

# Band structure of neutron rich Se and Ge isotopes

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**Abstract.** The band structure of the neutron-rich Se and Ge isotopes has been studied in terms of the full-fledged shell model. The monopole and quadrupole pairing plus quadrupole-quadrupole interaction is employed as an effective interaction. The model reproduces well the energy levels of high-spin states as well as the low-lying states. In order to investigate the structure of the high-spin states and low-lying collective states, the energy spectra in the shell model are compared with those in the quantum-number-projected generator coordinate method. It is shown that the triaxial components play essential roles in describing the  $\gamma$  bands.

## 1. Introduction

The intriguing properties of the even-even Se and Ge isotopes in the mass region  $A \sim 80$  have been investigated in a number of previous experimental and theoretical studies [1]. These isotopes belong to a typical transitional region. The structure of their low-lying states can be attributed to the interplay of rotational and vibrational collective motions. For high-spin states,  $\gamma$ -ray spectroscopy of the near-yrast states in the  $N = 44$  and 46 isotones of Se ( $^{80,82}\text{Se}$ ) was carried out for deep-inelastic reactions [2]. Recently, full-fledged shell-model calculations have been performed on the even-even and odd-mass nuclei in this mass region using the phenomenological monopole and quadrupole pairing plus quadrupole-quadrupole ( $P+QQ$ ) interaction [3]. One year later large scale shell model calculations were performed to determine effective interactions in the  $fpg$ -shell (JUN45) [4]. In this mile-stone paper Otsuka, Honma and others did a lot of efforts to obtain effective interactions in the whole range of the  $fpg$ -shell. Their shell model describes very well the expected properties of the  $fpg$ -shell nuclei such as the triaxiality, isomer states, the oblate ground-state deformation and the shape coexistence.

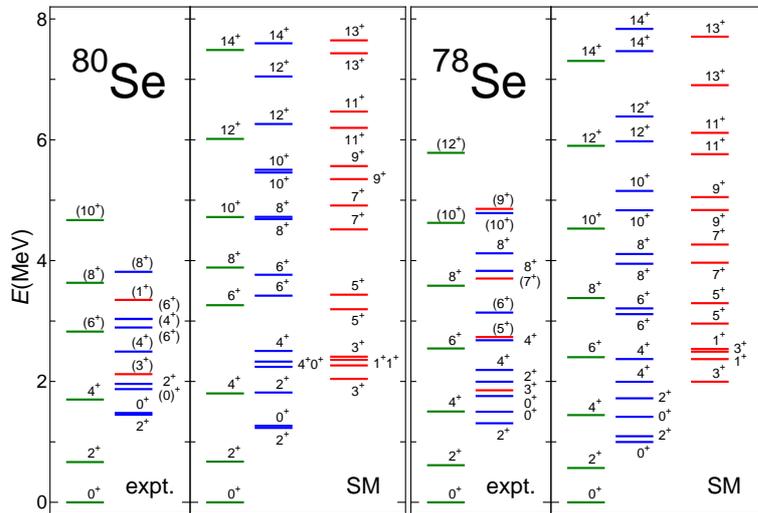
## 2. Shell model framework

In the shell model calculation we take  $Z=28$  and  $N=50$  as magic cores. Then, Se isotopes have 6 valence protons, and Ge isotopes, 4 valence protons. Here neutrons are treated as holes. We use the  $P+QQ$  interaction:

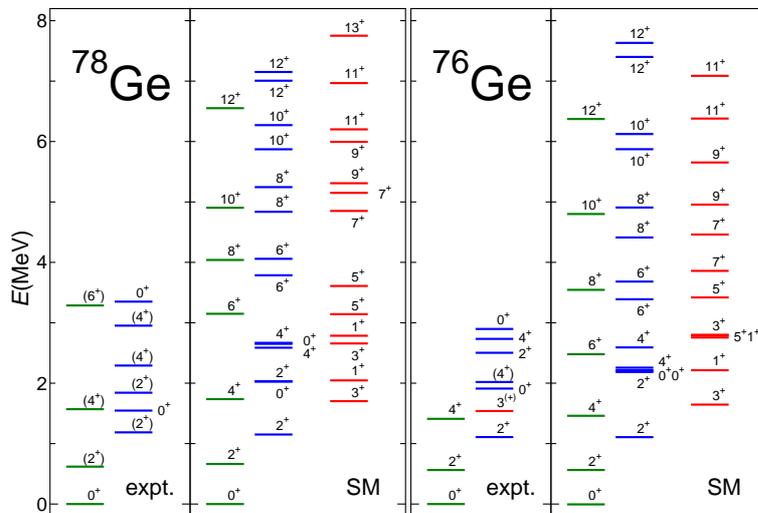
$$\hat{H} = \sum_{jm\tau=\nu,\pi} \varepsilon_{j\tau} c_{jm\tau}^\dagger c_{jm\tau} - \sum_{\tau=\nu,\pi} \left[ G_{0\tau} \hat{P}_\tau^\dagger(0) \hat{P}_\tau(0) + G_{2\tau} \hat{P}_\tau^\dagger(2) \tilde{\hat{P}}_\tau^{(2)} + \kappa_\tau : \hat{Q}_\tau \cdot \hat{Q}_\tau : \right] - \kappa_{\nu\pi} \hat{Q}_\nu \cdot \hat{Q}_\pi, \quad (1)$$



