Classical and quantum-mechanical axioms with the higher time derivative formalism

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Abstract. A Newtonian mechanics model is essentially the model of a point body in an inertial reference frame. How to describe extended bodies in non-inertial (vibration) reference frames with the random initial conditions? One of the most generalized ways of descriptions (known as the higher derivatives formalism) consists in taking into account the infinite number of the higher temporal derivatives of the coordinates in the Lagrange function. Such formalism describing physical objects in the infinite dimensions space does not contradict to the quantum mechanics and infinite dimensions Hilbert space.

1. Introduction

Classical Newtonian mechanics is essentially the simplest way of mechanical system description with second-order differential equations, when higher order time derivatives of coordinates can be neglected. The extended model of Newtonian mechanics with higher time derivatives of coordinates is based on generalization of Newton's Laws onto arbitrary reference systems (both inertial and non-inertial ones) with body dynamics being described with higher order differential equations. Newton's Laws, constituting, from the mathematical viewpoint, the axiomatic of classical physics, actually postulate the assertion that the equations describing the dynamics of bodies in inertial reference frames are second-order differential equations. However, the actual time-space is almost without exception non-inertial, as it is almost without exception that there exist (at least weak) fields, waves, or forces perturbing an ideal inertial reference frames. It corresponds to Mach's principle [1] with a general statement "Local physical laws are determined by the large-scale structure of the Universe." Non-inertial nature of the actual time-space is also supported by observations of the practical astronomy that expansion of the Universe occurs with an acceleration. In other words, actually any real reference frame is a non-inertial one, and such physical reality can be described by a differential equation with time derivatives of coordinates of an order exceeding two, which play the role of additional variables. Aristotle's physics considered velocity to be proportional to the applied force, hence the body dynamics is described by a first-derivative differential equation. Classical Physics in the inertial reference frames describe a free body maintains the constant velocity of the translational motion. In this case, the body dynamics is described by a second-order differential equation, with acceleration being proportional to the force [2]. This corresponds to the Lagrange function depending on coordinates and their first derivatives (velocities) of the body, and the Euler–Lagrange equation resulting from the principle of the least action. This model of the physical reality describes macrocosm fairly well, but it fails to describe microparticles. Both Newtonian axiomatic and the second Newton law are not valid in the microcosm. Only averaged values of observable physical quantities yield in the microcosm an approximate analogue of the second Newton law; this is the so-called Ehrenfest's theorem. The Ehrenfest's equation yields an averaged, rather than precise, ratio between the second time derivative of coordinate and the force, while to describe the scatter of quantum observables, the probability theory apparatus is required. Since Newtonian dynamics is restricted to the second-order derivatives, while microobjects must be described by equations with additional variables, making the Planck's constant tend to zero corresponds to neglecting these variables. Hence, offering the model of extended Newtonian dynamics, we consider the classical and quantum theories with additional variables, describing the body dynamics with higher-order differential equations. In our model, the Lagrangian is considered to be dependent not only on coordinates and their first time derivatives, but also on higher-order time derivatives of coordinates. Classical dynamics of a test particle's motion with higher-order time derivatives of the coordinates was first described in 1850 by Ostrogradski [3] and is known as Ostrogradski's Formalism. Being a mathematician, Ostrogradski considered the coordinate systems rather than the reference frames. This is just the case corresponding to a real reference frame comprising both inertial and non-inertial reference frames. In the general case, the Lagrangian takes on the form

$$L = L(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...). \tag{1}$$

Attempts to build a unified theory for both quantum and classical mechanics, exemplified by the Hybrid theory of classical and quantum mechanics [4], are natural and make sense. However, constructing a theory without an axiomatic consistency with both theories resembles a construction without foundation. The point is that the systems of axioms of classical and quantum theories are mutually incompatible and even contradictory. Obviously, the mathematical constructions of united theory may prove questionable without presenting a framework which is conceptually consistent both with classical and quantum mechanics. For example, a natural question arises: "Can the phase space be used, and can the momentum and the coordinate exist simultaneously in quantum mechanics?" The Heisenberg uncertainty principle tells us that it is not possible.

At the same time, quantum theory describes objects in Hilbert space, i.e. in terms of an infinite number of variables, and thus it gives a more detailed description as compared to classical theory. Thus, the question of quantum-mechanical description being incomplete should be answered so that any theory as a model of physical reality is incomplete following Godel's theorem.

