

# Computation and modeling of Ionospheric Plasma Pressure for F<sub>2</sub> layer at Pakistan air space

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**Abstract:** This paper deals with the computation of Ionospheric plasma pressure for F<sub>2</sub> layer at Pakistan air space. The physical processes of plasmas have been studied extensively by now both in laboratory as well as in space environment as in the present study. This paper deals with the computation of Ionospheric plasma pressure for F<sub>2</sub> layer at Pakistan air space. Modeling plasma pressure has been a part in terms of instabilities in the ionosphere. We use stochastic approach to model and analyze the stability properties of plasma. These models are used to sustain our investigations of instabilities due to plasma pressure at Pakistan air space.

**Key words:** Plasma pressure, Plasma instabilities, Stochastic process Stability properties.

## 1. Introduction

Gauss 1839 speculated on the existence of electrical phenomena in the upper atmosphere, and Balfour Stewart in 1883 postulated a conducting layer as the site of the currents that cause the small daily variations of the earth's magnetic field. This conducting layer now called ionosphere to have been discovered by Marconi when he sent radio signals across the Atlantic in 1901. This ionosphere forms space plasma that could be defined as the quasi neutral gas of charged particles that can exhibit collective behavior and also follow the three criteria:

- (a)  $\lambda_D \ll L$
- (b)  $N_D \gg \gg 1$
- (c)  $\omega_i > 1$

It has been known that magnetohydrodynamics (MHD) explains the process that deals with the motion of a highly conducting fluid like in the present case ionospheric plasma due to presence of magnetic field. In formulating this description of plasma comprising the momentum equation, Debye shielding, mean free path or thermal gyro-radius etc. Our assumption that the time scales of interest are long compared to the microscopic particle motion and that of spatial scales with respect to Debye length and mean free path [1-5]

For a fluid like ionospheric plasma the particles are tied with the field lines. In the magnetic intensity  $B$  the magnetic stresses are equivalent to a tension  $B^2/4\pi$  and hydrostatic pressure  $B^2/8\pi$ . The magnetic pressure is balanced by the fluid pressure (plasma pressure) so that the lines of force behave as taut string under tension. The magnetic force in the form of

$\frac{1}{c} J \times B$  on the right hand side of the momentum equation and the momentum equation to



$$\rho \left( \frac{d}{d_t} + u \cdot \nabla \right) = - \nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} (B \cdot \nabla) B$$

It is evident from the above expression that  $\frac{B^2}{8\pi}$  indicates magnetic pressure and has dimension of kinetic

pressure. The ratio of plasma pressure to the magnetic field pressure is represented by the symbol  $\beta$  where  $\beta = p / B^2 / 8\pi$ . If  $\beta \ll 1$  then the plasma is cold and if  $\beta \gg 1$  then the plasma is considered as warm and the plasma currents may become important.

The plasma gas may be created, either by the natural ionization of the upper atmosphere at Pakistan air space. The Ionospheric plasma is coupled formation by the earth magnetic field. It is possible to obtain a state of equilibrium for plasma in presence of field, that interact with plasma, we have already shown that there can exist a simple equilibrium configuration in which plasma pressure is balanced by magnetic pressure. Now given perturbation to this system, it is important to know whether this equilibrium is stable or unstable. Numerical models are used to maintain our examine instabilities due to plasma pressure. The particles have maxwellian velocity distributions and the states are not perfectly in thermodynamic equilibrium. Albeit they are in the equilibrium in the sense that all the forces are in balance and a time-independent solution is possible. The free energy that is present can cause waves to be self-excited equilibrium then an unstable one. Thus an instability is always a motion that decreases the free energy and brings the plasma closer to true thermodynamic equilibrium. Instabilities are classified according to the kind of free energy available to derive them.

## 2. Plasma Pressure

We have known the plasma instability in which the effect of plasma pressure has been considered. We considered warm plasma the elastic waves exist, where a plasma gradient can build up where plasma pressure is considerable.

