Journal of Physics: Conference Series **445** (2013) 012015 doi:10.1088/1742-6596/445/1/012015

Interacting boson model with energy density functionals

K Nomura

Institut für Kernphysik, Universität zu Köln, D-50937 Köln, Germany

E-mail: nomura@ikp.uni-koeln.de

Abstract. A brief overview of the recent advancement in the microscopic study of the interacting boson model (IBM) is given. A new nucleon-boson mapping method has been proposed recently, that derives the IBM Hamiltonian based on the self-consistent mean-field model with the microscopic energy density functional (EDF). The mean-field total energy surface computed with a given EDF is mapped onto the analogous energy expectation value in the boson condensate, thereby the energy spectrum and the transition rates are generated. Since the EDF framework allows an accurate global description of nuclear intrinsic properties, the IBM is derived in a unified way, basically for any situation of low-energy quadrupole collective states of nuclei. The basic notion of the new mapping technique is sketched.

1. Introduction

Since its advent in the mid-1970s, the interacting boson model (IBM) of Arima and Iachello [1] has made tremendous successes in the study of low-lying collective states of medium-heavy and heavy nuclei. The ingredients of the IBM are monopole s and quadrupole d bosons, which correspond respectively to collective nucleon-pairs with angular momenta $J^\pi = 0^+$ and 2^+. The boson Hamiltonian is composed of only essential interactions acting among the bosons, and measurable quantities like energy spectrum and electromagnetic transition rates are obtained through the diagonalization of the Hamiltonian, either numerically or analytically when the Hamiltonian is written as a particular combinations of interaction terms (dynamical symmetry) [1]. Despite its success, however, the IBM has lacked in the microscopic foundation (and hence the predictive power) since the parameters of the boson Hamiltonian have been determined only from the phenomenological fit to the known experimental data.

From a microscopic point of view, the IBM system represents a vast truncation of the full Hilbert space of nuclear shell model, and conventionally the parameters of the boson Hamiltonian have been calculated by associating the matrix element of a given fermion operator in the truncated shell-model space, comprising collective S and D (plus G, if needed) pairs, to the matrix element of the corresponding boson operator in the sd boson space [2–4]. Since a nucleus is consisting of protons and neutrons, the proton-neutron version of the IBM (IBM-2) has been introduced as a natural consequence, that is composed of neutron (proton) s_ν (s_π) and d_ν (d_π) bosons [2, 3]. The number of neutron (proton) bosons, denoted here as N_ν (N_π), equals the number of the collective pairs of valence neutrons (protons) outside of an inert core. Although the shell-model derivation of the IBM has worked out in many realistic cases of spherical vibrational and γ unstable systems [4, 5], since the underlying shell-model configuration

becomes too complicated to be handled with the simple truncation scheme like the generalized seniority, the unified framework of deriving IBM Hamiltonian for general situations including rotational deformed system has long been missing.

Recently a novel robust way of deriving the IBM Hamiltonian has been developed, in conjunction with the self-consistent mean-field model based on the microscopic energy density functional (EDF) [6, 7]. In this technique, a given boson Hamiltonian has been fixed so that the energy expectation value of the nucleon many-body Hamiltonian in the mean-field intrinsic state with varying deformation is mapped onto the analogous energy expectation value in the boson condensate. The self-consistent mean-field method with a fixed EDF parametrization allows an accurate global description of the nuclear bulk properties and collective excitations, e.g., mass, radii, deformation, shell structure, giant resonance, ... etc. [8], and can be thus exploited as a starting point from which the IBM Hamiltonian is constructed.

In this contribution, the basic notion of the new fermion-boson mapping technique is reviewed and we shall demonstrate how it works by taking a simple example. This contribution is based on Refs. [6, 7, 9] but discusses only a few essential results. For a complete review of the proposed methodology and its various extensions, the reader is referred to Ref. [10].

2. Formulation

To construct the boson Hamiltonian for general situations associated to various types of deformation, the energy surface of the collective deformation variables can be appropriate to start with. The energy surface is generated with a set of the constrained self-consistent mean-field methods, such as Hartree-Fock, Hartree-Bogoliubov or Hartree-Fock-Bogoliubov models, etc., using a microscopic energy density functional. The mean-field solution of the total energy surface at each collective coordinate (β, γ), denoted here as $E(\beta, \gamma) = \langle \Phi_F(\beta, \gamma) | \hat{H}_F | \Phi_F(\beta, \gamma) \rangle$ with $|\Phi_F\rangle$ being the corresponding intrinsic wave function, are obtained through the minimization of the total energy normally as a sum of kinetic energy, a given model of EDF, pairing functional and Coulomb energy with spurious center-of-mass motion being subtracted. The constraint means the one for the mass quadrupole moment associated to the deformation variables of the geometrical model (β,γ) [11]. The effective interaction for the particle-hole channel are typically of the Skyrme [12, 13] and the Gogny [14] types, and other parameterizations based on the relativistic mean-field Lagrangian [15]. Note that, in this framework, only the total energy at the level of the mean-field approximation is of relevance here. This means that neither collective potential energy surface nor mass parameters are considered explicitly and that any symmetry projection is not performed.

