V (dashed line). We mark the value $\theta_{\rm bf}$ of the parameter θ for which the χ^2 functions have a global minimum with a red dot. For better guidance of the eye we also mark $\theta = \pi/n$, $n = 4, 3, 2$ on the curves. The shown $3\,\sigma$ ranges for the mixing angles and the best fit values of the atmospheric mixing angle are taken from [5]. The plot for $\sin^2\theta_{12}$ and $\sin\theta_{13}$ is the same as for Case I and Case IV and can be found in figure 1.

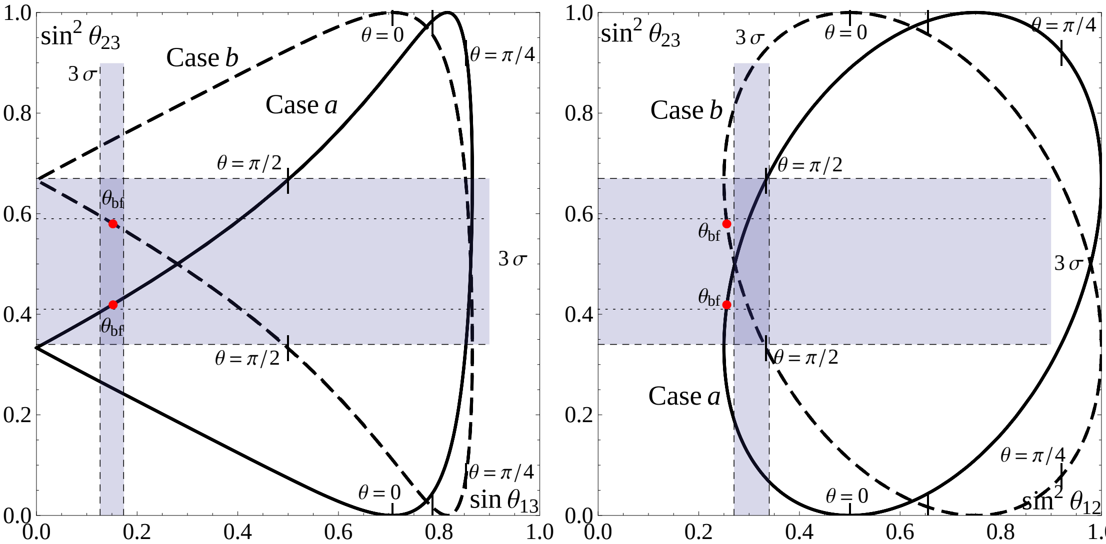
agreement with the data. Notice that the $\mu\tau$ reflection symmetry is not explicit in the chosen basis, where the CP transformation X_1 in the $3'$ representation is canonical. To make a direct contact with the $\mu\tau$ reflection symmetry we have to move to the basis where the T generator and the combination $m_l^\dagger m_l$ are diagonal. In a concrete model realizing either case I or case IV the predicted value of the Dirac phase would only depend on the symmetry breaking pattern, and not on the specific values of Lagrangian parameters realizing it. In this sense cases I and IV could provide examples of "geometrical" CP violation.

Not all the cases considered in our analysis necessarily lead to non-trivial CP phases. For instance in cases II and V, defined in table 4, we find that all phases are trivial.

case	Q_i	Z	X
II	T	S	X_3
V	T	SU	X_2

Table 4. Choice of (Q_i, Z, X) defining cases II and V. The subgroup G_e generated by T is Z_3.

Such a result is counterintuitive, since CP is not assumed to be part of the residual symmetry of the charged lepton sector and thus we might expect that CP is always broken in our construction. Trivial CP phases can however arise from a CP symmetry of accidental type. In cases II and IV it happens that the charged lepton sector, invariant under G_e, is also accidentally invariant under the CP transformation associated to X_3. In case II this suffices to conclude that CP is conserved, at least at the level of the lepton masses and mixing angles. In case IV we should further notice that X_3 is also accidentally a symmetry of the neutrino sector, even though we originally asked invariance under the CP transformations generated by X_2. Cases II and V make the same prediction of the solar and reactor angles as cases I and IV, respectively. They are given



