Four- and five-body cluster structure of hypernuclei

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Abstract. We review interesting phenomena of hypernuclear structure from view points of clustering physics. The glue like role of Λ particle provide with size contraction of hypernuclei. For an example, we review the structure of $^{7}_{\Lambda}$Li, $^{13}_{\Lambda}$C. As an example, the energy gain by the addition of a Λ particle is dependent on the shell-like states or clustering states in core nuclei, we review the structure of ^{12}C and $^{13}_{\Lambda}$C. We show a very preliminary result of $^{6}_{\Lambda}$H within the framework of ^{5}H $+ n + n + \Lambda$ four-body cluster model to demonstrate the stabilization of the state due to the attraction of ΛN interaction. Finally, we review the structure of double Λ hypernucleus, $^{11}_{\Lambda\Lambda}$Be with the $\alpha\alpha + \Lambda + \Lambda + n$ five-body cluster model.

1. Introduction

One of the purpose of hypernuclear physics is to study the new dynamical features induced by the Λ particle. For example, in light p-shell hypernuclei, the remarkable role of the Λ particle has been pointed out in Refs. [1]: the Λ participation gives rise to more bound states and the appreciable contraction of the system (this stabilization is called the 'gluelike' role of the Λ). In light nuclei near the neutron drip line, interesting phenomena concerning the neutron halo have been observed. If a Λ particle is added to a very weakly bound halo nucleus the resultant hypernucleus will become substantially stable against the neutron decay. Thanks to the gluelike role of the Λ particle, there is a new chance to produce a hypernuclear neutron (proton) halo state if the core nucleus has a weakly unbound (resonant) state with an appropriate energy above the particle decay threshold. In this paper, we show two examples such as $^{7}_{\Lambda}$He and $^{6}_{\Lambda}$H within the framework of $\alpha + \Lambda + N + N$ and $t + \Lambda + n + n$ four-body cluster model. As mentioned above, it is considered that the gluelike of Λ produce shrinkage effect of the core nuclei. Now, we have a question that all nuclei are compressed or not. In order to answer it, we review the structure of $^{13}_{\Lambda}$C within the framework of $3\alpha + \Lambda$ four-body model. Third, we have interesting property of the injection of Λ particle as follows: When a Λ particle is added to core nuclei with $A \geq 11$, the energy gain by addition of a Λ particle is dependent on the clustering states or shell-like states. For a example, we show the energy spectra of $^{13}_{\Lambda}$C. Another goal in hypernuclear physics is to understand the baryon-baryon interaction. The baryon-baryon interaction is fundamental and important for the study of nuclear physics. In order to understand the baryon-baryon interaction, two-body scattering experiments are the most useful tool. For this purpose, many NN scattering experiments have been done and the total number of NN data are more than 4,000. However, due to the difficulty of performing two-body hyperon(Y)-nucleon(N) and hyperon(Y)-hyperon(Y) scattering experiments, the total

number of YN scattering data are very limited. Namely, the number of differential cross section are only about 40 and there is no YY scattering data. Then, YN and YY potential models so far proposed have large ambiguity. Therefore, as a substitute for the two-body limited YN and non-existent YY scattering data, the systematic investigation of light hypernuclear structure is essential. In fact, at J-PARC facility, they are planning to produce many double Λ hyprnuclei by emulsion experiment. However, it is difficult to determine the spin and parity and whether the observed state is the ground state or an excited state. Then, it is necessary to compare the data with the theoretical calculation for the identification of the state. For successful examples, we review the clustering structure of $^{10}_{\Lambda\Lambda}$Be and $^{11}_{\Lambda\Lambda}$Be.

2. Shrinkage effect in $^{7}_{\Lambda}$Li and $^{13}_{\Lambda}$C.

When a Λ particle is injected into the light nucleus, it is considered that nuclear size is reduced due to the ΛN interaction. Historically, Motoba *et al.* pointed out that the shrinkage effect was seen in the value of $B(E2)$ [1]. In Ref.[2], we suggested to experimentalist to measure $B(E2; 5/2^+ \rightarrow 1/2^+)$ in $^{7}_{\Lambda}$Li ,and to propose a prescription to derive the size of the ground state of $^{7}_{\Lambda}$Li from that $B(E2)$ value with the aid of the empirical $B(E2; 3^+ \rightarrow 1^+)$ and the size of the ground state of ^{6}Li. Within the framework of $^{5}_{\Lambda}$He $+ N + N$ three-body model, we predicted that core nucleus ^{6}Li will be shrunk by 25 %. The first observation of the hypernuclear $B(E2)$ strength was recently performed in the KEK-E419 experiment for $B(E2; 5/2^+ \rightarrow 1/2^+)$ in $^{7}_{\Lambda}$Li; the observed $B(E2)$ value was 3.6 ± 0.5 e^{2}fm^4 [3] and they reported the size of $\bar{R}_{\text{core}-(np)}$ was shrunk by 19 ± 4 %, which is consistent with our prediction. This confirms, for the first time, the shrinkage of the nuclear size induced by the Λ particle.

