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Alpha-particle condensation in nuclear systems: present status and perspectives

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Abstract. Alpha clustering and α condensation in lighter nuclei are presently strongly and increasingly discussed in the literature both from the experimental and from the theoretical side. A discussion of the present status of the theory as well as outlooks for future developements will be presented.

1. Introduction

For about ten years, since the idea of the possible existence of α condensate type of states in $n\alpha$ nuclei was formulated for the first time [1], many exciting new theoretical and experimental results have been produced. In this contribution, we would like to assess where we stand and what possible future extensions may be.

Let us start with the reminder that nuclear clustering and, in particular, α clustering would not exist, if we did not have in nuclear physics *four* different types of fermions (proton/neutron spin up/down), all attracting one another. We should be aware of the fact that this is a rather singular situation in fermionic many-body systems. However, the possibility of future trapping of four different kinds of cold fermionic atoms may open a new field of cluster physics with similar features. In a mean field description of an isolated α particle (which, with, e.g., Skyrme forces, gives reasonable results, if the c.o.m. motion is treated correctly) the four fermions can occupy the lowest (0S) level. Were there only neutrons, only two of them could be in the 0S-level, and the other two neutrons would have to be in the energetically very penalising 0P-state. That is why α particles exist, tetra-neutrons not. The ensuing fact is that α particles are very strongly bound ($E/A \sim 7$ MeV), almost as strongly as the most strongly bound nucleus, which is ^{56}Fe ($E/A \sim 8$ MeV). In addition the first excited state of the α particle (~ 20 MeV) is by factors higher than that of any other nucleus. The α particle can, therefore, be considered as an almost inert ideal bosonic particle. As we will see in the discussion below, in spite of its strong binding, α particle condensation can only exist in the so-called BEC (Bose-Einstein Condensation) phase, which implies low density. There is no analogue to the BCS phase of pairing where the Cooper pairs can have very large extensions, strongly overlapping with one another, still being fully antisymmetrised. This is the reason why α condensation can only be present at low densities where the α particles do not overlap strongly. (This holds, if the system consists of protons and neutrons and α's. If other clusters as t, ^{3}He, d are around, the situation may change, see below.) These considerations apply to nuclear matter as well as to finite nuclei. The Hoyle state in ^{12}C which can, to a good approximation, be described as a product of three α particles occupying all the 0S state of their bosonic mean field has a density which is by a factor 3-4 lower than the one of the ground state of ^{12}C. In the ground state there exist α-type of correlations but there is no condensation phenomenon. Let us start our considerations with infinite matter.

2. Alpha particle condensation in infinite matter

The in-medium four-body equation can be written in the following form

$$(E_{\alpha,\mathbf{K}} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4)\Psi^{\alpha,\mathbf{K}}_{1234} = (1 - f_1 - f_2)v_{121'2'}\Psi^{\alpha,\mathbf{K}}_{1'2'34} + (1 - f_1 - f_3)v_{131'3'}\Psi^{\alpha,\mathbf{K}}_{1'23'4} + ... \quad (1)$$

In total, there are six terms coming from permutations. The ε_i are kinetic energies plus mean field corrections; v_{1234} are the matrix elements of the two-body interaction, and f_i is a Fermi-Dirac distribution of the uncorrelated nucleons accounting for phase space blocking. Repeated indices are summed over and index numbers comprise momenta and spins. The above equation describes *one* quartet in a gas of uncorrelated nucleons at temperature T. The analogous two-body equation can be used to determine the critical temperature T_c for the onset of superfluidity or superconductivity, where T_c has to be determined so that the eigenvalue comes at two times the chemical potential μ. This is the famous Thouless criterion of BCS theory. In analogy with pairing, one has to find the critical temperature T_c^α so that the eigenvalue of the four-body equation (1) comes at 4μ. The in-medium four-body equation is very difficult to solve. None the less, the solution has been found employing the Faddeev-Yakubovsky equations and using the Malfliet-Tjon bare nucleon-nucleon interaction which yields realistic nucleon-nucleon phase shifts and properties of an isolated α particle [2]. To simplify the problem, we made in addition a very easy-to-handle variational ansatz of the four-body wave function in (1). It consists of a mean field ansatz for the α particle projected on good total momentum. In momentum space this is

$$\Psi_{1234} \propto \delta(\mathbf{K} - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)\varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2)\varphi(\mathbf{k}_3)\varphi(\mathbf{k}_4) \quad (2)$$

