

## Summary talk – from a theoretical point of view

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**Abstract.** This summary is based on the theoretical talks presented at the Tenth International Conference on Clustering Aspects of Nuclear Structure and Dynamics, Debrecen, Hungary, 24–28 September, 2012.

### 1. Introduction

The concept of clustering in nuclei is already fairly old but still frequently employed in studies on both nuclear structure and reactions. The basic idea of clustering is that a group of nucleons forms a spatially localized substructure and behaves as an entity rather than individual nucleons. The clustering assumption leads to simplicity of the description of complicated nuclear phenomena. Its physical foundation is primarily based on the saturation property of the nucleus though its microscopic foundation is a challenge even today.

Many of the plenary session titles of the Debrecen conference are the same as the keywords of the previous cluster conferences. They are more or less traditional subjects, but compared, e.g., to the Kyoto conference in 1988 [1], some new titles appear, such as *Few-body and ab initio approaches*, *Structure and reactions of exotic nuclei* and *Clustering in nuclear matter*. These obviously reflect a current trend in the nuclear physics community.

Some basic questions addressed include where do we expect cluster states, what about the coupling of cluster states to other degrees of freedom, how are cluster states formed in reactions, what role do cluster states play in reactions and fission, and what about a microscopic foundation of clustering with the use of realistic nuclear forces.

In his introductory talk on recent developments in cluster physics Horiuchi [2] has referred to four works, viz. on neutron-rich nuclei, cluster gas phase, coexistence of cluster and mean-field dynamics, and *ab initio* approaches.

### 2. Cluster structure in light nuclei

#### 2.1. Alpha and exotic clustering

Alpha clustering plays a central role in cluster physics. As suggested by the Ikeda diagram [3], some cluster states may appear near the threshold for  $\alpha$  breakup.

Using an orthogonality condition model of  $4\alpha$  for  $^{16}\text{O}$ , Funaki has reported the existence of six  $0^+$  states [4], which are understood to arise from the excitation of  $^{12}\text{C}$  and of the  $^{12}\text{C}-\alpha$  relative motion. In particular, the sixth state, which appears at 2 MeV above the  $4\alpha$  threshold is indicated to have a gas-like structure, which is a candidate for the observed  $0^+$  level at 15.1



MeV with a decay width of 170 keV. The small width has been explained by a small overlap of that special structure with the decaying-channel wave functions.

Motivated by the finding of resonances that preferentially decay to  ${}^6\text{He}$  and  ${}^8\text{He}$ , Ito has studied exotic cluster structure in Be isotopes using the two-centre cluster model of  $\alpha + \alpha + xn$  [5]. Besides the two bound  $0^+$  states, two  $0^+$  resonances above the  ${}^6\text{He} + {}^8\text{He}$  threshold were obtained in  ${}^{14}\text{Be}$  by solving a scattering problem that takes into account the coupling to the continuum.

Since the  $\alpha$  cluster is a composite particle consisting of two neutrons and two protons, it is subject to the Pauli principle in a microscopic cluster model. Its effect is especially important when the  $\alpha$  cluster interacts with the other nucleons of the system at short distances. A calculation with a fully microscopic cluster model becomes increasingly hard when more than two clusters are employed though such calculations usually produce rewarding results; see, for example, the multi-cluster calculations for  ${}^{9,10,11}\text{Li}$  [6]. Therefore, an approximate version has often been employed for many years.

In a Faddeev formalism Dufour has treated a  $3\alpha$  model for  ${}^{12}\text{C}$  using local and non-local  $\alpha$ - $\alpha$  potentials, and has shown that cluster-cluster potentials of the latter type appear to work better if the effect of the Pauli principle is taken into account [7]. It is worthwhile recalling that a non-local  $\alpha$ - $\alpha$  potential that renormalizes the energy-dependent kernel of the resonating group method has already been constructed and found to lead to a fair description of both the ground and the Hoyle state of  ${}^{12}\text{C}$  [8].

A semi-microscopic cluster model has been considered by Hess introducing  $\sigma$  and  $\pi$  bosons, and its geometry and phase transition have been discussed [9].

The techniques and lessons of cluster physics can be applied to hypernuclear physics as well. Since the hyperon-nucleon and hyperon-hyperon interactions are only poorly known, a theoretical analysis of light hypernuclei is expected to play an important role. A review talk on light  $\Lambda$  hypernuclei has been given by Hiyama [10]. Some examples of four- and five-body calculations have been presented and the glue-like nature of  $\Lambda$  has been stressed.