One may ask the next question "Are all nuclei compress by the injection of a Λ particle?" Our answer is No. We predict that the nuclear density of the ground states in the stable nuclei with $A \leq 11$ is not compressed due to the addition of a Λ particle. On the other hand, the matter radius in some excited states are shrunk by as much as 30 % due to the injection of a Λ particle. This was precisely studied for ^{12}C and $^{13}_{\Lambda}$C within the framework of 3α and $3\alpha + \Lambda$ three- and four-body model [4].

It is well known that the 0^+_1 state in ^{12}C is shell-like ground state while the 0^+_2 state at $E_x = 7/65$ MeV is well-developed clustering state and that both states are simultaneously well described by the microscopic 3α cluster model. The calculated Λ separation energies of $1/2^+_1$ and $1/2^+_2$ states in $^{13}_{\Lambda}$C are 11.69 MeV and 8.59 MeV, respectively. The $1/2^+_1$ is dominantly composed of the $0^+_1(^{12}$C) and $0s$ Λ particle and the $1/2^+_2$ is composed of the $0^+_2(^{12}$C) and $0s$ Λ particle. We can expect to observe a very characteristic $E2$ transition from the second $1/2^+$ state to the spin-doublet $3/2^+$-$5/2^+$ states of the ground-state band. This transition corresponds, in the core nucleus ^{12}C, to the $E2$ transition which intervenes between the well-developed clustering state (0^+_2) and the shell-model dominating state (2^+_1). Calculation of the $B(E2)$ strength is in progress, but, referring to the size-shrinkage of the second $1/2^+$ state, we can speculate it to be about a half of that of the corresponding transition in ^{12}C.

We also calculated r.m.s. distance between two α clusters, $\bar{r}_{\alpha-\alpha}$ in the $^{13}_{\Lambda}$C and ^{12}C. The r.m.s. distance between two α clusters in the ground state of $^{13}_{\Lambda}$C is $\bar{r} = 2.87 - 2.90$fm, which represents little shrinkage from $\bar{r} = 3.02$ fm in the ground state of ^{12}C. This contraction is more vividly seen in Fig.1, which illustrates the probability density of finding two α at a distance $r_{\alpha-\alpha}$. The little change in $\bar{r}_{\alpha-\alpha}$ is also visualized in Fig. 1 by drawing two similar distributions of the probability to find two α clusters at a distance $\bar{r}_{\alpha-\alpha}$. This reasonably implies that shell-like states are not easily contracted when a Λ particle joins.

On the other hand, in the second $1/2^+$ state of $^{13}_{\Lambda}$C, we obtain $r_{\alpha-\alpha} = 4.46 - 4.58$ fm, and therefore a sizable contraction of the $\alpha - \alpha$ distance is seen as compared with $r_{\alpha-\alpha} = 6.25$ fm in