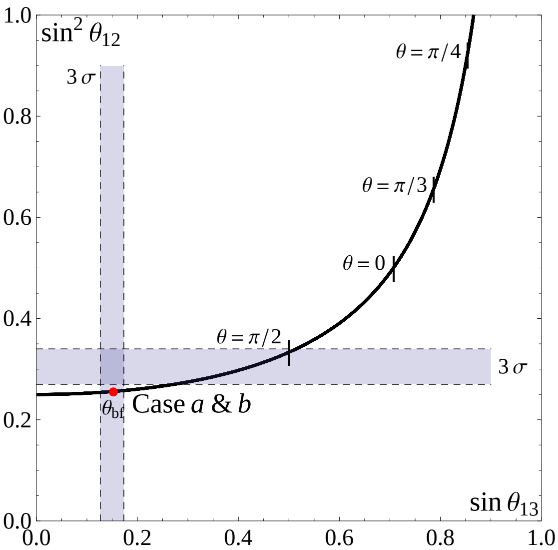


Figure 3: Results for the mixing angles $\sin\theta_{13}$, $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ for Case a (straight line) and Case b (dashed line) for $G_e = Z_4$ or $G_e = Z_2 \times Z_2$. We mark the value $\theta_{\rm bf}$ of the parameter θ for which the χ^2 functions have a global minimum with a red dot. For better guidance of the eye we also mark $\theta = 0, \pi/4, \pi/3, \pi/2$ on the curves. The shown 3σ ranges for the mixing angles and the best fit values of the atmospheric mixing angle are taken from [5].

in eq. (18). The atmospheric angle is related to the other angles by

$$\sin^2\theta_{23} = \begin{cases} \dfrac{1}{2}\left(1 - \dfrac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right) & \text{(case II)} \\ \dfrac{1}{2}\left(1 - \dfrac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right) & \text{(case V)} \end{cases} . \tag{19}$$

Despite the absence of CP violating effects form non-trivial phases, cases II and V provide a nice fit to the mixing angles, once the free parameter θ is optimized by minimizing the χ^2 function. Such a fit is illustrated in fig. 2.

Other two cases very similar to cases II and V are those called VI and VII and defined in table 5, through the choice of (Q_i, Z, X). At variance with the previous cases, the residual symmetry in the charged lepton sector is $Z_4(Z_2 \times Z_2)$ for case VI(VII). Also in these cases trivial CP phases are predicted as a consequence of accidental CP symmetries. Case VI and case VII predict the same mixing angles:

$$\sin^2\theta_{13} = \frac{1}{4}\left(\sqrt{2}\cos\theta + \sin\theta\right)^2 \tag{20}$$

$$\sin^2\theta_{12} = \frac{2}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta} \tag{21}$$

$$\sin^2\theta_{23} = \begin{cases} \dfrac{4\sin^2\theta}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta} & \text{(case a)} \\ 1-\dfrac{4\sin^2\theta}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta} & \text{(case b)} \end{cases} . \tag{22}$$

The two possibilities a and b arise from exchanging the second and third rows of U^0_{PMNS}. The correlations among the three mixing angles are shown in fig. 3. We see that also in these cases the experimental mixing angles can be well approximated by a suitable choice of the parameter θ.

case	Q_i	Z	X
VI	STU	U	X_2
VII	(TST^2S, UT^2)	U	X_1

Table 5. Choice of (Q_i, Z, X) defining cases VI and VII. The subgroup G_e generated by STU is Z_4, while that generated by (TST^2S, UT^2) is $Z_2 \times Z_2$.

We have not mentioned other cases leading to unrealistic lepton mixing angles. Among them we have also found possibilities where both the angles and the CP phases carry a non-trivial dependence from the parameter θ.

6. Conclusion

In neutrino physics we have recently witnessed a decisive progress on the experimental side. The reactor angle is now precisely measured and it is away from zero by many standard deviations. We also have a first indication favoring a non-maximal atmospheric mixing angle. While these steps have been effective in ruling out many models of fermion masses and mixing angles, it is fair to say that no compelling and unique theoretical picture has emerged so far. Perhaps the most disturbing aspect is that present data can still be described within widely different frameworks. The new data have strengthened the case of anarchy. We cannot exclude that neutrino mass ratios and mixing angles are just random $O(1)$ quantities, reflecting no special pattern. Flavour symmetries remain important tools to address the flavour problem, especially when quarks and leptons are analyzed in a common framework. Models based on discrete symmetries, tailored to reproduce the features of the early data from neutrino oscillations, are less supported by data now. Modifications of the simplest realizations are required. A lot of theoretical effort has been made in this direction. Several possibilities to accommodate a large θ_{13} have been devised. The simplest one consists in adding large corrections to the schemes predicting a vanishing θ_{13} at the LO. In alternative, several discrete groups leading to realistic LO mixing matrices have been identified. It is also possible to relax symmetry requirements to avoid $\theta_{13} = 0$ as LO prediction. In this talk we have reviewed a promising approach where CP is included in the symmetry breaking pattern. From a theory viewpoint, combining CP and a discrete symmetry G_f leads to non-trivial consistency conditions that the admissible definitions of CP should satisfy. In our construction we have assumed residual symmetries G_e and $G_\nu = Z_2 \times CP$, G_e and Z_2 being subgroups of a discrete group G_f, and shown that such a requirement determines all mixing angles and phases in terms of a single real parameter. By making a comprehensive analysis of the case $G_f = S_4$ we have identified several new realistic mixing patterns, thus proving the viability of our approach.

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