## 2.2. Molecular states and deformed states

The  ${}^{12}\text{C} + {}^{12}\text{C}$  system is a well-known classical example that exhibits a number of narrow resonances near the Coulomb barrier [11]. The appearance of the narrow resonances is explained by a coupling to the inelastic channel involving the  $2^+$  state of  ${}^{12}\text{C}$  as well as to other reaction channels, e.g.,  ${}^8\text{Be} + {}^{16}\text{O}$  and  $\alpha + {}^{20}\text{Ne}$ .

It is interesting that the  ${}^{24}\text{Mg} + {}^{24}\text{Mg}$  and  ${}^{28}\text{Si} + {}^{28}\text{Si}$  systems present even narrower high spin resonances well above the Coulomb barrier. These examples indicate that a di-nuclear molecule can be formed provided the entrance channel satisfies a specific condition on the energy and the angular momentum. Uegaki has discussed a mechanism of the resonances in these two cases by analysing the normal modes of two deformed molecules (a prolate  ${}^{24}\text{Mg}$  and an oblate  ${}^{28}\text{Si}$ ) around some equilibrium [12]. He has taken into account the wobbling motion perpendicular to the beam direction. The  $\gamma$ -ray yields from the resonance of the  ${}^{28}\text{Si} + {}^{28}\text{Si}$  case at 55.8 MeV were successfully accounted for.

The possibility of producing strongly deformed (hyperdeformed) states near the Coulomb barrier together with the cooling by neutron emission has been discussed by Adamian [13] for several combinations of isotopes such as  $\text{Ca} + \text{Sn}$ ,  $\text{Ca} + \text{Xe}$ ,  $\text{Ca} + \text{Ba}$  etc.

The approaches mentioned above describe nuclear deformation in terms of geometrical shape parameters. In contrast, a microscopic approach has been undertaken by Taniguchi [14] to study possible largely deformed states in the  $A \sim 40$  region. In particular, some excited states of  ${}^{42}\text{Ca}$  have been described in deformed-basis Gaussian wave packets (antisymmetrized molecular dynamics) together with a mixing of  $\alpha + {}^{38}\text{Ar}$  cluster configurations. Since various deformed states may appear more or less at the same energy, a generator coordinate method has been employed to take into account the coexistence of the rotational bands, which can be tested in

a comparison to experimental data on electric quadrupole transitions. It is desirable that this type of microscopic approach may be further extended to an analysis of molecular resonance phenomena.

### 3. Coupling of cluster states to other degrees of freedom

The clusters are usually frozen to their model ground states, and only their relative motion needs to be determined carefully. Furthermore, the intrinsic wave functions of the constituent clusters are quite often assumed to be the simplest possible shell model wave functions. This is implicitly related to the fact that the nucleon-nucleon potential used in the cluster model is more or less phenomenological, and, for example, contains no tensor component. This appears to be reasonable because otherwise cluster model calculations become extremely hard if the number of nucleons is not very small. Under these conditions it is possible to study the effects of the coupling of the cluster states to structures of different types.

Core excitations in exotic nuclei have been studied by Descouvemont [15]. He showed that there are several states in  $^{16}\text{B}$  that cannot be reproduced in a single-channel cluster model of  $^{15}\text{B}+n$  but the inclusion of many excited states of  $^{15}\text{B}$  can improve the description significantly. He has also shown that the core excitation has a crucially important role in explaining the  $3p$  emission from  $^{17}\text{Na}$ .

Kimura has discussed nuclear structure in the island of inversion with  $N \approx 20$  in antisymmetrized molecular dynamics calculations with deformed Gaussian wave packets [16]. Using a variety of deformed configurations, he has stressed the importance of cluster correlation near the neutron drip line, particularly in the description of the halo property of  $^{31}\text{Ne}$  and  $^{37}\text{Mg}$ , which is indicated by recent experimental findings of the enhanced reaction cross sections [17].

An approach similar to the above has been employed by Kanada-En'yo [18] to discuss cluster correlation in lighter nuclei, such as Be and C isotopes. She has pointed out that the correlation can be different, depending on whether the system of interest has a normal or dilute density.