the 0_2^+ state of ^{12}C. This significant contraction is seen in Fig.1. It is to be stressed that these theoretical results on the size-contraction in the excited $1/2^+$ state is owing to the four-body frame which is suited for describing the response of the 3α system to the addition of the Λ particle.
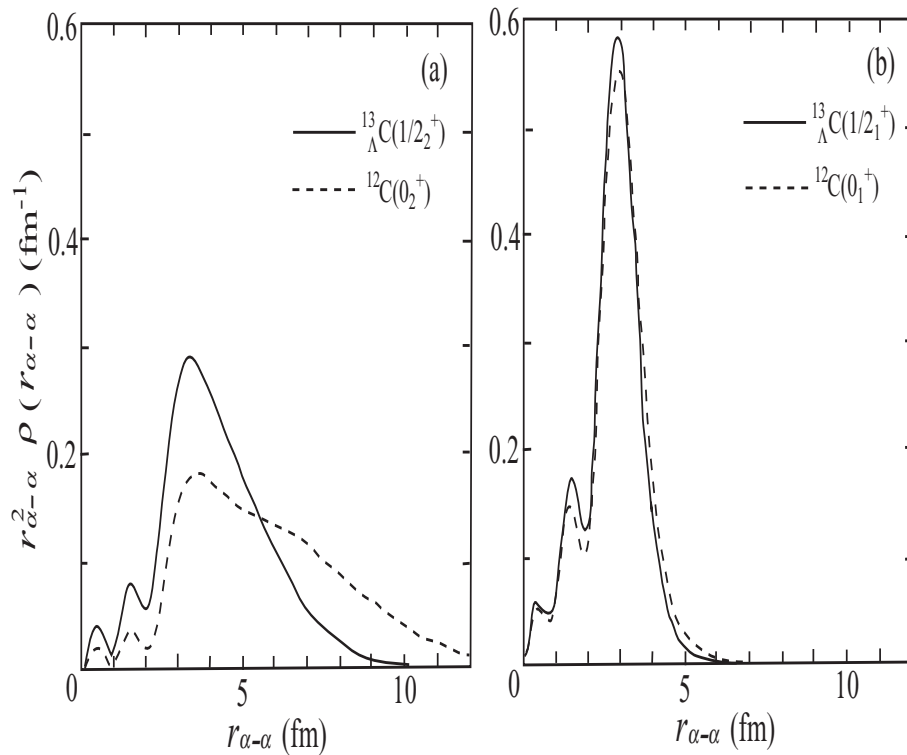

Figure 1. Probability densities of finding two α at a distance $r_{\alpha-\alpha}$ in the states (a) ^{12}C(0_2^+) and $^{13}_\Lambda$C($1/2_2^+$), and (b) ^{12}C(0_1^+) and $^{13}_\Lambda$C($1/2_1^+$). Shrinkage of the excited 0_2^+ in ^{12}C is drastic, but that in the ground state is little. This figure is taken from [4].

The characteristic feature of ^{12}C is the existence of cluster states, together with the ground band states $(0_1^+, 2_1^+, 4_1^+)$ with a shell-model-like nature. The 0_2^+ state at excitation energy, $E_x = 7.64$ MeV, called 'Hoyle state', has a loosely bound three α-cluster structure, for which a shell-model type of calculation would have great difficulties in reproducing its properties. In fact, the microscopic 3α cluster model succeeded in reproducing the Hoyle state, including the ground-band states and negative-parity states. Figure 1 shows the density distribution of the 0_1^+ and 0_2^+ states of ^{12}C calculated by the microscopic 3α cluster model. One can see a clear difference between the structures of the 0_1^+ and 0_2^+ states. On the other hand, the negative-parity states, 3^- and 1^-, are known to have intermediate characters between the shell-model compact structure and the loosely bound 3α structure. Therefore, it is a new issue in $^{13}_\Lambda$C *not* considered in $^9_\Lambda$Be what kinds of structures appear in addition of a Λ particle into ^{12}C with the three different types of structures, i.e. shell-model compact one, loosely bound 3α cluster one, and the intermediate one.

3. Neutron-rich Λ hypernucleus, $^{6}_{\Lambda}$H

Recently, a neutron-rich Λ hypernucleus, $^{6}_{\Lambda}$H has been observed as a bound state by FINUDA experiment [5]. They reported the observed B_Λ to be 4.0 ± 1.1 MeV. Before measurement, some authors [6–8] studied the structure of this hypernucleus by shell model and G-matrix theory.

Now, we have an important issue: we can reproduce the observed data or not. Motivating the experiment, we calculated the binding energy of $^{6}_{\Lambda}$H within the framework of $t + n + n + \Lambda$ four-body model and here report a very preliminary result.

In order to discuss the binding energy of $^{6}_{\Lambda}$H, it is important to reproduce the observed energy of the core nucleus, ^{5}H, $E = 1.7 \pm 0.3, \Gamma = 1.9 \pm 0.4$ MeV.

To study the core nucleus, ^{5}H, R. De Diego *et al.*, calculated the energy and width within the framework of $t + n + n$ three-body model [9] We employ the $t - n$ potential employed by Ref.[9]. However, it should be noticed that they employed partial dependent $t - n$ potential. It is difficult to employ this potential directly. Then, here we employ parity dependent potential starting from $t - n$ potential by Ref.[9]. The two-body interaction does not reproduce the observed data of ^{5}H, then, we use a phenomenological $t - n - n$ three-body force introduce by Ref.[9]. Using this potential, the calculated energy of ^{5}H is 1.45 MeV with $\Gamma = 2.1$ MeV, which is in good agreement with the observed data within the error bar.

