

Initial Conditions with Flow from a McLerran Venugopalan model with Transverse Dynamics

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Abstract. Using a recursive solution of the Yang-Mills equation, we calculate analytic expressions for the gluon fields created in ultra-relativistic heavy ion collisions at small times τ . We have worked out explicit solutions for the fields and the energy momentum tensor up to 4th order in an expansion in τ . We generalize the McLerran-Venugopalan model to allow for a systematic treatment of averaged charge densities μ^2 that vary as a function of transverse coordinates. This allows us to calculate radial, elliptic and directed flow of gluon fields. Our results can serve as initial conditions for hydrodynamic simulations of nuclear collisions that include initial flow.

1. Introduction

Nuclear collisions at high energies at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) can create and probe novel forms of QCD matter. One example is a deconfined phase of QCD called Quark Gluon Plasma (QGP). A thermalized QGP is created in nuclear collisions at RHIC and LHC after a time of about 1 fm/c. Relativistic hydrodynamics is the tool of choice to describe the evolution of QGP in this stage [1, 2]. Large uncertainties in the hydrodynamic evolution come from the incomplete understanding of the initial conditions, i.e. the evolution of the nuclear collision prior to 1 fm/c. This is where another new phase of QCD, Color Glass Condensates (CGC) plays an important part. The natural limit of a nucleus at very high energy is characterized by a saturated gluon density in which the gluon field can be described by a quasi-classical field [3, 4]. The earliest phase of the collision of two nuclei can thus be described as the collision of two sheets of color glass [5]. The classical field resulting from the collision then decays and equilibrates to a plasma of quarks and gluons.

To be more precise, in the color glass formalism partons are distinguished by momentum fraction. Sources have large Bjorken- x while the small- x gluons are treated through the classical field $F^{\mu\nu}$. They are related by the Yang-Mills equation $[D_\mu, F^{\mu\nu}] = J^\nu$. Before the collision, the two incoming nuclei on the positive and negative light cone are dominated by transverse chromo-electric and chromo-magnetic fields which are perpendicular to each other [3, 4]. After the collision, the fields in the forward light cone can be parameterized as $A^\pm = \pm x^\pm A$ and a transverse field A^i , $i = 1, 2$. A and A^i depend on the proper time $\tau = \sqrt{t^2 - z^2}$ and transverse coordinates but not on the space-time rapidity η . For small τ , a series expansion $A^\mu = \sum_n \tau^n A_{(n)}^\mu$ in powers of τ can be employed. A few lowest orders in time should give an adequate and analytic description of the gluon fields immediately after the collision [6].



The collision initially ($\tau \rightarrow 0$) creates longitudinal chromo-electric and magnetic fields

$$E_0 = ig[A_1^i, A_2^i], \quad B_0 = -ig\epsilon^{ij}[A_1^i, A_2^j]$$

between the nuclei, where A_1^i and A_2^i are the fields of nucleus 1 or 2 before the collision in light cone gauge. The next order in τ gives us linearly growing transverse electric and magnetic fields

$$E_{(1)}^i = -\frac{1}{2}(\sinh \eta[D^i, E_0] + \cosh \eta \epsilon^{ij}[D^j, B_0]), \quad B_{(1)}^i = \frac{1}{2}(\cosh \eta \epsilon^{ij}[D^j, E_0] - \sinh \eta[D^i, B_0]).$$

We have calculated analytic expressions for the fields up to fourth order in τ explicitly and will discuss those results elsewhere [7]. The behavior we find is qualitatively consistent with numerical studies of classical field QCD [8].

The energy momentum tensor of the gluon field after the collision can be easily calculated. Up to order $\mathcal{O}(\tau^3)$ — we skip higher orders here for brevity — it can be written in the form

$$T_f^{mn} = \begin{pmatrix} A + C - D & B_1 & B_2 & E/\tau \\ B_1 & A + C & F & B'_1/\tau \\ B_2 & F & A + C & B'_2/\tau \\ E/\tau & B'_1/\tau & B'_2/\tau & (-A - C + D)/\tau^2 \end{pmatrix} \quad (1)$$

in the τ, x, y, η coordinate system. Here $A = (E_0^2 + B_0^2)/2$ is the initial energy density at $\tau = 0$, and B_i and B'_i are the coefficients of the Poynting vector $T^{0i} = B_i \cosh \eta + B'_i \sinh \eta$ and linear in τ ($i = 1, 2$ are transverse indices). $C, D, E, F \propto \tau^2$ are coefficients quadratic in τ . All these coefficients have been calculated analytically [7]. One can check that this energy momentum tensor is boost invariant owing to the original boost-invariant setup of the colliding nuclei.