It is well-known that, owing to the Pauli principle, some harmonic oscillator shell-model configurations are expressible in terms of very few cluster-model wave functions. As one of the simplest examples, we recall that the closed-shell configuration of  $^{16}\text{O}$  can be equivalently rewritten as an  $\alpha+^{12}\text{C}$  cluster wave function. Based on this fact, most of the levels of  $^{16}\text{O}$  were accounted for in the unified framework of the  $\alpha+^{12}\text{C}$  cluster model [19]. Yamada [20] has discussed the isoscalar monopole excitation of  $^{16}\text{O}$  in a  $4\alpha$  cluster model, and obtained considerable strength below 16 MeV excitation energy. The monopole operator can excite not only the cluster relative motion but also the cluster internal motion, so that the experimentally observed strength at higher excitation energy is attributed to the latter type.

The monopole operator belongs to the generators of the symplectic group  $Sp(6, R)$  together with the quadrupole operator [21, 22]. Though the  $4\alpha$  model is more extensive than the  $\alpha+^{12}\text{C}$  cluster model, Yamada's result appears to be consistent with the old cluster-symplectic mixed basis calculation [23].

An ambitious approach to the no-core shell model guided by the symplectic algebra (or more generally some suitable symmetry) has been reported by Draayer [24]. With an increasing number of  $N_{\text{max}}$  (the number of oscillator shells included), the energies of the second  $0^+$  and  $2^+$  states of  $^{12}\text{C}$  come down dramatically, closely approaching the observed energies. It will be interesting to analyse a mechanism of this behaviour and a connection with the cluster model, especially that with localized clustering.

It is of great importance to realize spatially localized clustering in microscopic theories that have no *a priori* cluster assumption. In this respect the description of the first excited  $0^+$  states of  $^{12}\text{C}$  and  $^{16}\text{O}$  are still challenging even with effective nuclear forces, let alone with realistic forces. Some interesting attempts to describe clustering have been reported. One of such approaches is the description of  $^{12}\text{C}$  by Fukuoka, who superposed a number of Slater determinants consisting

of Gaussian wave packets chosen in a stochastic way [25]. Another work of this type is a five-body calculation of  $p + p + n + n + {}^{12}\text{C}$  presented by Horiuchi [26]. He uses correlated Gaussians whose parameters are chosen by a stochastic procedure and their coefficients are determined variationally. He has demonstrated that the experimental energies of both the ground and the first excited  $0^+$  state of  ${}^{16}\text{O}$  can be reproduced in this model almost perfectly.

#### 4. Clustering in reactions

Nuclei far from stability have been explored since the advent of research with the use of radioactive nuclear beams. Since they are studied through scattering and reactions, sound reaction theories are needed to extract structure information. Especially breakup effects are known to be quite important in scattering and reactions induced by weakly bound nuclei.

Capel has compared various reaction models like continuum-discretized coupled channels, time-dependent approach and eikonal approximations for a projectile of a core plus a nucleon [27]. He has found that the ratio of two angular distributions, breakup and elastic, is insensitive to the reaction mechanism, which may enable one to get structure information.

In a multi-cluster model Katō has presented several results including resonances of multi-particle systems,  $\alpha + d$  scattering and Coulomb breakup reactions of  ${}^6\text{He}$ , and, furthermore,  $3\alpha$  and  $4\alpha$   $0^+$  states [28]. The relevant continuum states have been treated in a unified way with the use of the complex scaling method.

In nuclear reactions at intermediate energies hot nuclear matter is formed and subsequently decays into a number of fragment combinations. This reaction of nuclear multifragmentation has been discussed in two different approaches. Ono has described the fragmentation in the antisymmetrized molecular dynamics on Gaussian wave packets with the Skyrme interaction [29]. With the inclusion of the nucleon-nucleon collision, the multifragmentation data have been reproduced very well. The symmetry energy of the asymmetric nuclear matter has also been discussed. Raduța has treated the multifragmentation in a statistical model with cluster degrees of freedom included, and discussed the finite-size effects on the liquid-gas phase transition [30].

#### 5. Cluster decay in heavy nuclei and fission dynamics

It is an intriguing question what evidence we have for clustering in heavy nuclei. The enhancement of  $\alpha$ -decay rate is a well-known fact. For example, the theoretical understanding of the  $\alpha$  decay of  ${}^{212}\text{Po}$  has a long history. A most probable explanation so far has been achieved by admixing an  $\alpha$ -cluster configuration explicitly near the  ${}^{208}\text{Pb}$  surface to the shell-model configurations [31]. The recent observation of enhanced  $E1$  transitions in  ${}^{212}\text{Po}$  may also indicate the importance of  $\alpha$  clustering in heavy nuclei [32].