To associate the energy surface obtained through the EDF calculation to the analogous energy surface in the boson system, the so-called coherent-state framework [16] is used. The coherent state represents the intrinsic state of the boson system and is given as the following boson condensate $|\Phi_B\rangle$

$$|\Phi_B\rangle = \frac{1}{\sqrt{N_\nu! N_\pi!}} (\lambda_\nu^\dagger)^{N_\nu} (\lambda_\pi^\dagger)^{N_\pi} |-\rangle \tag{1}$$

with $|-\rangle$ being inert core, and $\lambda_\rho^\dagger$ ($\rho = \nu$ or π) reads

$$\lambda_\rho^\dagger = \frac{1}{\sqrt{1+\beta_\rho^2}} \Big[s_\rho^\dagger + \beta_\rho \cos\gamma_\rho d_{\rho 0}^\dagger + \frac{1}{\sqrt{2}} \beta_\rho \sin\gamma_\rho (d_{\rho+2}^\dagger + d_{\rho-2}^\dagger) \Big]. \tag{2}$$

The IBM analogue of the mean-field energy surface is obtained as an expectation value of the boson Hamiltonian $\langle \Phi_B | \hat{H}_B | \Phi_B \rangle$. The variables β_ρ and γ_ρ in Eq. (2) are equivalent to geometrical quadrupole deformation (β, γ). Since the following discussion is restricted to isoscalar motion

and both neutrons and protons are supposed to have the same shape due to their strong coupling, $\beta_\nu = \beta_\pi \equiv \beta_B$ and $\gamma_\nu = \gamma_\pi \equiv \gamma_B$ can be the first good approximation. The variables (β_B, γ_B) can be related to the geometrical variables (β,γ) in the following way:

$$\beta_B = C\beta \quad \text{and} \quad \gamma_B = \gamma, \tag{3}$$

the former of which comes from the difference in the model space [16], the fact that the IBM is comprised of only valence nucleons while the collective model entire nucleus. Thus, β_B is always larger than β with $C \geq 1$. The latter relation in Eq. (3) implies that the angle γ should have the same physical meaning in both models. One can then map each point of the microscopic energy surface $E(\beta, \gamma)$ onto the IBM analogue of the mean-field energy surface. This requires that the following approximate equality be satisfied over a certain range of the deformation (β, γ) corresponding to the low-energy excitation (typically up to several MeVs from the minimum).

$$\langle \Phi_F(\beta,\gamma)|\hat{H}_F|\Phi_F(\beta,\gamma)\rangle \sim \langle \Phi_B(\beta_B,\gamma_B)|\hat{H}_B|\Phi_B(\beta_B,\gamma_B)\rangle, \tag{4}$$

which means that the boson energy surface should be calculated to be identical in topology around the mean-field minimum to the microscopic energy surface as much as possible [1]. By mimicking the basic topology of the microscopic energy surface most relevant to the low-energy excitation, effects of the essential ingredients of nuclear many-body system, e.g., Pauli principle and nuclear force, are expected to be incorporated into the IBM system by renormalizing them in the values, and/or the boson-number dependence, of the strength parameters.

For the boson Hamiltonian $\hat{H}_B$, the possible simplest but the most essential form can be taken

$$\hat{H}_B = \epsilon \hat{n}_d + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi \tag{5}$$

where $\hat{n}_d = \hat{n}_{d\nu} + \hat{n}_{d\pi}$ stands for the d boson number operator, which drives nucleus into spherical shape, and the second term the quadrupole-quadrupole interaction between neutron and proton bosons, which makes dominant contribution to deformation, with quadrupole operator being $\hat{Q}_\rho = s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho s_\rho + \chi_\chi [d^\dagger_\rho \times \tilde{d}_\rho]^{(2)}$. ϵ, κ, χ_ν and χ_π are parameters to be determined from the mapping. The boson Hamiltonian could be more general, but the simplified form Eq. (5) already embodies rich aspects of quadrupole collective states. The energy spectra and transition rates are obtained through the diagonalization of the mapped Hamiltonian in the laboratory frame.