The employed $t - \Lambda$ potential reproduce the binding energies pf the 0^+ and 1^+ states of $^{4}_{\Lambda}$H. In this case, $\Lambda N - \Sigma N$ coupling is renormalized into ΛN interaction. And we employ AV8 NN realistic interaction. The ΛN potential reproduces the binding energies of $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H which is employed for the discussion on charge symmetry breaking effect of $^{4}_{\Lambda}$H and $^{4}_{\Lambda}$He.

Using all potential above, the preliminary result of $^{6}_{\Lambda}$H B_Λ is 2.41 MeV, which is unbound with respect to $^{4}_{\Lambda}$H$+ n + n$ breakup threshold.

The preliminary result concludes that we have no bound state as long as we take only Λ channel. When we take $\Lambda N - \Sigma N$ coupling effect, our conclusion might change. The further analysis is in progress.

4. $S = -2$ hypernuclei and YY interaction

It is interesting to investigate the structure of the multi-strangeness system when one or more Λ's are added to a $S = -1$ nucleus. It is conjectured that extreme limit, which includes many Λ's (and other hyperons) in nuclear matter is the core of a neutron star. In this sense, the sector of $S = -2$ nuclei, double Λ hypernuclei and Ξ hypernuclei, is just the entrance to the multi-strangeness world. However, we have hardly any knowledge of the YY interaction because there exists no YY scattering data. Then, in order to understand the YY interaction, it is crucial to study the structure of double Λ hypernuclei and Ξ hypernuclei. The equation of state with the strangeness degree of freedom is a crucial component in understanding neutron stars.

Recently, the epoch-making data has been reported by the KEK-E373 experiment. Namely, the double Λ hypernucleus $^{6}_{\Lambda\Lambda}$He was observed [10]. This observation was called NAGARA event. The formation of $^{6}_{\Lambda\Lambda}$He was uniquely identified by the observation of sequential weak decays, and the precise experimental value of the 2Λ binding (separation) energy, $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}$ MeV, was obtained.

We studied double Λ hypernuclei with $A = 6 - 10$ [11]. Firstly, (1) we employed the $\Lambda\Lambda$ interaction of Nijmegen model D and performed an $\alpha + \Lambda + \Lambda$ three-body calculation for $^{6}_{\Lambda\Lambda}$He. (2) By comparing the theoretical result with the experimental data of the binding energy of $^{6}_{\Lambda\Lambda}$He, (3) we suggested reducing the strength of $^{1}S_0$ term of the $\Lambda\Lambda$ interaction by half to reproduce the data. Again, (2) using the improved potential, we predicted energy spectra of new double Λ hypernuclei with $A = 7 - 10$ [11], which is discussed below.

In fact, it is planned at J-PARC to produce many double Λ hypernuclei by emulsion experiment [12]. However, it will be difficult to determine spin-parities and to know whether the observed state is the ground state or an excited state. Therefore, it will be necessary to

compare the data with any theoretical study for the identification of the state. The author's role is to contribute to the theoretical calculation using few-body techniques.
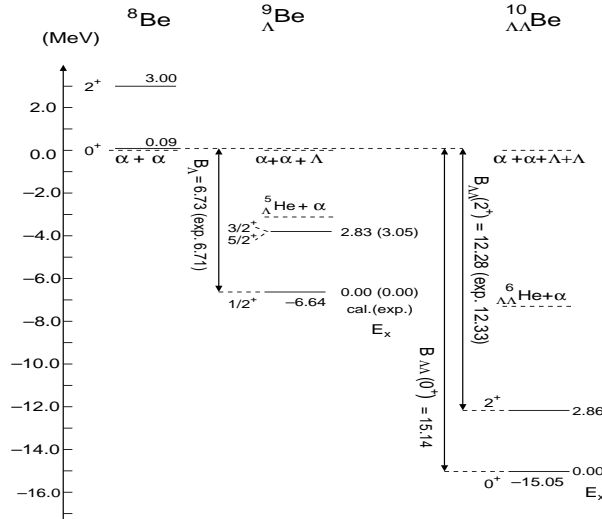

