Fluctuating Hydrodynamics Confronts the Rapidity Dependence of Transverse Momentum Fluctuations

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Abstract.

Interest in the development of the theory of fluctuating hydrodynamics is growing. Early efforts suggested that viscous diffusion broadens the rapidity dependence of transverse momentum correlations. That work stimulated an experimental analysis by STAR. We study the hydrodynamic evolution using second order causal viscous hydrodynamics including Langevin noise. We obtain a deterministic evolution equation for the transverse momentum density correlation function. We use the latest theoretical equations of state and transport coefficients to compute the STAR observables. The results are in excellent accord with the measured broadening. In addition, we predict features of the distribution that can distinguish 2nd and 1st order diffusion.

1. Introduction

Relativistic Heavy Ion Collider (RHIC) experiments have provided evidence of formation of deconfined state of quark and gluons, or quark-gluon plasma (QGP). It was shown that QGP behaves like a most perfect liquid known. The main reason behind this conclusion was the successful use of relativistic ideal hydrodynamic model to describe the flow data. The quantity used to indicate the perfectness of QGP is the ratio of viscosity to entropy density η/s. Of course, for perfect fluid it is zero. Current consensus is that for QGP it is in the range between the conjectured lower bound (the "KSS" bound) of $1/4\pi$ and ~ 0.3 (in units where $\hbar = k_B = c = 1$). This range is much smaller than the value ($= 0.7$) of superfluid liquid helium. In Ref.[1] Gavin and Abdel-Aziz propose an independent method for estimating η/s using rapidity distribution of transverse momentum correlations. They calculate the difference in the widths of these distributions between peripheral and central collisions, using first order diffusion of p_t fluctuations. They estimate η/s in the range 0.08 - 0.3, which is the same as that obtained from the flow data. Recently STAR has measured [2] the widths of correlations and p_t covariance profile in rapidity. Here we show that causally constrained diffusion equation with temperature dependent η/s is required to explain the data. We start with a brief discussion of diffusion of transverse momentum fluctuation in Section 2, before applying the concepts to two-particle p_t correlations in Section 3. We then present the results and discuss them in Section4.

2. Diffusion of Transverse Flow Fluctuations

Equations of dissipative relativistic hydrodynamics are derived from the basic principles of conservations of charges and of energy-momentum. Depending on the order of gradients included in the dissipative part of the energy momentum tensor, relativistic hydrodynamics is said to be of first or second order. The first order is the relativistic version of the famous Navier-Stokes theory and the second order is the well known Israel-Stewart theory. If we take small fluctuations on quantities (like the flow u^μ) over equilibrium values, we can see that transverse modes decouple from the longitudinal modes. The longitudinal modes are propagating sound modes while the transverse modes are solely the diffusing shear modes. As we are more interested in shear viscosity we focus on the transverse modes. Note that shear viscosity is the most important mode of dissipation in QGP. Starting with relativistic hydrodynamic equations and constitutive relation for shear tensor (see [3], for example, for such equations and relations) and then linearizing, one arrives at [4] diffusion equations for a transverse momentum current:

$$\frac{\partial \delta T_{0y}}{\partial t} = \nu \nabla_z^2 \delta T_{0y}, \qquad \tau_\pi \frac{\partial^2 \delta T_{0y}}{\partial t^2} + \frac{\partial \delta T_{0y}}{\partial t} = \nu \nabla_z^2 \delta T_{0y} \tag{1}$$

These two equations follow from the first order and second order theories, respectively. Here $T_{0y} \approx (\epsilon_0 + p_0)\delta u^y$. δu^y is a transverse fluctuation in flow, ϵ_0 and p_0 are energy density and pressure at equilibrium. Note that the diffusion coefficient $\nu = \eta/(\epsilon_0 + p_0) = \eta/Ts$ contains the ratio η/s and thus encapsulates the information on viscosity. In [5] Hirano and Gyulassy have put together the latest information on η: a combination of results from perturbative QCD, kinetic theory and $\mathcal{N} = 4$ Super Yang-Mills (SYM) at infinite coupling strengths, in the respectively applicable temperature ranges. This information has been used here for shear viscosity. For entropy density we have looked at values from lattice QCD calculations, specifically s95p-v1 from [6, 7].