Inserting this ansatz into (1), one obtains a nonlinear HF-type equation for the S-wave function $\varphi(\mathbf{k})$. Of course, for quartet condensation, we choose $\mathbf{K} = 0$. With the mean field ansatz (2), one cannot use a bare force. We adjusted an effective separable force with two parameters which are chosen to reproduce the binding energy and radius of the free α particle. The full Faddeev-Yakubovsky solution of (1) is shown for symmetric matter in Fig. 1 (crosses). We see

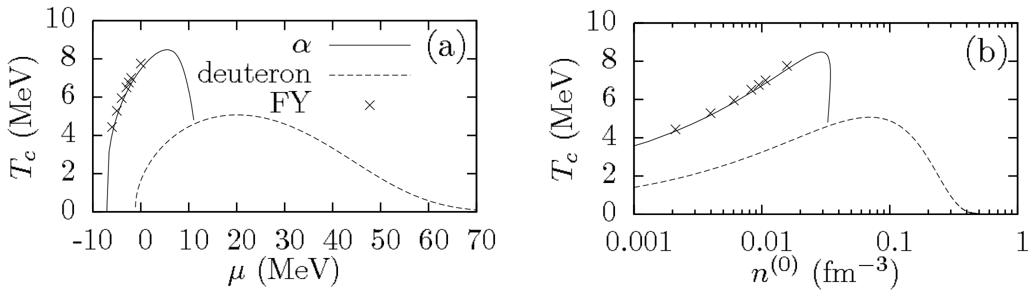

Figure 1. Critical temperatures for α particle and deuteron condensation in symmetric nuclear matter as a function of μ (a) and density $n^{(0)}$ (b).

that the ansatz (2) which very much eases the otherwise difficult solution of (1) works very well (continuous line). Also shown is the critical temperature for deuteron condensation. The striking feature is that α particle condensation abruptly breaks down already at very low density which approximately coincides with the point where the α's start to overlap appreciably (this fact was already found in [3] using a somewhat different variational ansatz for Ψ_{1234}). On the other hand deuteron condensation goes on up to very high densities and the limit is only triggered by the range of the effective force (which has been readjusted to reproduce the deuteron properties). This is so for symmetric nuclear matter. For strong asymmetry, deuteron condensation breaks down earlier than α condensation because the α particle according to its much stronger binding is less sensitive to asymmetry [4]. This is the afore-mentioned phenomenon that α condensation only exists in the BEC phase, i.e., at low density, whereas the deuteron continuously goes from negative to positive chemical potentials where for the latter the deuterons turn into large-size Cooper pairs. More on this can be found in [2, 3]. We should mention that our calculation of T_c^α is only reliable rather close to the breakdown point. For lower densities, T_c^α should join the one for condensation of ideal bosons (α's). To describe this feature, one should extend our theory to the so-called Nozières–Schmitt-Rink (NSR) theory [5] for pairing, see also [6], to α particle condensation. This, however, has not been worked out so far and remains a task for the future.

At zero temperature, there are many α's which go into the condensate phase. For this, we have to set up an approach analogous to the nonlinear BCS theory. Equation (1) corresponds to the linearised version and only describes *one* α particle in an otherwise uncorrelated gas (at finite T) of fermions. In finite nuclei, there may exist such a situation even at zero temperature. This is the case of ^{212}Po which can, to a certain extent, be viewed as an α particle sitting on top of the doubly magic core of ^{208}Pb which can be well described by a HF mean-field approach, i.e. a Fermi gas in a container. We will come back to this later when we discuss finite nuclei.