The classical description of physical reality contains an incomparably fewer number of variables. This raises the question: "How the classical description can be completed?" While a possibility of supplementing the quantum mechanical description with additional ("hidden") variables has been debated for long, the question as to how to complete the classical description to make it compatible with the quantum mechanical one has not received a due attention.

The classical and quantum theory could be based on the following common axioms:

1. Any reference frame is subject to random external influences. Hence, every reference frame is individual and a transition from one to another reference frame may lead to jump like changes. The notion of the inertial frame in classical mechanics is valid only on the average and, correspondingly, the Galilean relativity is an average notion as well. Then there are many trajectories of a particle corresponding to different reference frames; the Heisenberg uncertainty can be understood as a consequence of the nonexistence of ideal inertial frames; and the Ehrenfest theorem can be seen as a consequence of the inertial frame being an average notion.

Correspondingly, *the free body preserves the same order of its time derivative as the constant kinematic characteristics of the class of reference frames*, e.g. in the uniformly accelerating reference frame the free body preserves its acceleration.

2. In Hybrid theory [4] the generalized Ehrenfest relations for the QM observables are defined by coordinates average. Compare the averaging procedure in this paper with the time averaging. Within the above framework were the ideal inertial frames are non-existent, we can consider the averaging of the classical equations of motion over the time interval Δt:

$$-\frac{\partial U}{\partial q} = \frac{d}{dt}\frac{[p(t+\Delta t)+p(t-\Delta t)]}{2}.$$

Using the Taylor expansion

$$p(t \pm \Delta t) = p(t) \pm \dot{p}\Delta t + \frac{1}{2!}\ddot{p}(t)\Delta t^2 + ... + \frac{(-1)^n}{n!}\overset{\cdot(n)}{p}(t)\Delta t^n + ..$$

the function $F = -\frac{\partial U}{\partial q}$ can be expanded as follows:

$$F(q.\dot{q}, \ddot{q}, \dddot{q}..., \overset{\cdot(k)}{q}) = \dot{p}(t) + \frac{1}{2!}\overset{\cdot(3)}{p}(t)\Delta t^2 + \frac{1}{4!}\overset{\cdot(5)}{p}(t)\Delta t^4 + ...,$$

where $\overset{\cdot(n)}{p}$ denotes n-th time derivative of momentum p. *It is the Extended Law of Dynamics in arbitrary reference frames* including the case of the vibration non-inertial reference frames. Correspondingly, the free body preserves the same order of its time derivative like the constant kinematic characteristics of the reference frames. For example, in the uniformly accelerating reference frame the free body preserves its acceleration.

3. The de-Broglie waves $\psi = \psi_0 \exp(-iS/\hbar)$ with the actions functions $S = S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...)$ can be considered as having the gravity-inertial nature following from the fact that every *reference frame is vibrational due to the influence of random gravitational fields and waves* so that every free particle appears to be oscillating.

4. As the action function $S = S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...)$ is a convergent series in high derivatives of q the difference $S(q, \dot{q}, \ddot{q}, ..., \dot{q}^n, ...) - S(q, \dot{q}) = h$ is finite and can be identified with the constant h. Within the presented framework the variables of *the (high order extension of the) phase space do describe the completed dynamics of a particle*, but they cannot be measured because the ideal inertial reference frames do not exist in really. The infinite dimensionality of Hilbert space can also be understood as a consequence of all high order time derivatives being taken into account in the description of the dynamics.

2. How to complete the quantum-mechanical description?

If the statement by Einstein, Podolsky and Rosen on incompleteness of Quantum-Mechanical description of nature is correct, then we can regard Quantum Mechanics as a Method of Indirect Computation. The problem is, whether the theory is incomplete or the nature itself does not allow complete description? And if the first option is correct, how is it possible to complete the Quantum-Mechanical description? Here we try to complement de-Broglie's idea on wave-pilot the stochastic inertial-gravitation gives origin to. We assume that de-Broglie's wave-pilots are inertial-gravitational stochastic ones with the high derivatives, and we shall regard micro-objects as test classical particles being subject to the influence of de-Broglie's waves stochastic inertial-gravitation. The Quantum Theory exists for many decades. But is everything OK with it completeness[5]? To our opinion, it is not just so. The incompleteness of Quantum-Mechanical description gives rise to various paradoxes, such as Einstein-Podolsky-Rosen (EPR) one, the paradox of the Schrodinger's cat, the Paradox of Quantum Non-locality and Paradox of the Quantum Teleportation. In this study we shall call the phenomena of quantum nonlocal behavior and teleportation of the quantum states as paradoxes because they follow from Stochastic