We make use of Bjorken boost invariant coordinates (τ, x, y, η), where τ and η are proper time and space-time rapidity: $\tau = \sqrt{t^2 - z^2}$ and $\eta = (1/2)\ \ln((t+z)/(t-z))$. [1] The entropy production equation with boost invariance can be written as [3] $\frac{ds}{d\tau} + \frac{s}{\tau} = \Phi/\tau$, where $\Phi = 4\eta/3\tau$ for the first order case, and for second order case $\tau_\pi \frac{d\Phi}{d\tau} + \left(1 + \frac{\tau_\pi}{2\tau} + \frac{1}{2}\eta T \frac{d}{d\tau}(\frac{\tau_\pi}{\eta T})\right)\Phi = \frac{4\eta}{3\tau}$, where $\Phi = \pi_0^0 - \pi_z^z$.
The first equation of (1) is a regular diffusion equation. There is a well-known problem with the first order theory: this diffusion equation violates causality and is not suitable for relativistic particles or fluid cells. It has been shown that second order theory gives relativistically consistent diffusion equation. Equation (1), which follows from second order theory, does not violate causality. The relaxation time τ_π in the second order diffusion equation restores causality. For details on causality and diffusion see [3], [8] and the references therein.

3. Two-Particle Transverse Momentum Correlations

Correlation function of two-particle transverse momentum current is $r = \langle T_1^{0y} T_2^{0y}\rangle - \langle T_1^{0y}\rangle\langle T_2^{0y}\rangle$, where $T_1^{0y} \equiv T^{0y}(\mathbf{x}_1)$ and $T_2^{0y} \equiv T^{0y}(\mathbf{x}_2)$ are momentum currents of particle pairs at points $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Considering small perturbations about the average, $T^{0y} = \langle T^{0y}\rangle + \delta T^{0y}$ and treating them as stochastic quantities, we can write $r = \langle \delta T_1^{0y} \bar{T}_2^{0y}\rangle + \langle \bar{T}_1^{0y} \delta T_2^{0y}\rangle + \langle \delta T_1^{0y} \delta T_2^{0y}\rangle$. The bars over the letters represents the same average - the average over the ensembles of events. The last term becomes important in stochastic methods. This term vanishes in the absence of noise. Using the second of (1) and the expression for r and manipulating the terms we can [4] subtract away the noise term to obtain

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial t^2} + \frac{\partial \Delta r}{\partial t} = \nu(\nabla_{z1}^2 + \nabla_{z2}^2)\Delta r. \tag{2}$$

[1] Note that the same letter η is used for shear viscosity and for the longitudinal space-time rapidity here. The meaning should be clear from the context of its use.

We can get similar equation for the first order case [1]. Here $\Delta r = r - r_{eql}$.
If we use Bjorken boost invariance, (2) we can obtain this equation in terms of τ and η:

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial \tau^2} + \frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2}(\nabla^2_{\eta 1} + \nabla^2_{\eta 2})\Delta r \tag{3}$$

Clearly, η_1 and η_2 are space-time rapidities for particle 1 and particle 2. This equation and its first order version [1] (i.e., (1) without the first term) are the equations we numerically solve. The diffusion coefficient and the relaxation time are both temperature (and time) dependent.

4. Results and Discussion

The main observables we compute from the numerical solutions (Δr) are the width σ of the correlation function and an important observable $\mathcal{C}$ given by [1] $\mathcal{C} = \langle N\rangle^{-2}\int \Delta r_g(\mathbf{x_1}, \mathbf{x_2})dx_1 dx_2 = \langle N\rangle^{-2}\langle\sum_{i\neq j} p_{ti}p_{tj}\rangle - \langle p_t\rangle^2$, where i labels particles from each event. It can be seen that $\mathcal{C}$ connects Δr to experimentally measurable p_t-covariance. The relaxation time is $\tau_\pi = \beta\frac{\eta}{Ts} = \beta\nu$. Suggested value of β are 5-6 [9] and 6.32 [10]. Here we have used $\beta = 6$. Initial (thermalization) time is taken to be $\tau_0 = 1$ fm/c. Initial Δr is taken to be a gaussian in relative rapidity $\Delta\eta$, with initial width $\sigma_0 = 0.54$, making it consistent with experimental data for the most peripheral collisions. We have assumed a fixed temperature freezout, at 150 MeV. Glauber model has been used to connect impact parameters and multiplicity of the collisions. The freezeout proper time is assumed as $\tau_F - \tau_0 \propto (R - R_0)^2$, with R the rms participant radius. Freezeout time for most central collisions is taken to be 9 fm/c. The computed σ and $\mathcal{C}$ are compared with the values measured recently by STAR [2].