The plasma pressure is product of some constant, plasma concentration and temperature

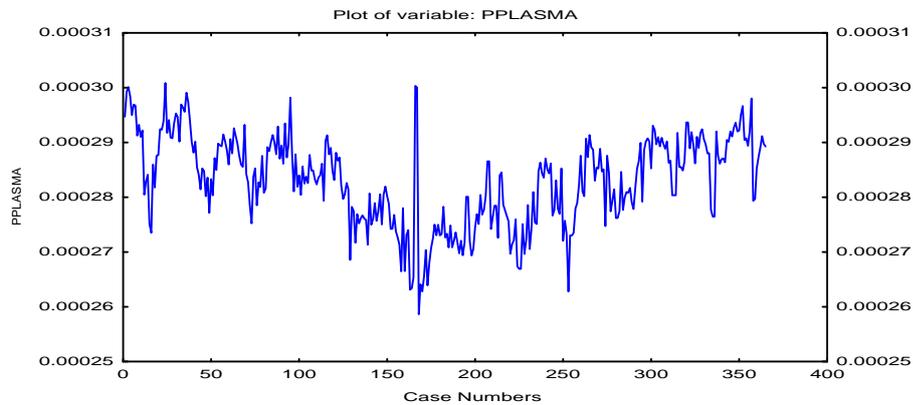
$$p = \gamma nKT$$

Where  $\gamma = 5/3$  for long wave, the distance traversed by an electron during one wave period much smaller than the wavelength and thus the effect of temperature perturbations will be negligible. Plasma pressure can not be ignored. The plasma gradient term in the momentum equation has to be included. Even where there are no obvious drive force such an electric or a gravitational field, plasma is not in perfect thermodynamics equilibrium as long as it is confined. The plasma pressure tends to make the plasma expand and the expansion energy can drive instability. This type of free energy is always present in any finite plasma, and the resulting waves are called universal instability.[6-7]

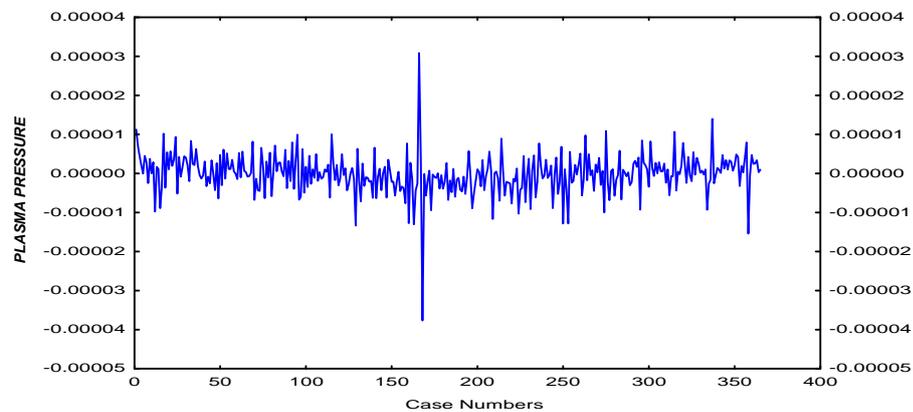
## 3. Methodology

Our purpose of this study to construct a mathematical model and predict plasma pressure at upper atmosphere. The technique that has become the standard for estimating AR models, was proposed by making successive approximations through various stages, Identification, estimation, and diagnostic checking. It can be used for forecasting for the period the process for developing this model begins by examining whether the time series under study is stationary non-stationary. [8-9]

The data collected from SUPARCO for virtual height in km and critical frequency in MHz. Figure 1 shows time plot of the plasma pressure for Ionospheric layers for Pakistan air space. The residual analysis has been done and plot is shown in figure 2.



**Figure 1.** The temporal plot for plasma pressure of Ionospheric F<sub>2</sub> layer



**Figure 2.** Residual analysis for plasma pressure

#### 4. Test of Hypothesis

Mean of plasma pressure of Ionospheric layer is 0.000283. the standard deviation is known to be 0.000008 a random sample of month of Jan 30 days plasma pressure points are taken and the mean of plasma pressure is computed as 0.000291 and level of significance value  $\alpha = 0.05$  then test of hypothesis is accept  $H_0$  or reject  $H_1$ .

$$H_0: \mu \leq 0.000283 \quad (\mu = 0.000283)$$

$$H_1: \mu > 0.000283$$

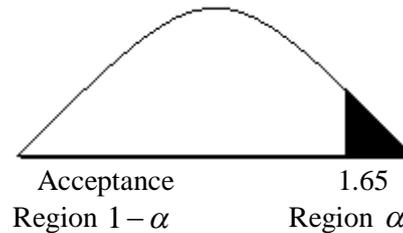
$$\alpha = 0.05$$

Test of statistic since standard deviation of population  $\sigma = 0.000008$  known, therefore Z-statistics is suitable to use.