Gravitation Model of Quantum Mechanics. It can be easily seen that these are paradoxes, and indeed they are brought about by the drawbacks in the Quantum Theory rather than being actual properties of nature. This is due to the fact that time in Quantum Theory plays the role not conforming to the physical reality. In particular, the Quantum Theory employs the concept of Hilbert Space, in which time acts as a parameter. Henceforth, this parameter (i.e. the time) may be the same in different points of the Hilbert Space. This property of time in the Hilbert Space brings about the effects of simultaneous quantum states of microobjects at different space points (or transfer of the state from one Hilbert Space point to another with velocities exceeding the velocity of light). These effects of the Quantum Theory that are apparently real we call here the Quantum Non-locality Paradox. The Paradox of Quantum Teleportation is a sort of Quantum Non-locality Paradox. These paradoxes do not exist in the Classical Physics and in the Stochastic Gravitation Model of the Quantum Mechanic, and General Relativity Theory (employing the 4-dimensional space), in which different points of time-space correspond to different values of the time. And another question is whether the quantum-mechanical wave-function interpretation of micro-objects is complete? Let us select harmonic coordinates (the condition of harmonicas of coordinates mean selection of concomitant frame $\frac{\partial h_n^m}{\partial x^m} = \frac{1}{2}\frac{\partial h_m^m}{\partial x^n}$) and let us take into consideration that $h_{\mu\nu}$ satisfies the gravitational field equations

$$\Box h_{mn}(j) = -16\pi G S_{mn}(j), \tag{2}$$

which follow from the General Theory of Relativity; here S_{mn} is energy-momentum tensor of gravitational field sources with d'Alemberian $\Box$ and gravity constant G. Then, the solution shall acquire the form

$$h_{\mu\nu}(j) = e_{\mu\nu}(j)\exp(ik_\gamma(j)x^\gamma) + e^*_{\mu\nu}(j)\exp(-ik_\gamma(j)x^\gamma), \tag{3}$$

where the value $h_{\mu\nu}(j)$ is called metric perturbation, $e_{\mu\nu}(j)$ polarization, and $k_\gamma(j)$ is 4-dimensional wave vector.

We shall assume that this metric perturbation $h_{\mu\nu}(j)$ is distributed in space with an unknown distribution function $\rho = \rho(h_{\mu\nu})$. Relative oscillations ℓ of two particles in classic gravitational fields are described in the General Theory of Relativity by deviation equations, which we can write for the stochastic case as

$$\frac{D^2}{D\tau^2}\ell^\mu(j) + R^\mu_{\nu mn}(j)\ell^m \frac{dx^\nu}{d\tau}\frac{dx^n}{d\tau} = F(j), \tag{4}$$

being $R^\mu_{\nu mn}(j)$ the gravitational field Riemann's tensor with gravitational field number j of the stochastic gravitational fields and $F(j)$ is the stochastic constant (for the non-stochastic case this constant is zero $F(j) = 0$).

Specifically, the deviation equations give the equations for two particles oscillations

$$\ddot{\ell}^1 + c^2 R^1_{010}\ell^1 = 0, \quad \omega = c\sqrt{R^1_{010}}. \tag{5}$$

The solution of this equation has the form

$$\ell^1(j) = \ell_0 \exp(k_a x^a + i\omega(j)t), \tag{6}$$

being $a = 1, 2, 3$. Each gravitational field or wave with index j and Riemann's tensor $R^\mu_{\nu mn}(j)$ shall be corresponding to the value $\ell^\mu(j)$ with stochastically modulated phase $\Phi(j) = \omega(j)t$. If we to sum the all fields, we can write the stochastic phase $\Phi(t) = \omega(t)t$, where t is the time coordinate.

3. Corrected Bell inequalities in random gravity-inertial fields

We shall consider the physical model with the Stochastic Inertial-Gravitational Background [i.e. with the background of inertial-gravitational fields and waves]. This means that we assume the non-inertial vibration reference frames due to existence of fluctuations in inertial-gravitational waves and fields expressed mathematically by metric fluctuations.