where the strengths of the parameters are the same as in the previous shell model calculations [3]. As shown in figures 1 and 2 our calculation reproduces quite well the energy levels for the Se and Ge isotopes. We show  $B(E2)$  transition rates in figure 3. The reduction of  $B(E2)$  occurs



**Figure 1.** Comparison between the experimental spectra (expt.) and shell model results (SM) for  $^{80,78}\text{Se}$  isotopes.

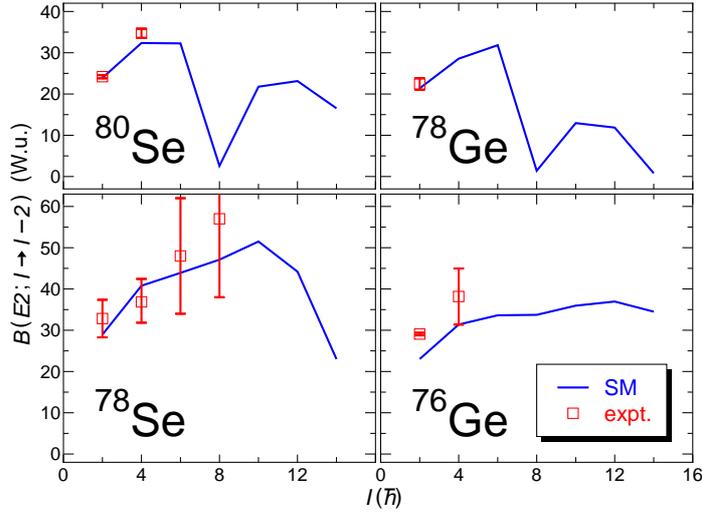


**Figure 2.** Same figure as in figure 1, but for  $^{78,76}\text{Ge}$  isotopes.

from spin 8 to 6 for  $N=46$  isotones ( $^{80}\text{Se}$  and  $^{78}\text{Ge}$ ), but no abrupt change is seen for  $N=44$  isotones ( $^{78}\text{Se}$  and  $^{76}\text{Ge}$ ).

Two questions arise here. (i) Why the  $8_1^+$  states seem to be irregular and  $B(E2)$  transitions become small from  $8_1^+$  to  $6_1^+$  states such as in  $N=46$  for Se and Ge isotopes? (ii) Is the triaxiality important for a description of  $\gamma$  bands? If so, is the system either  $\gamma$ -unstable or  $\gamma$ -rigid?

To answer the first question, we have compared the results of the shell model (SM) calculations with the pair truncated shell model (PTSM) calculations [5, 6, 7]. In this model the collective  $S$ ,  $D$ ,  $G$  nucleon pairs, and the non-collective  $(0g_{9/2})^2$  pairs ( $H$  pairs) are assumed to be the building blocks. The  $S$  and  $D$  pairs play essential roles in describing the low-lying states, while the pair of  $0g_{9/2}$  neutrons is indispensable for a description of high-spin states. We have found that for nuclei with  $N=46$  a sudden alignment of two  $0g_{9/2}$  neutrons occurs, but only a smooth change in structure occurs for  $N=44$ . All the details are given in [3]. The ground state energies



**Figure 3.** Comparison of the calculated  $B(E2)$  values (solid lines) for the yrast states of  $^{80,78}\text{Se}$  and  $^{78,76}\text{Ge}$  with experimental data (open squares).

for  $^{78}\text{Ge}$  are found to be  $E_{\text{SM}} = -11.0529$  MeV and  $E_{\text{PTSM}} = -10.6827$  MeV, respectively, for the SM and the PTSM calculations.

### 3. Quantum-number-projected generator coordinate method

To answer the second question we apply the quantum-number-projected generator coordinate method (GCM) under the same interaction as used in previous SM studies [3].