3. Transition from spherical vibrational to deformed rotational states

The examples for the microscopic energy surfaces are shown on the left-hand side of Fig. 1 for axially deformed Sm isotopes, obtained through the HF+BCS method solved in a coordinate space [17] with the parametrization Skyrme SkM* [18] and the density-dependent zero-range pairing. One finds in the figure that the energy surfaces of the ^{146,148}Sm nuclei, which are in the vibrational regime, exhibit the minima close to the origin. Coming to the transitional nuclei ^{150,152}Sm, the minimum departs from the origin to $\beta \approx 0.3$ and also looks flat in β. Even larger deformation ($\beta \approx 0.35$) is suggested in the ^{154,156}Sm nuclei, where the minimum becomes well isolated being very sharp with respect to both β and γ. The mapped IBM energy surfaces on the right-hand side of Fig. 1 reproduce the topology of the microscopic energy surface around the minimum and the overall systematic trend with N.

If one looks at the resultant spectra in Fig. 2, the basic empirical features are reproduced: lowering of yrast levels with N, the phase-transitional rapid changes in the side-band levels at $N \approx 88$ or 90. The present model well describes the vibrational spectra around $N \approx 86$, being

[1] For technical details behind the mapping the reader is referred to Refs. [7, 10].

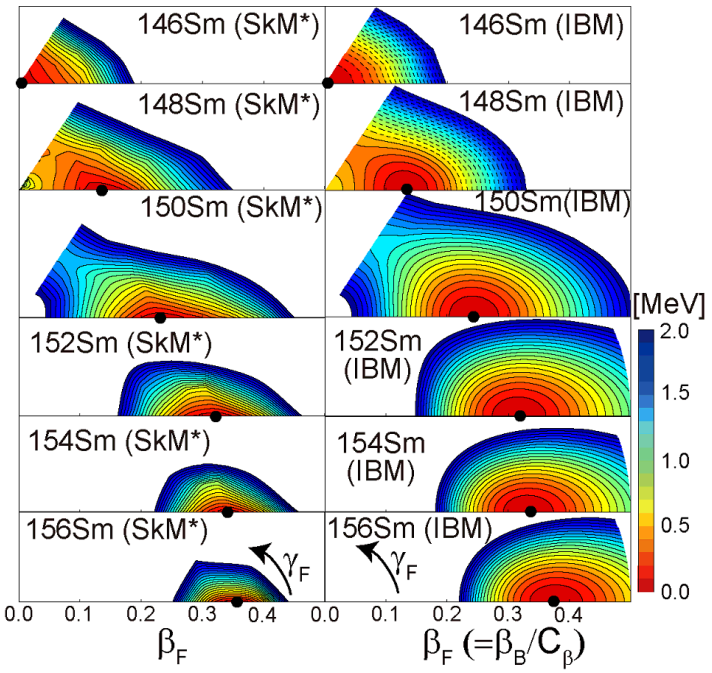

Figure 1. (Color online) Total (β, γ) energy surface of $^{146-156}$Sm isotopes. On the left-hand side (SkM*) shown are microscopic energy surfaces with Skyrme SkM* interaction, while on the right-hand side the mapped IBM energy surfaces. The minimum is identified by the filled circle. β_F and γ_F stand for the geometrical deformation variables.

close to the U(5) limit of IBM [1], and the typical rotational band around $N \approx 92$, or the SU(3) limit of the IBM [1]. It is also worth noting that the result of the level energy systematics correlates with the variation of the deformation properties with neutron number indicated by the microscopic energy surface in Fig. 1.

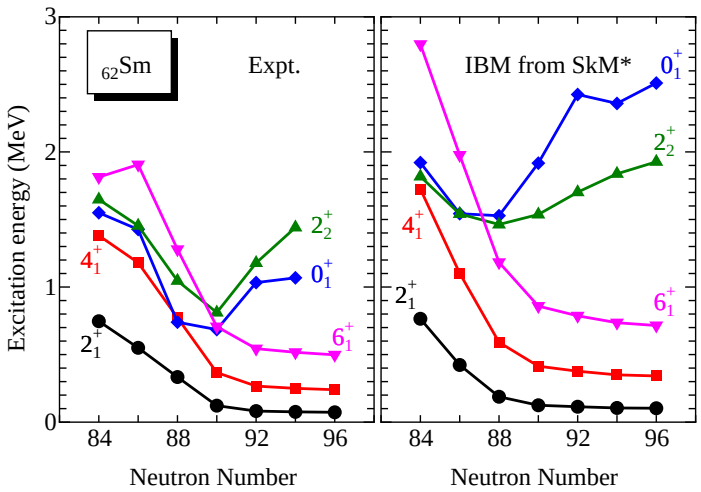

Figure 2. (Color online) Level energies of 2_1^+, 4_1^+, 6_1^+, 0_2^+ and 2_2^+ excited states with neutron number. Experimental data have been taken from Ref. [19].