Figure 2. Calculated energy levels of ^{8}Be, $^9_\Lambda$Be and $^{10}_{\Lambda\Lambda}$Be on the basis of the $\alpha + \alpha$, $\alpha + \alpha + \Lambda$, and $\alpha + \alpha + \Lambda + \Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds or are given by the excitation energies $E_{\rm x}$. The calculated 2^+ state of $^{10}_{\Lambda\Lambda}$Be explains the Demachi-Yanagi event. This figures is taken from [11]

A successful example to determine spin-parity of double Λ hypernuclei is the case of $^{10}_{\Lambda\Lambda}$Be. There was one more event found in the E373 experiment named the 'Demachi-Yanagi' event [13]. The most probable interpretation of this event is the production of a bound state of $^{10}_{\Lambda\Lambda}$Be having $B^{\rm exp}_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV. But the experiment could not determine whether this state was the ground state or any excited state. In order to determine this, our calculation [11] mentioned above was useful. We studied $^{10}_{\Lambda\Lambda}$Be by employing an $\alpha + \alpha + \Lambda + \Lambda$ four-body model. The $\Lambda\Lambda$ interaction is the one improved from the Nijmegen Model D as mentioned above. The $\Lambda\Lambda$, $\alpha\Lambda$ and $\alpha\alpha$ interactions were chosen so as to reproduce the binding energies of all the subsystems, $^6_{\Lambda\Lambda}$He, $^5_\Lambda$He, ^{8}Be and $^9_\Lambda$Be. As shown in Fig.2, it is striking that our calculated value of $B_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}Be(2^+))$ is 12.28 MeV that agrees with the experimental data. Therefore, the Demachi-Yanagi event can be interpreted most probably as the observation of the 2^+ excited state in $^{10}_{\Lambda\Lambda}$Be.

Furthermore, in the KEK-E373 experiments, they observed one more event, Hida event [13]. This observation is for $^{11}_{\Lambda\Lambda}$Be or $^{12}_{\Lambda\Lambda}$Be. The observed $B_{\Lambda\Lambda}$ for $^{11}_{\Lambda\Lambda}$Be is 20.49 ± 1.15 MeV and $B_{\Lambda\Lambda}$ for $^{12}_{\Lambda\Lambda}$Be is 22.06 ± 1.15 MeV [13]. The important issue is to interpret the Hida event as a ground state or excited state in $^{11}_{\Lambda\Lambda}$Be or $^{12}_{\Lambda\Lambda}$Be. We assume Hida event as $^{11}_{\Lambda\Lambda}$Be and calculate the $B_{\Lambda\Lambda}$ with $\alpha\alpha n\Lambda\Lambda$ five-body problem. This five-body calculation is numerically difficult since we have three kinds of particles such as α, Λ and neutron, and we have five different kinds of interactions such as $\Lambda\Lambda$, Λn, $\Lambda\alpha$, $n\alpha$ and $\alpha\alpha$, and we have Pauli principle between α and α, and between α and neutron. Recently, we succeeded in performing this calculation.

In the present $\alpha + \alpha + n + \Lambda + \Lambda$ five-body model for $^{11}_{\Lambda\Lambda}$Be, it is absolutely necessary that all sub-cluster systems composed of two α's, a neutron and two Λ's are described reasonably with the interactions among these units. In our previous work [11], our interactions, which include the $\alpha\alpha$, αn, $\alpha\Lambda$, Λn and $\Lambda\Lambda$ interactions, were determined so as to reproduce reasonably well the following observed quantities: (i) Energies of the low-lying states and scattering phase shifts in

the $\alpha + n$ and $\alpha + \alpha$ systems, (ii) Λ-binding energies B_Λ in $^5_\Lambda$He ($= \alpha + \Lambda$), $^6_\Lambda$He ($= \alpha + \Lambda + n$) and $^9_\Lambda$Be ($= \alpha + \alpha + \Lambda$), (iii) double-$\Lambda$ binding energies $B_{\Lambda\Lambda}$ in $^6_{\Lambda\Lambda}$He ($= \alpha + \Lambda + \Lambda$), the Nagara event. Then, as mentioned above, the Demachi-Yanagi event for $^{10}_{\Lambda\Lambda}$Be ($= \alpha + \alpha + \Lambda + \Lambda$) was simultaneously reproduced with no additional adjustable parameter.