After having considered the linearised version of the equation for the quartet order parameter at the critical temperature, let us now try to write down, in analogy to the BCS case, the fully non-linear system of equations for the quartet order parameter. To clearly see the analogy to the BCS case, let us repeat the latter equations in a slightly unusual way. The pairing order parameter, allowing for non-zero c.o.m. momentum of the pairs, reads

$$(\varepsilon_{k_1} + \varepsilon_{k_2})\kappa_{\mathbf{k}_1\mathbf{k}_2} + (1 - n_{k_1} - n_{k_2})\Delta_{\mathbf{k}_1\mathbf{k}_2} = 2\mu\kappa_{\mathbf{k}_1\mathbf{k}_2} \tag{3}$$

where $\Delta_{\mathbf{k}_1\mathbf{k}_2} = \sum v_{k_1k_2k_{1'},k_{2'}}\kappa_{\mathbf{k}_{1'},\mathbf{k}_{2'}}$ and $\kappa_{\mathbf{k}_1\mathbf{k}_2} = \langle\mathrm{BCS}|c_{\mathbf{k}_1}c_{\mathbf{k}_2}|\mathrm{BCS}\rangle = u_{\mathbf{k}_1}v_{\mathbf{k}_2}$ is the usual pairing tensor (with spin indices suppressed) and $n_k = v_k^2 = 1 - u_k^2$ are the BCS occupation numbers. The ε_k's are, as before, the kinetic energies, eventually including a mean field correction. The occupation numbers can be obtained from the Dyson equation

$$G_k^\omega = G_k^0 + G_k^0 M_k^\omega G_k^\omega, \tag{4}$$

with $M_k = \Delta_k\Delta_k^*/(\omega + \varepsilon_k)$ the BCS mass operator where Δ_k is the diagonal part of the gap for cases where the pairs are at rest. From the single-particle Green's function, obviously we can calculate the occupation numbers thus closing the typical BCS self-consistency cycle. Inspired by the BCS case, we then write for the quartet order parameter [see (1)]

$$(4\mu - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4)\kappa_{1234} = (1 - n_1 - n_2)v_{121'2'}\kappa_{1'2'34} + (1 - n_1 - n_3)v_{131'3'}\kappa_{1'23'4} + \dots \tag{5}$$

with $\kappa_{1234} = \langle c_1c_2c_3c_4\rangle$. Again, we have to close the equation with the Dyson equation for the occupation numbers. However, the mass operator now contains the quartet order parameter

$$M_1^\alpha = \sum_{234} \frac{\Delta_{1234}[\bar{n}_2^0\bar{n}_3^0\bar{n}_4^0 + n_2^0 n_3^0 n_4^0]\Delta_{1234}^*}{\omega + \varepsilon_2 + \varepsilon_3 + \varepsilon_4}\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \tag{6}$$

with $\bar{n}^0 = 1 - n^0$ and n_i^0 being the uncorrelated Fermi function, i.e., the Fermi step and $\Delta_{1234} = \sum V_{123'4'}\kappa_{3'4'34}$ where V_{1234} is an effective coupling vertex linking the single-particle motion to the order parameter (for more details and derivation, see [7]). Before trying to solve this equation, let us discuss the differences between the pairing and the quartet case. The first thing which strikes is that the three 'holes' only have to have total momentum $\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4 = -\mathbf{k}_1$ and, therefore, we have a remaining sum over momenta. In the pairing case with only one 'hole', there is no sum. Furthermore in the pairing case the hole propagator has no phase space factor because the 'forward' and 'backward' going parts add up to one: $\bar{n}^0 + n^0 = 1$. In the quartet case there are three hole propagators and the corresponding sum of phase space factors does *not* add up to one, i.e. $\bar{n}^0_1\bar{n}^0_2\bar{n}^0_3 + n^0_1 n^0_2 n^0_3 \neq 1$! This makes a dramatic difference from the pairing case. In order to understand this a little better, let us compare the level density of a single hole with the one of three holes:

$$g^{1h}(\omega) = \sum_k [\bar{n}^0_k + n^0_k]\delta(\omega + \varepsilon_k) = \sum_k \delta(\omega + \varepsilon_k) \tag{7}$$

$$g^{3h}(\omega) = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} [\bar{n}^0_{k_1}\bar{n}^0_{k_2}\bar{n}^0_{k_3} + n^0_{k_1}n^0_{k_2}n^0_{k_3}]\delta(\omega + \varepsilon_{k_1} + \varepsilon_{k_2} + \varepsilon_{k_3}) \tag{8}$$