The  $\alpha$  decay of  ${}^{212}\text{Po}$  and  ${}^{104}\text{Te}$  has been studied by Id Betan in the framework of a complex-energy shell model [33]. The idea was to take into account the continuum components to improve the tail behaviour of the  $\alpha$ -formation amplitude.

In the region of the even larger masses of superheavy nuclei,  $\alpha$  decay may compete with heavy-particle radioactivity. The current status of comprehensive and detailed calculations for the competition in the region of  $Z=104$ – $124$  has been reported by Poenaru. He treats radioactivity in a fission-type model [34]. He has stressed that further improved accurate atomic mass models are needed to make a reliable prediction for the half lives.

Nuclear fission has been known for many years but still offers interesting observations. The recent experiment that  ${}^{180}\text{Hg}$  shows an asymmetric mass distribution of fission fragments rather than a symmetric distribution peaked at  ${}^{90}_{40}\text{Zr}$  has been explained by Antonenko [35] by comparing the potential energy surfaces calculated for different fragment pairs in a scission-point model.

A quantum mechanical description of fission, especially the scission point, has not been realized yet. Schunck talked about an attempt toward the understanding of the dynamics of

fission in the density functional theory approach [36]. In particular, he has discussed the fission barrier and the temperature of the fission fragments for the neutron-induced fission of  $^{240}\text{Pu}$ .

## 6. Clustering in nuclear matter

Schuck has first summarized the study of  $\alpha$  condensation in light nuclei and then discussed the  $\alpha$  condensation or quartetting in infinite nuclear matter as well as the critical temperature for  $\alpha$  and deuteron condensation [37].

Usmani has discussed the binding energy per nucleon of nuclear matter in the limit of zero density and its connection with the clustering in the surface region [38].

## 7. Few-body and *ab initio* approaches

Few-body systems offer unique opportunities to test the interaction between the nucleons and theoretical methods of solving bound and continuum problems etc. Among others, *ab initio* calculations with realistic nuclear forces are appreciated very much because of their high predictive power. It should be recalled that some theoretical tools such as the resonating group method, the generator coordinate method and *R*-matrix method are tailored to nuclear cluster physics and are frequently employed as standard machinery.

As is well-known, realistic nuclear forces are characterized by strong repulsion at short distances and strong long-ranged tensor components. The latter mixes *S* and *D* waves, so that, for example, the  $\alpha$  particle contains more than 10% *D* states [39]. It is particularly interesting and important to clarify whether or not we have deeper understanding of clustering phenomena starting from a realistic interaction.

Calculations with a direct use of the realistic potentials are obviously very hard. Arai has reported such type of calculations on *d*-induced reactions on *d* [40]. Compared to a pilot calculation using a tensor-force-free effective potential, he has convincingly shown that the tensor force plays an essential role in reproducing the astrophysical *S*-factors of reactions,  $d(d, \gamma)^4\text{He}$ ,  $d(d, p)^3\text{H}$  and  $d(d, n)^3\text{He}$ , at  $E \leq 0.1$  MeV. This is attributed to the fact that two deuterons initially approaching each other in the *S*-wave go to the final state dominantly through the *D*-wave.

Employing a realistic low-momentum interaction generated from the unitary correlation operator method, Neff has reported calculations for cluster states of  $^{12}\text{C}$  and for the  $^3\text{He}(\alpha, \gamma)^7\text{Be}$  radiative capture reaction [41]. He superposed many basis states of Gaussian wave packets according to fermionic molecular dynamics, and has succeeded in obtaining results that agree very well with the relevant experiment. It is interesting to note, however, that his calculation on  $^3\text{H}(\alpha, \gamma)^7\text{Li}$  gives slightly larger *S*-factor than experiment.

Fujiwara has presented *n*-*d* scattering cross sections calculated in a Faddeev formalism [42]. The potential used by him is generated from the quark cluster model. It is nonlocal and reproduces the nucleon-nucleon scattering phase shifts fairly well.

## 8. Conclusion

It is natural that I could not cover all the theory talks presented at this conference, mainly because some of them are beyond my expertise and partly because many of them were delivered in parallel sessions. I apologize even to those speakers whose talks I have covered for the possible incompleteness of my understanding, interpretation and appreciation of their work.