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}, n=30, \bar{X} = 0.000291, \sigma = 0.000008 \text{ and } \mu = 0.000283$$

$$Z = \frac{0.000291 - 0.000283}{\frac{0.000008}{\sqrt{30}}}$$

$Z_{\text{Calculated Value}} = 5.477$ , critical region at  $\alpha = 0.05$  for one tailed test is  $Z > 1.65$



**Figure 1:** Right tailed test  $H_1 : \mu > \mu_0$

Conclusion since the calculated value of Z falls in critical region, therefore we reject null hypothesis  $H_0$

$$|Z_{\text{Calculated Value}}| = 5.477 \text{ and } |Z_{\text{Tabulated Value}}| = 1.65$$

$$|Z_{\text{Calculated Value}}| > |Z_{\text{Tabulated Value}}|, \text{ then we reject } H_0$$

### 5. Model Fitting and diagnostics

We have used the computed plasma pressure for ionospheric  $F_2$  layer data that has been explained in last portion and fitted model AR(1) is suitable model whose parametric values are shown in table 1 and figure.2 that manifests the residual plot for AR(1) model as given below:

$$Y_t = 0.77 Y_{t-1} + 6.4 \times 10^{-5} + e_t \quad (1)$$

Table: 1 Model parametric values for AR (1) model of plasma pressure Ionospheric  $F_2$  layer

| Variable         | Constant | P(1)     |
|------------------|----------|----------|
| Coefficient      | 0.000283 | 0.773277 |
| Standard Error   | 0.000001 | 0.033709 |
| t-statistics     | 239.7984 | 22.9398  |
| Probability      | 0.00     | 0.00     |
| Lower 95 percent | 0.000281 | 0.706988 |
| Upper 95 percent | 0.000286 | 0.839566 |

## 6. Forecasting

Equation (1) shows prediction equation that is used to forecast series of plasma pressure values as shown in table 2 and the figure.3 manifests the forecasts values.

**Table 2:** Forecast values of plasma pressure for ionospheric F<sub>2</sub> layer

| No of forecasting | Forecast |
|-------------------|----------|
| 366               | 0.000288 |
| 367               | 0.000287 |
| 368               | 0.000286 |
| 369               | 0.000286 |
| 370               | 0.000285 |
| 371               | 0.000285 |
| 372               | 0.000284 |
| 373               | 0.000284 |
| 374               | 0.000284 |
| 375               | 0.000284 |