2. Flow in the McLerran Venugopalan Model

In the McLerran-Venugopalan (MV) model [3, 4], the simplest implementation of color glass, an observable O calculated from sources in the two nuclei has to be averaged over all possible configurations of color source distributions ρ . Each collision samples arbitrary ρ due to the short time scale of the collision compared to internal time scales of the nuclear wave function. The distribution of charge is usually assumed to be Gaussian around zero with an average value μ^2 of the variation. We use the definition

$$\langle \rho_a(\vec{x}_\perp) \rho_b(\vec{y}_\perp) \rangle = \frac{g^2 \delta_{ab}}{N_c^2 - 1} \mu^2(\vec{x}_\perp) \delta^2(\vec{x}_\perp - \vec{y}_\perp). \quad (2)$$

We have suppressed the dependence on the longitudinal coordinate for brevity. Note that we have generalized the original expression from the MV model such that μ^2 can be a function of transverse coordinate \vec{x}_\perp , instead of being an average charge density that is assumed to be homogeneous. Of course the latter is unrealistic and the generalization needs to be made if long-range transverse dynamics is to be described. The charge density μ^2 of a nucleus is related to the large- x parton distribution in the transverse plane. It can be simulated by sampling the nucleons inside the nucleus, for example using a Woods-Saxon distribution. We have shown explicitly that if μ^2 is varying slowly such that $|\mu^2(\vec{x}_\perp)| \gg m^{-1} |\nabla^i \mu^2(\vec{x}_\perp)| \gg m^{-2} |\nabla^i \nabla^j \mu^2(\vec{x}_\perp)|$, color glass dynamics and long-distance dynamics which are not described by CGC can be safely separated. Here m is an infrared regulator. We find that the generalized two gluon correlation function is [7]

$$\langle A_a^i(\vec{x}_\perp) A_b^j(\vec{x}_\perp) \rangle = \delta_{ab} \frac{g^2 \mu^2(\vec{x}_\perp)}{8\pi(N_c^2 - 1)} \left[\delta^{ij} \ln \frac{Q^2}{(1.47m)^2} + \frac{\nabla^k \nabla^l \mu^2(\vec{x}_\perp)}{m^2 \mu^2(\vec{x}_\perp)} \left(\frac{1}{6} \delta^{kl} \delta^{ij} - \frac{7}{12} \delta^{ik} \delta^{jl} \right) \right], \quad (3)$$

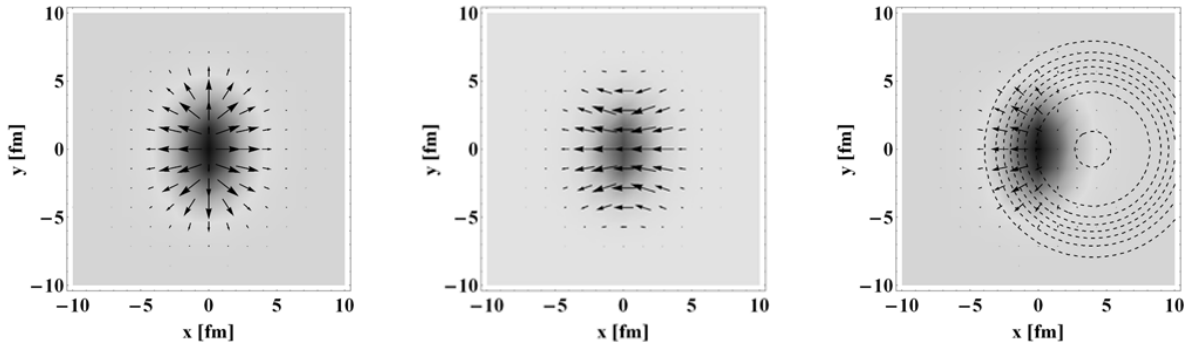


Figure 1. Left: The hydro-like flow component \vec{B} (arrows) and the underlying energy density for Au+Au collisions using Woods-Saxon profiles. Middle: The rapidity-odd flow component \vec{B}' . Right: The Poynting vector T^{0i} at $\eta = 1$ together with the energy density. The contour lines indicate the density profile of one of the nuclei.

where the first term is the same as in the original MV model [9]. Q here is the saturation scale.

