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Alpha-decay quantum-tunnelling calculations based on a folded Woods-Saxon potential

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Abstract. Assuming that the α particle is a structureless point particle with two protons and two neutrons, we construct a mean-field-type cluster potential based on the Woods-Saxon potential with a folding factor which is to satisfy the quantization condition of a quasibound cluster state. The folded Woods-Saxon cluster potential has been successfully applied to the calculations of α-particle decay in light and superheavy nuclei. The standard values of the Woods-Saxon parameters were used without any adjustment. The calculated α-decay widths or lifetimes agree generally with experiment. Such a cluster potential leads to a consistent description of single-particle and cluster motions.

1. Introduction

Alpha-particle emission is the most important decay mode of nuclei heavier than Pb. Also, α decay has been observed widely from the excited states of light nuclei. In light nuclei, α-cluster structures are favoured when nuclei are excited to the vicinity of the α-decay threshold [1]. Molecular structures with two or more α particles and covalent neutrons in light nuclei have become a hot topic currently in both experiment [2, 3] and theory [4, 5]. In heavy nuclei, α decay can happen from the ground states as well. For superheavy nuclei, detecting α decay is a unique method to identify new superheavy elements.

Theoretically, α-particle emission can be considered a process of quantum tunnelling of an α particle through a potential barrier, which is called the Gamow decay model [6]. A reasonable barrier is crucial for the calculation of decay width or lifetime. Several phenomenological α-cluster potentials have been proposed for the calculations of α-decay half-lives and spectroscopic properties [7, 8, 9]. Double-folding microscopic cluster potentials have also been successfully applied to α-decay and α-scattering calculations [10, 11, 12, 13, 14].

2. The model

In our previous works [15, 16, 17, 18, 19, 20], we constructed folded mean-field-type cluster potentials based on microscopic Skyrme-Hartree-Fock [15, 16, 17] or phenomenological Woods-Saxon potentials [18, 19, 20]. These cluster potentials have been successfully applied to the calculations of various cluster decays including α and heavier cluster decays [15, 16, 19, 20] and molecular structures as well [16, 18]. In this paper, we review the Woods-Saxon-potential-based calculations with focusing on α decay from excited states of the light nucleus ^{24}Mg and from the ground states of superheavy nuclei.

In the spherical case, the cluster potential can be decomposed as (e.g., [8])

$$V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2} l(l+1), \tag{1}$$

which contains the nuclear potential $V_N(r)$, the Coulomb potential $V_C(r)$ and the centrifugal potential with l and μ for the angular momentum carried by the α particle and the reduced mass of the α-core system, respectively [15]. The general folding procedure to derive a cluster potential can be written as [10]

$$V_N(r) = \lambda \int\int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{r}_2) v_{\mathrm{eff}}(|\mathbf{r}+\mathbf{r}_1-\mathbf{r}_2|)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2, \tag{2}$$

where λ is the folding factor, $v_{\mathrm{eff}}(|\mathbf{r}+\mathbf{r}_1-\mathbf{r}_2|)$ is an effective nucleon-nucleon interaction, and $\rho_1(\mathbf{r}_1)$ and $\rho_2(\mathbf{r}_2)$ are the densities of the daughter and the cluster, respectively. To simplify the equation above, we consider the cluster as a structureless point particle, then obtain

$$V_N(r) = \lambda \int [Z_c\rho_1^p(\mathbf{r}_1)v_{\mathrm{eff}}^p(|\mathbf{r}+\mathbf{r}_1|) + N_c\rho_1^n(\mathbf{r}_1)v_{\mathrm{eff}}^n(|\mathbf{r}+\mathbf{r}_1|)]\mathrm{d}\mathbf{r}_1, \tag{3}$$

where the superscripts 'p' and 'n' indicate the proton and neutron, respectively, and N_c and Z_c are the neutron and proton numbers of the cluster, respectively. For the α particle, $N_c = 2$ and $Z_c = 2$. In Eq. (3) the integral gives the mean-field single-particle potential. Therefore, the nuclear potential between the cluster and the remaining core can be simplified further,

$$V_N(r) = \lambda[N_c v_n(r) + Z_c v_p(r)], \tag{4}$$

where $v_n(r)$ and $v_p(r)$ are single-neutron and single-proton potentials (excluding the Coulomb potential) respectively, generated by the core. Single-particle potentials can be obtained by mean-field models, such as the Skyrme-Hartee-Fock (SHF) [15, 16, 17] or the simple Woods-Saxon potential [18, 19, 20]. The Coulomb potential $V_C(r)$ need not be calculated by a folding procedure; we take its usual form [21] with assuming a homogeneous charge distribution in the daughter.

In this paper, we adopt the Woods-Saxon potential for the single-particle mean-field, that is

$$v(r) = \frac{V_0}{1+e^{\frac{r-R}{a}}}, \tag{5}$$

with

$$V_0(r) = -V_{00}\left(1 \pm \kappa\frac{N_d - Z_d}{N_d + Z_d}\right), \tag{6}$$

where the sign is $+$ $(-)$ for the proton (neutron). The index 'd' indicates the daughter.