In the present scheme, the spins of the neutron and proton systems ( $I_\nu$  and  $I_\pi$ ) are projected out separately, and the total spin of each state is constructed by angular momentum coupling [8, 9]. To generate functions for the GCM, we employ the intrinsic Nilsson states for the neutron or proton system. The intrinsic Nilsson states are constructed by the following procedure. First we consider the intrinsic Nilsson hamiltonian for either neutron or proton space:

$$\hat{h}_{\text{Nil}} = \sum_{jm} \varepsilon_j c_{jm}^\dagger c_{jm} - \hbar\omega\beta \left[ \cos\gamma \hat{Q}_0 - \frac{\sin\gamma}{\sqrt{2}} (\hat{Q}_2 + \hat{Q}_{-2}) \right] \quad (2)$$

with

$$\hat{Q}_\mu = \sum_{jm, j'm'} \langle jm | \frac{r^2}{b^2} Y_\mu^{(2)} | j'm' \rangle c_{jm}^\dagger c_{j'm'}. \quad (3)$$

Here  $\beta$  and  $\gamma$  indicate axial and triaxial quadrupole deformations, respectively. Then  $N$ -particle Nilsson intrinsic states are written as

$$|\Phi(\beta, \gamma)\rangle = b_1^\dagger b_2^\dagger \cdots b_{N-1}^\dagger b_N^\dagger | - \rangle = \prod_{i=1}^N b_i^\dagger | - \rangle. \quad (4)$$

The intrinsic deformed state  $|\nu\rangle = b_\nu^\dagger | - \rangle$  is related with the spherical basis state  $|i\rangle = c_i^\dagger | - \rangle$  as

$$b_\nu^\dagger = \sum_i F_{i\nu} c_i^\dagger, \quad (5)$$

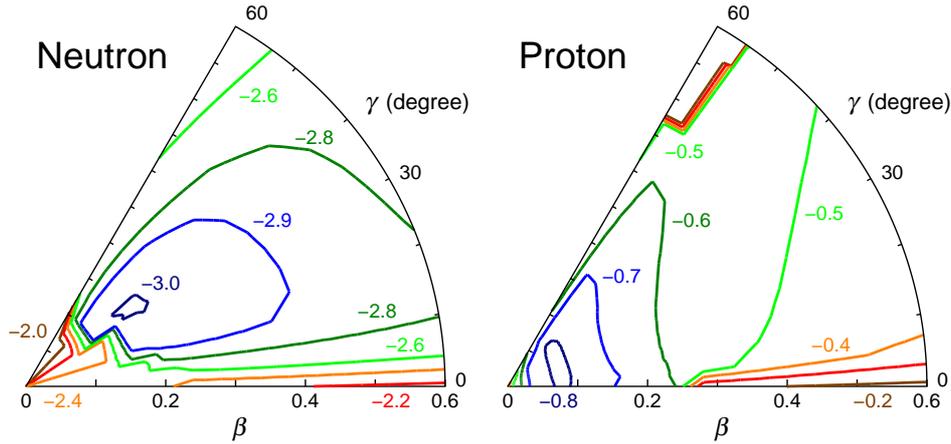
by diagonalizing the Nilsson hamiltonian  $\hat{h}_{\text{Nil}}$ . Then the GCM wavefunction for the  $\rho$ th state of a spin  $I_\tau$  in neutron or proton space ( $\tau = \nu$  or  $\pi$ ) is given by

$$|\Psi_{I_\tau M_\tau \rho}^{(\tau)}\rangle = \sum_k \sum_{K_\tau = -I_\tau}^{I_\tau} \mathcal{F}_{K_\tau \rho}^{I_\tau k} \hat{P}_{M_\tau K_\tau}^{I_\tau} |\Phi_\tau(\beta_k, \gamma_k)\rangle, \quad (6)$$

where  $\hat{P}_{M\tau K\tau}^{I\tau}$  is the spin projection operator,  $\mathcal{F}_{K\tau\rho}^{I\tau k}$ , the weight function to be determined by the GCM, and  $k$  stands for a representative point of deformation  $(\beta, \gamma)$ . Then, the many-body wavefunction for an even-even nucleus can be written as [8, 9]

$$|\Psi_{IM}(I\nu\rho, I\pi\sigma)\rangle = [|\Psi_{I\nu\rho}^{(\nu)}\rangle \otimes |\Psi_{I\pi\sigma}^{(\pi)}\rangle]_M^{(I)}, \quad (7)$$

where  $I$  is the total spin and  $M$ , its projection.



**Figure 4.** The contour plot of the energy gain calculated for neutrons (left panel) and protons (right panel) in  $^{78}\text{Ge}$ . The contour line separation is 0.1 MeV.