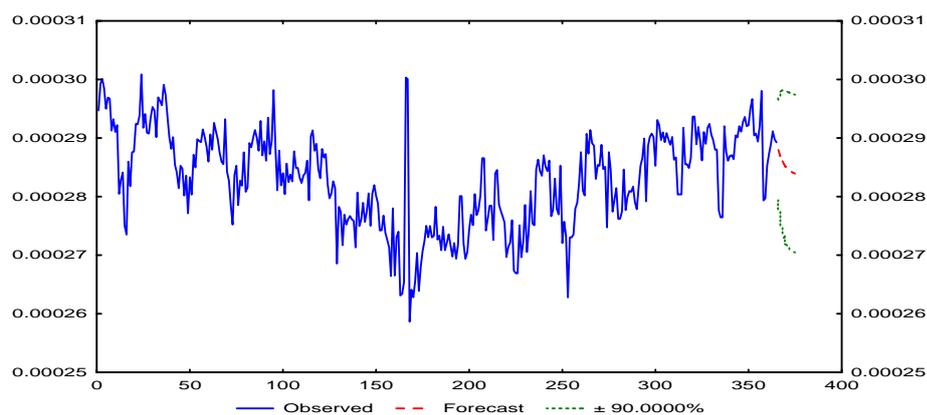
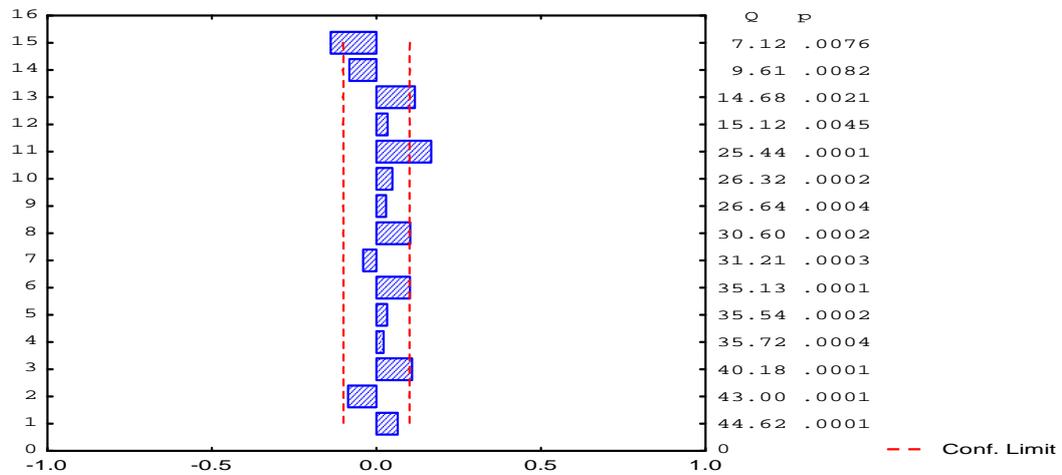


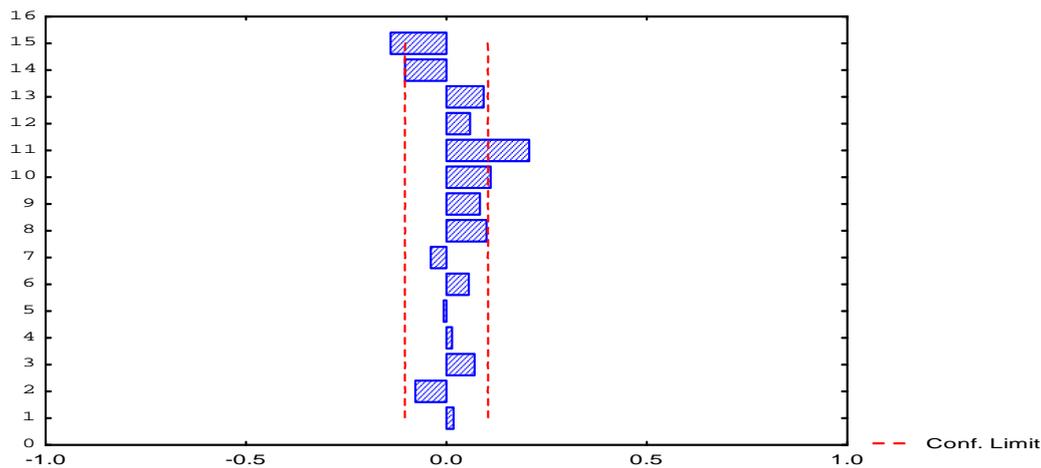
Figure 3. Forecasts of Ionospheric data showing random process plasma pressure ionospheric F<sub>2</sub> layer a first order AR

## 7. Validity of the constructed model

This model can be validated using the figures 4 and 5 that depict the autocorrelation function of the random process for the plasma pressure at Ionospheric F<sub>2</sub> layer and the partial autocorrelation function of the data of the same random process in the plasma pressure for ionospheric F<sub>2</sub> layer



**Figure 4.** autocorrelation function showing a random process the plasma pressure for Ionospheric F<sub>2</sub> layer a first order AR



**Figure 5.** The plot showing the partial autocorrelation coefficient of the data that exhibits a random process in the plasma pressure for ionospheric F<sub>2</sub> layer a first order AR

## 8. Conclusion

In this communication we have computed plasma pressure and explored an idea to quantify the plasma pressure and its instability in the ionospheric layers at this region. Forecast values have been obtained and more useful results were reported from the Analysis of autoregressive (AR) model of first order and the higher orders could also be verified based on the availability of data sets. The variation of the predicted values of plasma pressure depend on the ionospheric conditions

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