Let us now focus on the transverse Poynting vector T^{0i} in the energy momentum tensor which describes the energy flow of the gluon field. The term even in η after averaging reads

$$B_i = -\tau \frac{g^6 N_c (N_c^2 - 1)}{32\pi} \nabla^i (\mu_1^2 \mu_2^2) \ln^2 \frac{Q^2}{(1.47m)^2}. \quad (4)$$

This expression gives radial and elliptic flow of energy $\propto -\nabla^i \epsilon$. This is similar to what a hydrodynamic flow field would look like for the same energy density ϵ if the hydrodynamic pressure is a monotonous function of ϵ . The term odd in η is

$$B'_i = -\tau \frac{g^6 N_c (N_c^2 - 1)}{32\pi} \ln^2 \frac{Q^2}{(1.47m)^2} [\mu_2^2 \nabla^i \mu_1^2 - \mu_1^2 \nabla^i \mu_2^2]. \quad (5)$$

Interestingly this term can describe a directed flow of energy. Note that this term does not violate boost-invariance of the energy momentum tensor. On the other hand boost-invariant hydrodynamics would never be able to create such a flow field. Rather one can trace this term back to the QCD analogon of Gauss' Law. This makes this rapidity-odd flow field a curious phenomenon. The flow vector fields \vec{B} and \vec{B}' from two gold nuclei with Woods-Saxon profiles μ^2 colliding with impact parameter $b = 8$ fm are shown in Fig. 1. The net Poynting vector T^{0i} exhibits a clear dipole asymmetry in the transverse plane when $\eta \neq 0$. Flow in color glass at midrapidity has recently also been studied numerically in [10].

3. Hydrodynamic Initial Conditions

Since the classical gluon field only describes the very earliest stage of a nuclear collision we need to translate the energy momentum tensor into initial conditions for hydrodynamics. The dynamics of equilibration is outside the purview of either hydrodynamics nor classical field theory and would require a more complete description of quantum non-abelian dynamics. One can however derive a matching hydrodynamic energy momentum tensor simply using conserved currents both on the classical field side and the hydrodynamic side. For ideal hydrodynamics this was worked out in [11, 12] by just imposing energy and momentum conservation. The flow of energy generally translates into a flow of particles on the hydrodynamic side. The direction of the hydrodynamic transverse flow velocity is parallel to the Poynting vector and its size is a monotonous function of the magnitude of the Poynting vector. A similar correspondence can be worked out in the case of matching to viscous hydrodynamics [7].

4. Directed Flow Phenomenology

Without a further hydrodynamic evolution we can nevertheless check whether our somewhat surprising direct flow result is qualitatively consistent with experiments [13]. The first thing to check is the direction of the flow. It turns out that our directed flow vector \vec{B}' points away from the spectators for a given side $\eta > 0$ or $\eta < 0$ of a collision system with finite impact parameter. This is consistent with experiments. The rapidity dependence of particle directed flow v_1 is also roughly consistent with a $\sinh y$ shape as a function of particle rapidity y [14, 15].

Note that \vec{B}' vanishes for $b = 0$ if the two nuclei are equal. Interestingly, if an asymmetric system like Cu+Au collides with $b = 0$ the flow vector \vec{B}' becomes radial, but points outward in the direction of the smaller nucleus and inward in the direction of the larger nucleus. An asymmetric system with $b \neq 0$ would exhibit a striking blend of both effects. Looking for such a strong dependence of flow on rapidity in central collisions of asymmetric systems could be a signature for the eminence of strong gluon fields early in nuclear collisions.

5. Conclusion

To summarize, if the McLerran Venugopalan model is generalized to allow for variations of the average charge density across the nucleus flow phenomena in early stages of nuclear collisions can be studied. Here we use analytic results for early gluon fields and their energy momentum tensor that can be obtained in an expansion of the Yang-Mills equations around the time of collision. We find radial and elliptic flow patterns similar to hydrodynamic behavior. In addition we also obtain a rapidity-odd term in the flow of gluon energy which resembles directed flow. We have argued that this flow will translate into directed flow of particles in the further course of the collision. The results are in qualitative agreement with results from the STAR experiment.

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