A number of talks have demonstrated the importance of clusters in both nuclear structure and reactions. I am especially impressed with the talks that have exhibited the vital role of clusters and a clear link between the cluster concept and the nuclear force.

It is true that cluster physics has come a long way since the first cluster conference in 1969. Having listened to the many talks delivered at the conference, however, I realize that there are still many intriguing and unsolved problems that will keep us busy.

If my observation is right, no talks from Debrecen or Hungary were presented at the conference despite the fact that a centre for nuclear science, Atomki, has been producing a number of works relevant to cluster physics. I missed some of them, but hope to remember the pleasant Debrecen atmosphere.

## Acknowledgments

I thank the local organizing committee for sending me those manuscripts which were submitted to this Proceedings early enough. I am indebted to the bilateral collaboration project (No. 119), 2011.4–2013.3, between the Japan Society for the Promotion of Science and the Hungarian Academy of Sciences for its support.

## References

- [1] Ikeda K, Katori K and Suzuki Y (eds) 1989 Proc. Fifth Int. Conf. on Clustering Aspects in Nuclear and Subnuclear Systems, Kyoto, 1988 *J. Phys. Soc. Jpn. Suppl.* **58**
- [2] Horiuchi H 2012 Debrecen conference
- [3] Ikeda K, Takigawa N and Horiuchi H 1968 *Prog. Theor. Phys. Suppl.* **Extra Number** 464
- [4] Funaki Y *et al* 2012 Debrecen conference
- [5] Ito M 2012 Debrecen conference
- [6] Varga K, Suzuki Y and Lovas R G 2002 *Phys. Rev. C* **66** 041302(R)
- [7] Dufour M 2012 Debrecen conference
- [8] Suzuki Y, Matsumura H, Orabi M, Fujiwara Y, Descouvemont P, Theeten M and Baye D 2008 *Phys. Lett. B* **659** 160
- [9] Hess P O 2012 Debrecen conference
- [10] Hiyama E 2012 Debrecen conference
- [11] Bromley D A, Kuehner J A and Almqvist E 1960 *Phys. Rev. Lett.* **4** 515
- [12] Uegaki E 2012 Debrecen conference
- [13] Adamian G G 2012 Debrecen conference
- [14] Taniguchi Y 2012 Debrecen conference
- [15] Descouvemont P 2012 Debrecen conference
- [16] Kimura M 2012 Debrecen conference
- [17] Takechi M *et al* 2010 *Mod. Phys. Lett. A* **25** 1878
- [18] Kanada-En'yo Y 2012 Debrecen conference
- [19] Suzuki Y 1976 *Prog. Theor. Phys.* **55** 1751; *Prog. Theor. Phys.* **56** 111
- [20] Yamada T 2012 Debrecen conference
- [21] Rosensteel G and Rowe D J 1977 *Phys. Rev. Lett.* **38** 10
- [22] Rowe D J 1985 *Rep. Prog. Phys.* **48** 1419
- [23] Suzuki Y 1989 *J. Phys. Soc. Jpn. Suppl.* **58** 129; Suzuki Y and Hara S 1989 *Phys. Rev. C* **39** 658
- [24] Draayer J P 2012 Debrecen conference
- [25] Fukuoka Y 2012 Debrecen conference
- [26] Horiuchi W 2012 Debrecen conference
- [27] Capel P 2012 Debrecen conference
- [28] Katō K 2012 Debrecen conference
- [29] Ono A 2012 Debrecen conference
- [30] Raduța A R 2012 Debrecen conference
- [31] Varga K, Lovas R G and Liotta R J 1992 *Nucl. Phys. A* **550** 421
- [32] Astier A, Petkov P, Porquet M G, Delin D S and Schuck P 2010 *Phys. Rev. Lett.* **104** 042701
- [33] Id Betan R 2012 Debrecen conference
- [34] Poenaru D N 2012 Debrecen conference
- [35] Antonenko N V 2012 Debrecen conference
- [36] Schunck N 2012 Debrecen conference
- [37] Schuck P 2012 Debrecen conference
- [38] Usmani Q N 2012 Debrecen conference
- [39] Kamada H *et al* 2001 *Phys. Rev. C* **64** 044001
- [40] Arai K 2012 Debrecen conference
- [41] Neff T 2012 Debrecen conference
- [42] Fujiwara Y 2012 Debrecen conference