In figure 4, the contour plots of the potential energy surface (PES) in the  $\beta$ - $\gamma$  plane for Nilsson states using equation (4) are shown for neutrons and protons of  $^{78}\text{Ge}$ . For neutron space, triaxiality appears, but it is not clear for proton space. In the GCM calculations we take  $\beta = 0.1, 0.2, \dots, 0.6$  and  $\gamma = 10^\circ, 20^\circ, 30^\circ, 40^\circ, \text{ and } 50^\circ$ . In figure 5, the theoretical energy spectra of the GCM are compared with the SM results and the experimental data for  $^{78}\text{Ge}$ . It is found that the energy levels obtained by the SM is well reproduced by the GCM. The ground state energy obtained by the GCM for the  $^{78}\text{Ge}$  is  $E_{\text{GCM}} = -10.5059$  MeV.

Now let us improve our GCM calculations by including the pairing correlation. We introduce the nucleon pair creation operator as

$$\Lambda^\dagger(\beta, \gamma) = \sum_{\alpha>0} f_\alpha(\beta, \gamma) b_\alpha^\dagger \bar{b}_\alpha^\dagger, \quad (8)$$

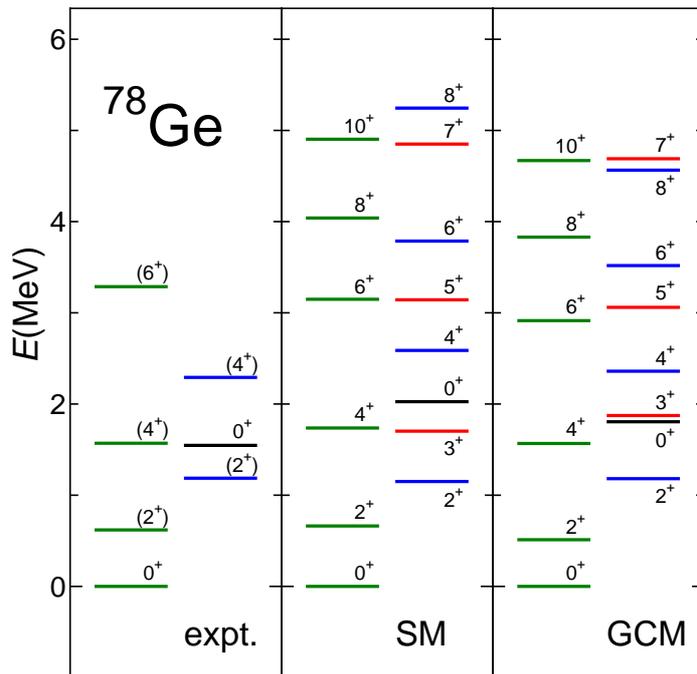
where  $\bar{\alpha}$  indicates the time reversal state of  $\alpha$ . Then the intrinsic  $N$ -particle states for neutrons or protons are written as

$$|\Phi(\beta, \gamma)\rangle = [\Lambda^\dagger(\beta, \gamma)]^{N/2} |-\rangle. \quad (9)$$

The structure coefficients  $f$ 's are determined by variation for a definite deformation  $(\beta, \gamma)$ :

$$\delta \frac{\langle \Phi(\beta, \gamma) | \hat{H} | \Phi(\beta, \gamma) \rangle}{\langle \Phi(\beta, \gamma) | \Phi(\beta, \gamma) \rangle} = 0. \quad (10)$$

In figure 6, the contour plots of the PES are shown for neutron space and proton space, respectively. Now they are very smooth functions of  $\beta$  and  $\gamma$ , and they have no definite minimum. The pairing interaction washes away the deformations.



**Figure 5.** Comparison of experimental energy spectrum (expt.) with that in the SM (SM) and that obtained by the GCM (GCM) for  $^{78}\text{Ge}$ .

