A note on the interpretation of the gamma evaluation index

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Abstract. The gamma evaluation method has become the gold standard for the comparison between measured and calculated absorbed dose distributions. However, test criteria and failure rate tolerance levels have hitherto normally been based on empirical evidence, rather than rigorous statistical analysis. In this work, it is proposed that the gamma-evaluation method could be reinterpreted such that the absorbed dose difference and distance-to-agreement criteria are replaced by the standard deviations of the associated uncertainties. By comparison between absorbed dose calculations and simulated measurements for clinically realistic test cases in 1D and 2D, it is then shown that the resulting squared gamma distribution follows a chi-squared distribution with one degree of freedom. This result can be used to test the statistical significance of measured deviations, and to determine proper failure rate tolerance levels in clinical radiotherapy quality control.

1. Introduction

One of the most important quality control (QC) procedures in radiation therapy is the experimental verification of calculated treatment plans by actual measurements, and the standard method for the evaluation of such comparisons is the so-called gamma-evaluation test method [1-2]. Due to the unavoidable uncertainties of absorbed dose measurements and detector positioning, some points in the gamma evaluation will always fail the test criteria even if there is no true deviation, and a certain failure rate has to be tolerated. Although test criteria and failure rate tolerance levels have hitherto normally been based only on empirical evidence [3], proper figures should preferably be chosen based on a statistical analysis of the actual uncertainty associated with the absorbed dose measurement and the positioning of the detector. For this purpose, some authors have adopted an error propagation approach, incorporating the local absorbed dose gradient into the analysis [4-7]. In practical implementations, however, such methods are generally limited to a linear gradient approximation, which may introduce anomalies in convex or concave parts of the dose distribution.

In case the complete probability distribution of the gamma index was known, the test criteria could be determined based on probability analysis [8-9]. Under certain conditions, an approximate gamma distribution can be derived theoretically, and a three-moment approximation of the squared gamma distribution has been developed by applying general results about quadratic forms of normal random variables and the chi-squared distribution [9]. This method could potentially be implemented in conventional gamma evaluation procedures, requiring only slight modifications and few extra calculations.

In this work, a further simplified method was presented, which could be used together with current gamma test procedures without modifications. The method was evaluated by using calculated dose distributions in 1D and 2D for clinically realistic QC situations, and by simulating measurements based on statistical uncertainties of absorbed dose measurements and detector positioning. The resulting squared gamma distribution was investigated and compared with the chi-squared distribution with one degree of freedom, and the results was used as a basis for the determination of gamma evaluation acceptance criteria.

2. Material and methods

The calculations in this work were based on clinically realistic absorbed dose distributions in 1D and 2D. In both cases, the relative absorbed dose was given as a continuous function of the spatial coordinates. For the 1D case, the equation provided by Low et al. [1] was used as a representation of a calculated relative absorbed dose profile, see figure 1a. The 2D case was based on an IMRT beam calculated by the Oncentra Master Plan treatment planning system, with a pixel resolution of 1mm and linear interpolation between the points of the dose matrix, see figure 1b.

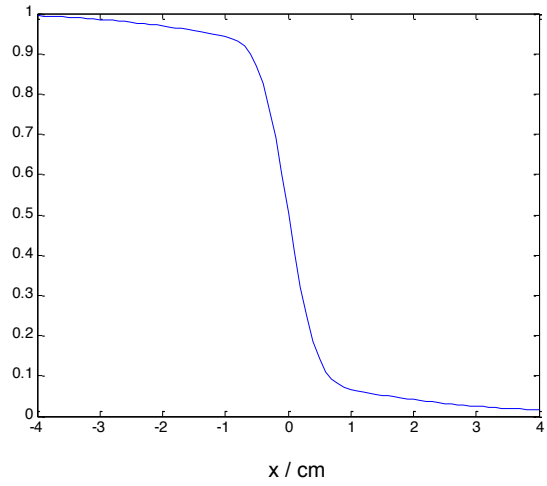

Figure 1a: The dose distribution in 1D.

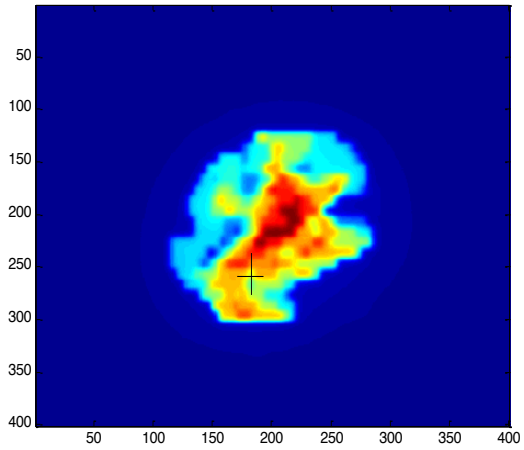

Figure 1b: The dose distribution in 2D.

Measured absorbed dose values were simulated by assuming that the calculated value at (and in the vicinity of) the measurement point represented the expectation value, and by adding a random measurement uncertainty. Thus, for each nominal measurement point, a measured value was simulated by taking the calculated dose at a randomly displaced position, and by adding a random measurement noise. Both the detector displacement and the measurement noise were drawn from normal distributions, with standard deviations σ_D=1cGy and σ_r=1mm, respectively. Then, the nearest calculation point was determined according to the gamma evaluation procedure, and the smallest gamma index was calculated. The acceptance criteria were set to $\Delta D_{crit}=\sigma_D$ for the dose difference and to $DTA_{crit}=\sigma_r$ for the distance-to-agreement. By repeating this simulation a large number of times (10^5 times in 1D and 10^4 times in 2D), a squared gamma (γ^2) distribution was obtained, which was then compared with the chi-squared distribution with one degree of freedom.