Figure 1 shows widths versus centralities computed using both first and second order theory with two different equations of state - EOS I and EOS II. EOS I is the one which contains the entropy density calculated from lattice QCD. EOS II uses the standard entropy density, summarized in [5]. We see that second order calculations agree very well with the experimental data. We also note that the choice of EOS makes a very small difference. However, there is significant difference between the first and second order diffusion, except for a few most central cases. The deviation of the first order results from data relates to how the relaxation time compares with the evolution time [4]. This clearly shows that one needs causally constrained hydrodynamic evolutions to better explain the experimental data. Figure.1 also compares results with NeXSPHeRIO calculations [11]. NeXSPHeRIO uses non-viscous (ideal) hydrodynamics in evolution of initial correlations. It reproduces most qualitative features of the two-particle correlations. It however does not reproduce the broadening of the rapidity width with increasing centrality, as one cane see in Fig. 1. We attribute that to the absence of viscosity in its hydrodynamic. Other explanations involving collective initial state behavior are also possible.

Figure 2 shows observable $\mathcal{C}$ vs $\Delta\eta$ for various centralities from our numerical calculations and the values measured by STAR [2]. The STAR measured values have error bars in the offsets (ridge), which we have not shown here. The plots show good agreement of the numerical results with the experiments. The single point at $\Delta\eta = 0$ in the most central case here has been attributed to the track merging and is just a detector artifact. One important observation we make here is the flattening of the peak (and to some extent double humps) in $\mathcal{C}$. The reason for the double hump (or rather the flattened peak) is a second order diffusion effect. We note that the second order diffusion equation has propagating wave part as well. Thus there is competing wave and diffusion effects, depending on the size of τ_π and ν. Wavefronts propagate in opposite directions and diffusion fills in the space in between. This can be seen more effectively in coordinate space (not shown here). In rapidity space, width saturates since the effective diffusion coefficient ν/τ^2 in (3) decreases rapidly with time. The first order diffusion obviously does not show such humps and flattening - there are no propagating waves. STAR data for other centralities [12] show the humps exactly at the same centralities. Shapes show fair agreement with the calculations. Here we have restricted ourselves with the published data only for the three centralities shown in Fig2. Finally, we just want to mention that the order of entropy production equations does not make practical difference in our results. Note that we have relatively large β in $\tau_\pi = \beta\eta/Ts$.

It would be interesting to measure the observable $\mathcal{C}$ from the p-A [13] and Pb-Pb [14] collisions data from LHC. In the pA case, it would be interesting to see whether there is a broadening with respect to pp. Experiments have identified a ridge in pA [13]. If hydrodynamics is applicable in the pA system, then viscous diffusion would broaden the rapidity width of $\mathcal{C}$.

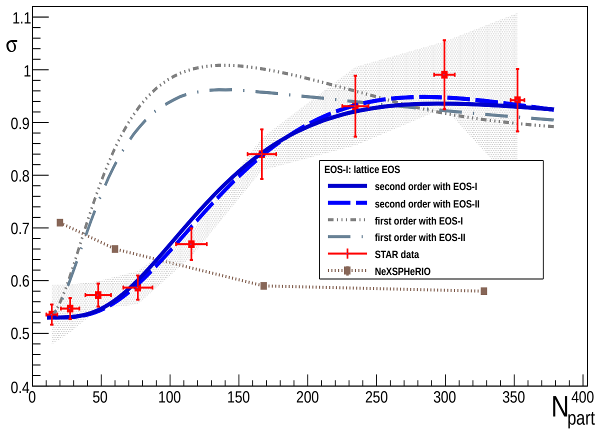

Figure 1. Second order, first order, with both EOS [4] and NeXSPHeRIO [11] calculations of width vs STAR [2] measurements.

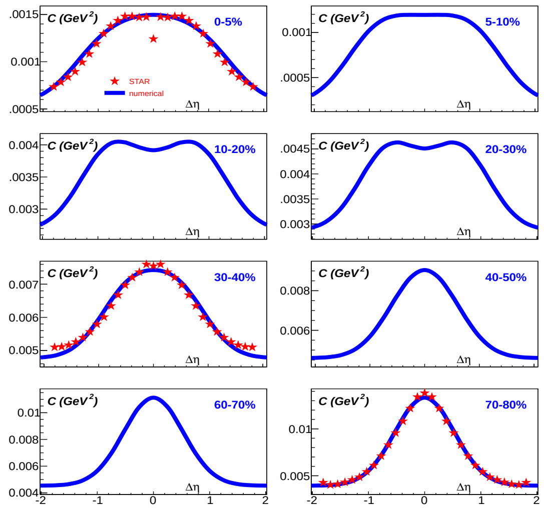

Figure 2. $\mathcal{C}$ vs $\Delta\eta$. Comparison with STAR data at different centralities. [2].

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