At this point, one could notice in Fig. 2 that the rotational moment of inertia for deformed nuclei with $N \geq 92$, calculated in the IBM, underestimates to a large extent the experimental data. The problem is highlighted in Fig. 3, where excitation energies calculated in the microscopic IBM (energy bands on the left-hand side in each panel (a) and (b), denoted as "w/o") are much larger than the experimental [19] values. A reason for this deviation of the moment of inertia can be the limited degrees of freedom in the IBM consisting of only s and d bosons. Nevertheless, as far as the result of the energy surface mapping in Fig. 1 is concerned, the

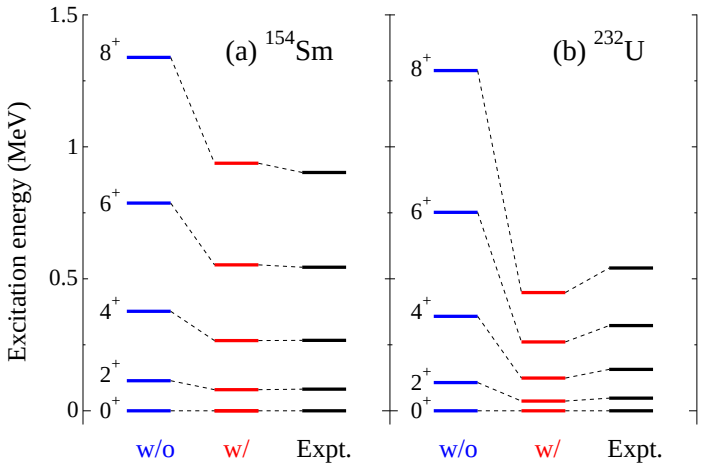

Figure 3. (Color online) Rotational bands of axially deformed (a) ^{154}Sm and (b) ^{232}U nuclei. The calculated spectra with (w/) and without (w/o) the $\hat{L}\cdot\hat{L}$ term are compared with experiment [19].

configuration of *sd* bosons appears to be rather sufficient. Thus, it would be more likely [9], that the problem occurs due to fact that the boson system differs in its rotational property from the nucleon system (see [9] for details). To settle the problem, we have then proposed to introduce an extended mapping procedure, that includes a specific rotational dynamics with non-zero angular frequency, in addition to the mapping of the energy surface with zero angular frequency (cf. Eq. (4)). The rotational dynamics could be incorporated through the standard cranking approximation [20]. Instead of the Hamiltonian $\hat{H}_X$ $(X = F, B)$, the following Hamiltonian (with axial symmetry)

$$\hat{H}'_X = \hat{H}_X - \omega \hat{L}_y, \tag{6}$$

where $\hat{L}_y$ and ω are the y component of the total angular momentum and cranking frequency, respectively, should be minimized. Given the values of the strength parameters already derived through the mapping of Eq. (4), the response (or the energy shift of the system due) to rotational cranking should be approximately identical between fermion and boson systems at a particular point on the $\beta\gamma$ surface, such as the equilibrium $(\beta^{(e)},\gamma^{(e)})$:

$$\Delta\langle\Phi'_F(\beta^{(e)},\gamma^{(e)})|\hat{H}'_F|\Phi'_F(\beta^{(e)},\gamma^{(e)})\rangle \sim \Delta\langle\Phi'_B(\beta^{(e)}_B,\gamma^{(e)}_B)|\hat{H}'_B|\Phi'_B(\beta^{(e)}_B,\gamma^{(e)}_B)\rangle, \tag{7}$$

where $|\Phi'_X\rangle$ represents the solution of $\hat{H}'_X$ in Eq. (6) at the equilibrium $(\beta^{(e)},\gamma^{(e)})$. To this end, we have further proposed to add the term to the boson Hamiltonian Eq. (5), that corrects the moment of inertia but does not alter the topology of the energy surface [9]. Such term should be of the form $\hat{L}\cdot\hat{L}$, with $\hat{L} = \hat{L}_\nu + \hat{L}_\pi$ being the angular momentum operator. In practice the approximate equality of Eq. (7) can be fulfilled by matching the cranking moment of inertia in the intrinsic state of IBM at the equilibrium $(\beta^{(e)},\gamma^{(e)})$ to the one resulting from the mean-field calculation, leading to the coupling constant of the $\hat{L}\cdot\hat{L}$ term [9].