Describing entangled photons in the stochastic curved space, we shall take into consideration the fact that the scalar product of two 4-vectors A^μ and B^ν equals $g_{\mu\nu}A^\mu B^\nu$, where for weak inertial-gravitational fields one can use the value $h_{\mu\nu}$, which is the solution of Einstein's equations for the case of weak inertial-gravitational field in harmonic coordinates.

Correlation factor M of random variables λ^i [6] which correspondent to higher-order time derivatives of coordinates are projections onto directions A^ν and B^n defined by polarizers (all these vectors being unit) is

$$|M_{AB}| = |\langle AB\rangle| = \left|\langle \lambda^i A^k g_{ik} \lambda^m B^n g_{mn}\rangle\right|. \tag{7}$$

The differential geometry gives

$$\cos\phi = \frac{g_{ik}\lambda^i A^k}{\sqrt{\lambda^i\lambda_i}\sqrt{A^k A_k}},$$
$$\cos(\phi+\theta) = \frac{g_{mn}\lambda^m B^n}{\sqrt{\lambda^m\lambda_m}\sqrt{B^n B_n}}.$$

Here i,k,m,n possess 0,1,2,3; θ is angle between polarizers, then

$$|M_{AB}| = \left|\frac{1}{\pi}\int_0^\pi \rho(\phi)\cos\phi\cos(\phi+\theta)\,d\phi\right| = |\cos\theta|, \tag{8}$$

here the integral of the distribution function of the metric ρ is $\left|\frac{1}{\pi}\int_0^\pi \rho(\phi)d\phi\right| = 1$.
Finally, the real part of the correlation factor is

$$|M_{AB}| = |\cos\theta|.$$

Then, we obtain the maximum value the Bell's observable S in Rieman's space for $\theta = \frac{\pi}{4}$

$$|< S >| = |[\langle M_{AB}\rangle + \langle M_{A'B}\rangle + \langle M_{AB'}\rangle - \langle M_{A'B'}\rangle]| =$$
$$= \left|[\cos(-\tfrac{\pi}{4}) + \cos(\tfrac{\pi}{4}) + \cos(\tfrac{\pi}{4}) - \cos(\tfrac{3\pi}{4})]\right| = \left|\sqrt{2}\right|,$$

which agrees fairly with the experimental data. The Bell in equality in Rieman's space shall take on the form $|\langle S\rangle| \le \sqrt{2}$.

Therefore, we have shown that the Classical Physics with the Stochastic Inertial-Gravitational Background gives the value of the Bell's observable matching both the experimental data and the quantum mechanical value of the Bell's observable. To sum it up, the description of microobjects by the classical physics accounting for the effects brought about by the Inertial-Gravitational Background is equivalent to the Quantum-Mechanical descriptions, both agreeing with the experimental data.

4. Conclusions

We are regarding the inertial-gravitational background of isotopic fields and waves as hidden variables. The inertial-gravitational background could be considered negligible and not affecting the behaviour of quantum microobjects. We are verified whether this is correct. The quantitative assessments of the inertial-gravitational background influence on the quantum microobjects' behaviour have not been performed due to the former having never been examined. The quantum effects are small as well, but their quantitative limits are known and are determined by the Heisenberg inequality. We have demonstrated the inertial-gravitational background being random and isotropic to affect the phases of microobjects. Our case corresponds to the Lagrange function $L(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}, ...)$, depending on coordinates, velocities and higher time derivatives, which we call additional variables, extra addends, or hidden variables. In arbitrary reference systems (including non-inertial ones) additional variables (addends) appear in the form of higher time derivatives of coordinates, which complement both classical and quantum physics. We call these additional addends, or variables, constituting the higher time derivatives of coordinates, hidden variables or hidden parameters, complementing the description of particles. The contemporary physics presupposes employment of predominantly inertial reference systems; however, such a system is very hard to obtain, as there always exist external perturbative effects, for example, gravitational forces, fields, or waves. In this case, the relativity principle enables transfer from the gravitational forces or waves to inertial forces. If the fact that the real reference frames is non-inertial and hence there exist additional variables in the form of inertial-gravitation effects is ignored, then non-local correlation of quantum states and quantum non-locality would seem surprising. The inertial-gravitational origin of quantum-mechanical wave functions in the form of non-local hidden variables described [7-9].

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