The 3h level density is shown in Fig.2. We see that for positive μ there is a striking difference

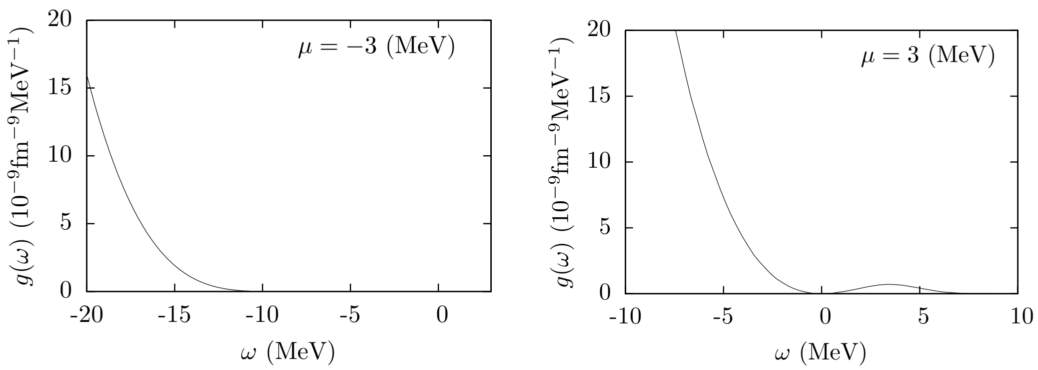

Figure 2. 3h level densities for negative and positive chemical potentials, respectively. Note that on the horizontal axis, the origin corresponds to $\mu = 0$.

from the 1h level density. At positive μ, $g^{1h}(\omega = \mu)$ is obviously finite (not shown), whereas $g^{3h}(\omega = 3\mu)$ goes through zero. This is because phase space constraints and energy conservation cannot be fullfilled simultaneously at the Fermi surface in the latter case as is easily verified. That is, in the quartet case, exactly at the point where the correlations should be built up, namely at the Fermi level, there is no level density! As a consequence, no quartet condensation is possible for positive μ. On the contrary, for negative μ, $n^0 = 0$ and thus the phase space factor in the case of g^{3h} is also equal to one and then there is no qualitative difference from the 1h case. This explains in a natural way why quartet condensation is not possible at positive chemical potentials. It is, by the way, well known that any mp–nh level density, besides the single-particle case, goes through zero at the Fermi level. For example, the origin of fermions at the Fermi energy having an infinite free mean path stems from the fact that the 2p-1h (2h-1p) level density is zero at the Fermi level and that is the equivalent to the imaginary part of the optical potential being zero at that energy. Also the 2p-2h level density, which plays an important role for the damping of zero sound modes, is zero at zero frequency. As first pointed out by Landau, it approaches zero as ω^2. In conclusion, for positive μ only pairing survives, whereas quartetting breaks down; it only exists in the BEC phase with negative μ. The situation may be different when other light clusters are present, i.e. in a mixed gas of, e.g., nucleons, tritons (^{3}He) and deuterons. Then a nucleon with momentum $\mathbf{k}$ may, eventually, directly pair with, e.g., a triton of momentum $-\mathbf{k}$

(or the other way round), rather similarly to the standard pairing situation apart from the fact that now two fermions with different masses have to pair up. Similar considerations hold for the pairing of two deuterons (pairing of 'bosons'). In compact star physics such situations may exist when the star is cooling down. The extension of our theory to this scenario is a task for the future. The full solution of the nonlinear set of equations (6) and (7) is again very much eased in taking for the order parameter the factorisation ansatz (2). The most interesting result is that the occupation number, e.g., for μ around zero is far from being close to saturation. At $k = 0$ it is only approximately $n_{k=0} \sim 0.30$. This scenario is analogous to pairing in the BEC regime. More results can be found in [7].