To see the impact of the $\hat{L}\cdot\hat{L}$ term on rotational band, we compare in Fig. 3 the energy levels calculated with the $\hat{L}\cdot\hat{L}$ term with those without the term for strongly deformed ^{154}Sm and ^{232}U nuclei as examples. By the inclusion of the $\hat{L}\cdot\hat{L}$ term, the agreement between theoretical and experimental rotational bands is remarkably improved for the nucleus ^{154}Sm so does for ^{232}U. It has to be stressed here that there is no adjustment to the experiment. It has been shown as well [9] that the effect of the $\hat{L}\cdot\hat{L}$ term is particularly important for axially deformed rotational nuclei but is normally much less for weakly deformed and γ-soft systems.

Given the results presented in Figs. 2 and 3, as well as in a number of applications [10] not shown here, we have come to the conclusion that the IBM can be derived in a unified

way for various situations of the low-energy quadrupole collective states, ranging from spherical vibrational to strongly deformed rotational states, etc.

4. Summary

In this contribution, a novel robust way of deriving the IBM Hamiltonian from the self-consistent mean-field model with microscopic EDF has been overviewed. By the proposed method, the basic collective excitation modes of nuclei are reproduced: not only vibrational and transitional states but the rotational deformed state. Particularly the latter point should more or less concern the question raised in the past by Bohr and Mottelson [21], as to whether the IBM is justified in the microscopic description of the rotational motion of deformed nuclei. While a conclusive answer to this question has still been missing, the present work sheds more light upon and should provide a new insight into this old problem. In practical applications, the IBM can be considered as an effective theory to solve complex many-fermion problems for heavy-mass system, which would be computationally much more costly with a large-scale calculation, and is granted a predictive power for the spectroscopy of heavy exotic nuclei.

Acknowledgments

Many thanks to Taka Otsuka for inspiring me to set out a work in this field and for pleasant discussions and collaborations for the past few years. I also acknowledge the financial support through the JSPS Postdoctoral Fellowships for Research Abroad.

References

[1] Iachello F and Arima A 1987 *The interacting boson model* (Cambridge: Cambridge University Press)
[2] Arima A, Otsuka T, Iachello F and Talmi I 1977 *Phys. Lett.* B **66** 205
[3] Otsuka T, Arima A, Iachello F and Talmi I 1978 *Phys. Lett.* B **76** 139
[4] Otsuka T, Arima A and Iachello F 1978 *Nucl. Phys.* A **309** 1
[5] Mizusaki T and Otsuka T 1996 *Prog. Theor. Phys. Suppl.* **125** 97
[6] Nomura K, Shimizu N and Otsuka T 2008 *Phys. Rev. Lett.* **101** 142501
[7] Nomura K, Shimizu N and Otsuka T 2010 *Phys. Rev.* C **81** 044307
[8] Bender M, Heenen P H and Reinhard P G 2003 *Rev. Mod. Phys.* **75** 121
[9] Nomura K, Otsuka T, Shimizu N and Guo L 2011 *Phys. Rev.* C **83** 041302
[10] Nomura K 2012 Ph.D. thesis University of Tokyo to be published in Springer Theses Physics (2013)
[11] Bohr A and Mottelsson B R 1975 *Nuclear Structure* vol 2 (New York: Benjamin)
[12] Skyrme T H R 1958 *Nucl. Phys.* **9** 615
[13] Vautherin D and Brink D M 1972 *Phys. Rev.* C **5** 626
[14] Dechargé J, Girod M and Gogny D 1975 *Phys. Lett.* B **55** 361
[15] Nikšić T, Vretenar D and Ring P 2011 *Prog. Part. Nucl. Phys.* **66** 519
[16] Ginocchio J N and Kirson M W 1980 *Nucl. Phys.* A **350** 31
[17] Bonche P, Flocard H and Heenen P H 2005 *Compt. Phys. Commun.* **171** 49
[18] Bartel J, Quentin P, Brack M, Guet C and Hakansson H B 1982 *Nucl. Phys.* A **386** 79
[19] Brookhaven National Nuclear Data Center ENSDF http://www.nndc.bnl.gov
[20] Ring P and Schuck P 1980 *The nuclear many-body problem* (Berlin: Springer-Verlag)
[21] Bohr A and Mottelson B R 1980 *Physica Scripta* **22** 468