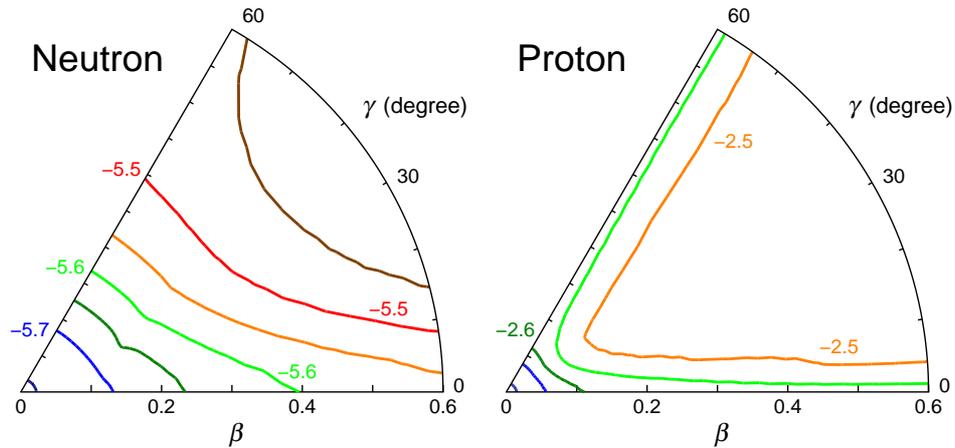
In the GCM calculations we take for axial deformations (31 points),  $\beta = 0.00, 0.04, 0.08, \dots, 0.60$  and  $\gamma = 0^\circ, 60^\circ$ . For triaxial deformations we take (20 points)  $\beta = 0.10, 0.20, 0.30, \gamma = 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ$ , and  $(\beta, \gamma) = (0.40, 20^\circ), (0.40, 40^\circ), (0.50, 10^\circ), (0.50, 30^\circ), (0.50, 50^\circ)$ . In figure 7 energy spectra are compared for experimental results, SM, GCM (triaxial) and GCM (axial), respectively. We compare between the SM and the GCM results (2 cases) for  $^{78}\text{Ge}$ . The ground state energies for the  $^{78}\text{Ge}$  are  $E_{\text{SM}} = -11.0530$  MeV,  $E_{\text{GCM(triaxial)}} = -11.0048$  MeV, and  $E_{\text{GCM(Axial)}} = -10.9643$  MeV, respectively for the SM, the GCM with triaxial deformations and the GCM only with axial deformations. A large improvement is seen for the triaxial deformation for the ground state energies. The triaxiality is important in this region, especially in the description of the  $\gamma$  band. The potential energy surface is shallow in the direction and it shows the  $\gamma$ -unstable nature or IBM O(6) like picture. The generator coordinate model calculations show the importance of triaxiality in describing the  $\gamma$ -band, but the PES is shallow, indicating the  $\gamma$ -unstable interpretation.

#### 4. Summary

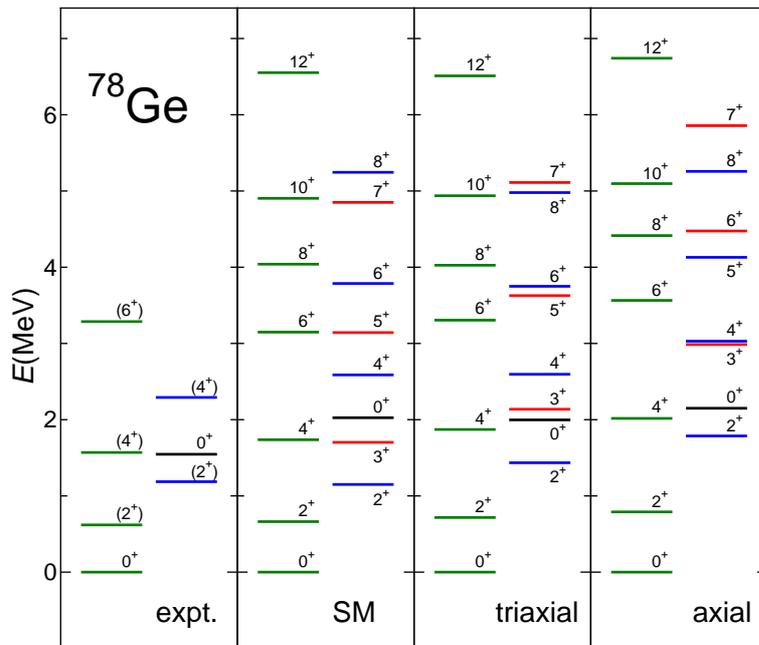
In this paper SM results are analyzed in terms of the PTSM and the GCM. The PTSM analysis shows the alignment of neutron in the  $g_{9/2}$  orbital at the yrast spin of  $8^+$ . The GCM calculations show the importance of triaxiality in describing the  $\gamma$ -band, but the PES is shallow, indicating the  $\gamma$ -unstable interpretation.

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**Figure 6.** Same as figure 4, but for those states with the pairing correlation included. The contour line separation is 0.05 MeV



**Figure 7.** Energy levels of the yrast and quasi- $\gamma$  bands in  $^{78}\text{Ge}$ . From the left, energy levels in the experiment (expt.), the SM (SM), the triaxial GCM (triaxial), and the axial GCM (axial) are displayed.

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