In the present work, we employ the same interactions of Ref.[11] so that those severe constraints are also successfully met in our two-, three- and four-body subsystems. But, as for the present core nucleus ^{9}Be ($= \alpha + \alpha + n$), which does not belong to the subsystems studied previously, use of the interactions that explain well the property of the $\alpha\alpha$ and αn subsystems do not well reproduce the energies of the low-lying states of ^{9}Be measured from the $\alpha + \alpha + n$ threshold (the same property of the calculated result was reported in another microscopic $\alpha + \alpha + n$ cluster-model study [14]). Therefore, we additionally introduce a phenomenological $\alpha\alpha n$ three-body force with a Gaussian shape, $v_0 e^{-(r_{\alpha-\alpha}/r_0)^2 - (R_{\alpha\alpha-n}/R_0)^2}$, having $r_0 = 3.6$ fm, $R_0 = 2.0$ fm and $v_0 = -9.7$ MeV ($+13.0$ MeV) for the negative-parity (positive-parity) state; we thus reproduce well the observed energies of the $3/2_1^-, 5/2_1^-, 1/2_1^-$ and $1/2_1^+$ states of ^{9}Be. The energy level is illustrated in Fig. 3 together with that of ^{9}Be. Interestingly enough, the
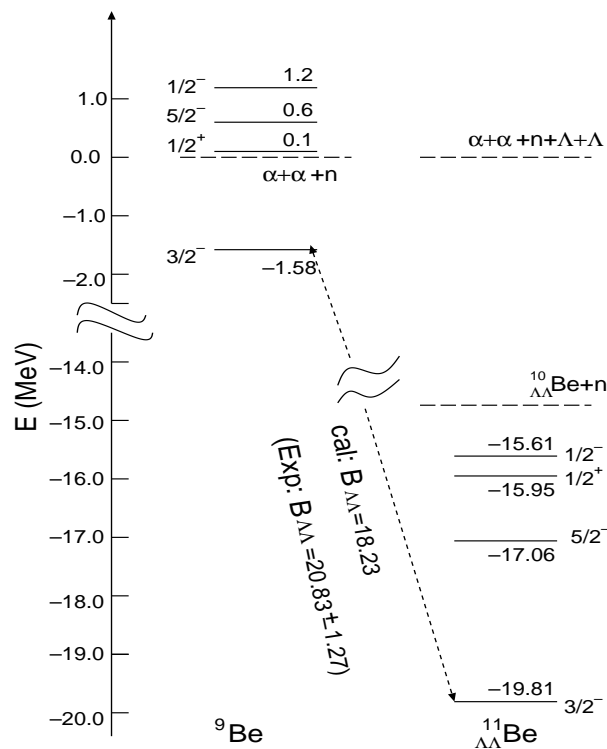

Figure 3. Calculated energy spectra of the low-lying states of $^{11}_{\Lambda\Lambda}$Be together with those of the core nucleus ^{9}Be.

order of the $1/2^+$ and $5/2^-$ states is reversed from ^{9}Be to $^{11}_{\Lambda\Lambda}$Be. This is because the energy gain due to the addition of the Λ-particle(s) is larger in the compactly coupled state ($5/2_1^-$) than in the loosely coupled state ($1/2_1^+$). Note that the same type of theoretical prediction was reported, in our early work [15] for $^{13}_\Lambda$C based on the $\alpha\alpha\alpha\Lambda$ model, for the Λ particle addition to the compactly coupled state (3_1^-) and to the loosely coupled state (0_2^+) in ^{12}C. As seen in Fig. 3 the calculated value of $B_{\Lambda\Lambda}(^{11}_{\Lambda\Lambda}$Be) is 18.23 MeV for the $3/2^-$ ground state, while for the excited states the $B_{\Lambda\Lambda}$ values are calculated to be less than 15.5 MeV. Therefore, the observed Hida event can be interpreted to be the ground state. When our calculated binding energy is

compared with the experimental value of 20.83 MeV with a large uncertainty of σ=1.27 MeV, we can say at least that our result does not contradict the data within 2σ. Motivated by the recent observation of the Hida event for a new double Λ hypernucleus, we have succeeded in performing a five-body calculation of $^{11}_{\Lambda\Lambda}$Be using an $\alpha\alpha n\Lambda\Lambda$ cluster model. The calculated $\Lambda\Lambda$ binding energy does not contradict the interpretation that the Hida event is an observation of the ground state of $^{11}_{\Lambda\Lambda}$Be.

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