3. Results and discussion

The resulting γ^2-distributions are shown (blue circles) for the 1D case in figure 2a (at the position x=0), and for the 2D case in figure 2b (at the position marked with a cross in figure 1b). In both cases it can be seen that the γ^2-distribution follows very well the chi-squared distribution with one degree of freedom (red curve).

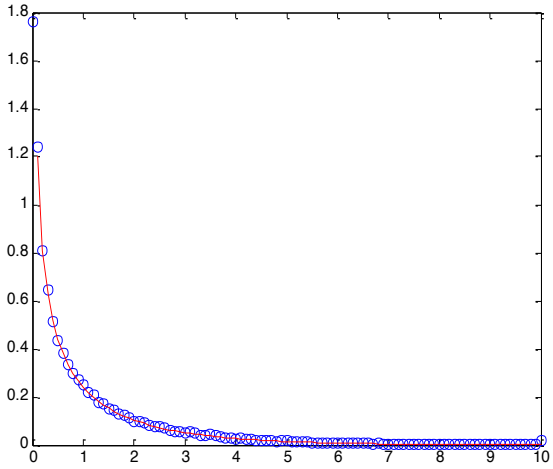

Figure 2a: Distribution of gamma values in 1D.

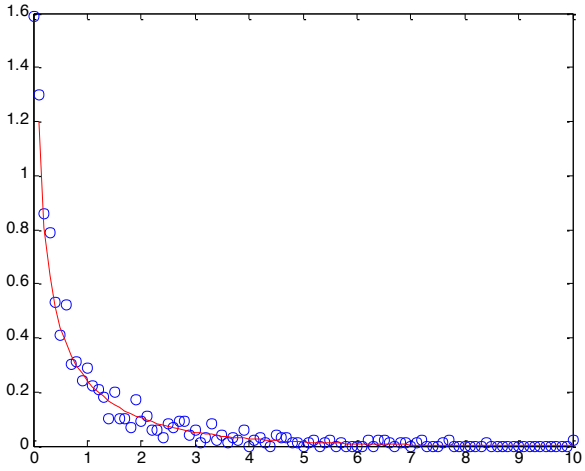

Figure 2b: Distribution of gamma values in 2D.

According to the chi-squared distribution with one degree of freedom, 5% of the values should be greater than 3.841 (given that there is no true deviation). The actual fraction of values greater than 3.841 for these particular simulations were 5.1% in the 1D case and 5.0% in the 2D case.

Importantly, these numbers remained close to 5% also at other measurement points. In figure 3, the fraction of values above 3.841 for the acquired γ^2-distribution is shown for positions from -2.0 cm to +1.0 cm along the x-axis. It can be observed that a 5% failure rate is approximately maintained (within about 0.5%) at all positions along the x-axis, even across the convex and concave parts of the penumbra region, where there is a non-linear absorbed dose gradient.

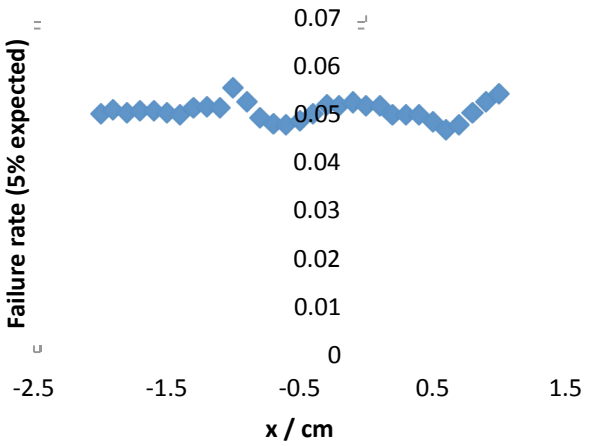

Figure 3: Failure rate for the 1D case as a function of position along the x-axis

The chi-squared distribution is intimately associated with quadratic forms of normally distributed variables. Assuming that the deviations between the observed and expected absorbed dose and detector position, relative to the test criteria, are independent standard normal random variables, the squared gamma index should follow (by definition) a chi-squared distribution with two degrees of freedom. However, since the absorbed dose and detector position are not entirely independent variables (and the relationship between them is unknown), this is no longer the case. While an approximate solution to this situation can be obtained by using more refined mathematics [9], a simpler approach using reduced degrees of freedom was tested in this work. The squared gamma distribution was investigated and found to agree well with the chi-squared distribution with only one degree of freedom. This result is very useful, as it makes it straightforward to implement a statistical interpretation of the gamma evaluation index into current gamma-evaluation procedures, without modifications.

4. Conclusions

The gamma evaluation method has become the gold standard for the evaluation of comparisons between measured and calculated absorbed dose distributions. However, in clinical practice the acceptance criteria have been based largely on experience, without any underlying statistical analysis. In this work, it is proposed that the gamma-evaluation method could be reinterpreted such that the absorbed dose difference and distance-to-agreement criteria are replaced by the standard deviations of the associated uncertainties. Then, the squared gamma index can be compared with the chi-squared distribution with one degree of freedom in order to test the statistical significance of the measured deviation. The results can be used to determine proper failure rate tolerance levels in clinical radiotherapy QC.

5. References

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