3. Finite Nuclei

As we know from pairing, a direct observation of condensation phenomena is only possible in finite nuclei. Of course, in such small systems, there cannot exist a condensation in the macroscopic sense. Nevertheless, as we know very well, only a handful of Cooper pairs suffices to show clear signatures of pairing in nuclei. For α particle condensation it is the same story. We can only expect that there exists about a handful of α particles, essentially in lighter $n\alpha$ nuclei, in a gaseous phase at low density. It is, indeed, surprising that such states at low density with about $\rho = \rho_0/3 - \rho_0/4$ with ρ_0 the density at saturation do exist as quite long lived excited states in those nuclei. The most famous example is the Hoyle state in ^{12}C at 7.65 MeV, just about 300 keV above the 3α threshold. We will not dwell much on the successful theoretical description of this state (and others, e.g., in ^{16}O) with the THSR wave function [1], since this has been presented in the recent literature a great number of times [8]. Let us only make a couple of remarks. The THSR wave function is schematically written for a finite number of quartets as

$$\Psi_{n\alpha}^{\mathrm{THSR}} = \mathcal{A}[\Phi_\alpha \Phi_\alpha ... \Phi_\alpha] \tag{9}$$

where the single-α wave function Φ_α depends on four spatial coordinates, the spin-isospin part being suppressed. This wave function is fully antisymmetrical due to the antisymmetriser $\mathcal{A}$ and is analogous to the number-projected BCS wave function

$$\Psi^{\mathrm{BCS}} = \mathcal{A}[\phi_{\mathrm{pair}} \phi_{\mathrm{pair}} ... \phi_{\mathrm{pair}}] \tag{10}$$

where ϕ_{pair} is the Cooper pair wave function depending on two spatial coordinates. The calculus with the α condensate wave function is very much facilitated by a variational ansatz where Φ_α is split into a product of a c.o.m. gaussian with a large width parameter B times another, intrinsic, gaussian depending only on the relative coordinates of the α particle and having a width parameter b which corresponds to the size of an isolated α particle. The first remark to be made is that this THSR wave function contains two important limits: if $B = b$ then it corresponds to a pure harmonic oscillator Slater determinant. If $B \gg b$, then the α's are so distant from one another that the Pauli principle among the different α's can be neglected and, thus, the antisymmetriser be dropped. The THSR wave function is then a pure product state of α particles, i.e., a condensate state of ideal bosons. Reality is, of course, in between those limits and one important task is to find out whether reality is closer to a Slater determinant or to a Bose condensate. That this question must be carefully investigated, can also be deduced from the fact that a number-projected BCS wave function always leads to a non-trivial pairing solution. For example for ^{208}Pb, one obtains a non-trivial BCS solution in spite of the fact that ^{208}Pb is certainly not superfluid. Only when the original particle-number-breaking BCS theory with $|\mathrm{BCS}\rangle = e^{\sum z_{kk'} c_k^+ c_{-k}^+} |\mathrm{vac}\rangle$ has a non-trivial solution, can we speak of a superfluid nucleus. For ^{208}Pb there is no such solution. One way to analyse whether the THSR approach leads primarily to an α condensate or to a Slater determinant, is to investigate the bosonic occupation numbers. An ideal Fermi gas has occupation numbers which are either one or zero.

In an ideal Bose condensate the bosons will occupy the lowest single-particle state with 100%. Of course, in real nuclei, neither the fermionic nor the bosonic occupation numbers agree with those of an ideal gas. For nucleons, we know that occupation numbers are around 70-80% down from 100%. For the α occupations in the condensate, we also have found a number around 70%, all the other occupancies being down by at least a factor of ten, whereas for the ground state the occupation numbers are almost equally distributed according to the SU(3) shell model scheme [8]. This is a typical condensate situation, though not totally the one of an ideal condensate. Residual antisymmetry effects between the slightly overlapping α particles makes that, with a certain probability, the α's are scattered out of the condensate. In addition there are also other correlations at work. One of the most important ones is certainly the formation of ^{8}Be clusters out of two α's. The answer is unclear even to the question whether α gas does not in reality consist of a gas of ^{8}Be's. The latter are, of course, also bosons and the old question arises whether in an attractive Bose gas the bosons condensate as singles or as molecules [9]. This is certainly a very interesting question which deserves further studies in the future.

Many other extensions of α condensation are presently discussed theoretically and experimentally. An intersting aspect is whether on top of the α condensate states excited α gas states exist. A long debate has recently been closed about the nature and existence of the second 2^+ state close to 10 MeV excitation energy in ^{12}C. Very nice experimental results by M. Freer, M. Itoh, and M. Gai have recently shown that this state is there and that it is a member of a family of α gas states [10, 11, 12]. Many more results, for instance, in ^{16}O are to be expected. One of the most exciting aspects is that one may be able to dismantle $n\alpha$ nuclei, with n a rather large number like $n = 10$ or more, into n α-particles. The first results in this direction have been reported at this conference on ^{56}Ni, a nucleus with 14 α-particles, by H. Akimune. A dream would be that all α particles be just excited to the Ikeda threshold and then they disintegrate in a very slow motion as a kind of coherent state driven by the Coulomb force, i.e., it would be some kind of soft Coulomb explosion. This would then be rather close to what happens with a trapped Bose condensate of cold atoms upon switching off the trapping potential. Other exciting perspectives are that α particles could exist in a gaseous phase on top of an inert core. For example four α's on top of ^{16}O in ^{32}S, or other variants, even in quite heavy nuclei. Because of space restrictions, we, unfortunately, cannot enlarge further upon these exciting aspects of α-gas type of states in nuclei.

Before closing, we would like to discuss, however, the question of a possible preformation of α particles in heavy nuclei. As is well known, this question is of great importance for the description of α decay rates. Let us take the example already alluded to in the infinite matter section, namely ^{212}Po. Since the lead core is doubly magic, one can treat it in mean field, i.e. as a Slater determinant with a suitable Skyrme- or Gogny-type force. That is we may view ^{212}Po as an α particle on top of a finite Fermi gas. If such a configuration exists, the α decay rate of ^{212}Po suggests, and it is clear from our experience with the Hoyle state as well, that such a cluster state cannot be described within the shell model alone, see [13, 14]. In the infinite matter section, we have already considered a situation where a single α particle is embedded in an uncorrelated Fermi gas. This was the case with respect to the critical temperature. For ^{212}Po we can consider our treatment of the single-α case even at zero temperature. Of course we cannot use the ansatz (2) as it is, since it corresponds to a c.o.m. wave function which is a plane wave $e^{i\mathbf{KR}}$. In position space (2) reads $\Psi_{1234} \propto e^{i\mathbf{KR}}\psi_{int.}(|\mathbf{r}_i - \mathbf{r}_j|)$ where $\psi_{int.}$ is the intrinsic wave function depending only on the relative coordinates. In a finite nucleus, instead of the plane wave, one would have to use a wave packet considering the following wave function

$$\Psi_{1234} \propto \Phi(\mathbf{R})\psi_{int.}(|\mathbf{r}_i - \mathbf{r}_j|) \tag{11}$$

where $\psi_{int.}$ and Φ are to be determined variationally upon inserting (11) into Eq. (1) where all indices and ingredients correspond now to the mean field of ^{208}Pb. For instance, ε_i correspond to the HF energies and the f_i are to be replaced by the HF occupation numbers at $T = 0$. In addition, for $\psi_{int.}$, one could use, as for the study of the Hoyle state, a gaussian wave function with the width parameter b. Then the single unknown would be the spherical wave function $\Phi(R)$ to be determined variationally where R is the c.o.m. coordinate of α with respect to the centre of the Pb core. Our knowledge that α's can only exist at very low density incites us to believe that this Φ wave function should be peaked rather far out in the surface of the Pb nucleus. If true, this would be a nice explanation of a preformed α particle in the nuclear surface. Adding more than one α to the Pb core may suggest that there could exist some sort of α condensate in the surface in a fluctuating state. However, as we know, adding more α's to the Pb core leads to deformed nuclei and the treatment of α's in the surface becomes a much more delicate subject.

4. Conclusions

As we have seen, the existence of α gas and α condensate states in nuclear systems where the α's play practically the role of elementary bosons, is fascinating. Nuclear physics is at the forefront of this kind of physics. In the future, experiments with cold atoms trapping four (or more) different kinds of fermions may also open wide perspectives in the field of cluster physics. For more reading on α cluster states, we invite the reader to consult our review article [8].

Acknowledgments

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