The folding factor λ is determined by the Bohr-Sommerfeld quantization condition, which, for the ground state, looks like

$$\int_{r_1=0}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}|Q_0 - V(r)|}\mathrm{d}r = (2n+1)\frac{\pi}{2} = (G+1)\frac{\pi}{2}, \tag{7}$$

where r_1, r_2 (and r_3 later) are classical turning points obtained from $V(r_i) = Q_l^*$ (the decay energy). The global quantum number $G = 2n$ (n is the node number in the radial wave function of the cluster tunnelling motion) is determined by the Wildermuth rule [22], giving $G = \sum_{i=1}^{A_c} g_i$ where A_c is the nucleon number of the cluster and g_i is the oscillator quantum number of a cluster nucleon orbiting the core. The g_i numbers are those of the single-particle states occupied in the parent nucleus by the nucleons constituting the α particle to be emitted.

The partial α-decay width is calculated by [8, 16, 19]

$$\Gamma = P\frac{\hbar^2}{4\mu}\frac{\exp\left[-2\int_{r_2}^{r_3} k(r)\mathrm{d}r\right]}{\int_{r_1}^{r_2} \mathrm{d}r/2k(r)}, \tag{8}$$

where $k(r) = \sqrt{(2\mu/\hbar^2)|Q_l^* - V(r)|}$ is the wave number, and P is the preformation factor of the α-particle being formed in the mother. For even-even nuclei, it has been well established that the $P = 1$ assumption under the use of the Bohr-Sommerfeld condition can well reproduce the experimental half-lives of various cluster decays [8, 15]. The decay half-life is calculated by $T_{1/2} = \hbar \ln 2/\Gamma$. In the equations above, Q_l^* is the α-decay energy from an excited state,

$$Q_l^* = Q_0 + E_{J_i}^* - E_{J_f}^*, \tag{9}$$

where Q_0 is the α-decay energy of the ground state, and $E_{J_i}^*$ and $E_{J_f}^*$ are the excitation energies of the mother with spin J_i and the daughter with spin J_f. The orbital angular momentum l carried by the α particle can be determined from the vector coupling of $\mathbf{l}$ and $\mathbf{J}_f$ to $\mathbf{J}_i$ and from parity conservation. Since the decay calculation is very sensitive to the Q_l^* value, experimental values have been used for Q_0 and the excitation energies.

3. Alpha-decay calculations for the excited states of ^{24}Mg and the ground states of superheavy nuclei

In α-decay calculations for ^{24}Mg, we take the Chepurnov parameters [23] of the Woods-Saxon potential, which work well for light nuclei. For the ^{24}Mg ground state, nucleons belonging to the α particle should occupy orbits immediately above the Fermi surface of the daughter ^{20}Ne, i.e., the $d_{5/2}$ shell, which gives a value of $G = 8$ [19]. Using the Bohr-Sommerfeld condition with taking the experimental $Q_0 = -9.316$ MeV for the α+^{20}Ne channel, we obtain a folding factor of $\lambda = 0.608$ for the α-cluster potential in ^{24}Mg. It has been known experimentally that the ^{24}Mg excited states in the energy range of $\approx 10 - 15$ MeV decay dominantly by α emission [24]. The occurrence of α decay requires decay energies satisfying $Q_l^* = Q_0 + E_{J_i}^* - E_{J_f}^* > 0$. Therefore, the excited states in this energy range decay mainly into the 0_1^+ (g.s.) and 2_1^+ (1.63 MeV) states of ^{20}Ne. Assuming a preformation factor of $P \approx 1$ for every α-decay transition, we calculated the α-decay widths of some excited states of ^{24}Mg. The widths obtained agree, in most cases, with the experimental data within two orders of magnitude, see [19] for detailed results. The preformation probabilities of an α particle in the excited states of ^{24}Mg range from 10^{-2} to 1 [25], which might explain the discrepancies between the calculated and experimental widths.

The folded Woods-Saxon cluster potential has also been applied in the α-decay calculations of the ground states of superheavy nuclei [20]. For superheavy nuclei, however, we took another set of the Woods-Saxon parameters, which has been widely used in cranking shell-model calculations of high-spin states in heavy and superheavy nuclei (see, e.g., [26] and references therein). They are

$$\begin{aligned} V_{00} &= 53.754 \text{ MeV}, \\ \kappa &= 0.791, \\ a &= 0.637 \text{ fm}, \\ r_0 &= 1.19 \text{ fm}. \end{aligned} \tag{10}$$

The main difference between this set of parameters and the Chepurnov parameters for light nuclei is in the radius parameter r_0. The Chepurnov parameterization takes $r_0 = 1.24$ fm [23].

Table 1 lists the calculated half-lives of α decays for even-even superheavy nuclei. They agree with the experimental values within one order of magnitude. For superheavy nuclei, we took the global quantum number $G = 22$ [17], which is consistent with the Wildermuth rule. The discrepancies between calculations and data might come from two main factors: 1) deformation effects (most superheavy nuclei are deformed, while our calculations were limited to spherical shapes); 2) possible large uncertainties in experimental half-lives due to poor statistics of decay events.

Table 1. Calculated half-lives of observed α decays for even-even superheavy nuclei. Data are from [27, 28, 29].

Nuclei	λ	$T^{\text{calc}}_{1/2,\alpha}$ (second)	$T^{\text{expt}}_{1/2,\alpha}$ (second)
^{246}Fm	0.764	7.33×10^{-1}	$1.1\times10^{+0}$
^{248}Fm	0.763	$1.28\times10^{+1}$	$3.6\times10^{+1}$
^{250}Fm	0.762	$5.16\times10^{+2}$	$1.8\times10^{+3}$
^{252}Fm	0.761	$2.02\times10^{+4}$	$9.1\times10^{+4}$
^{254}Fm	0.757	$4.32\times10^{+3}$	$1.2\times10^{+4}$
^{256}Fm	0.755	$5.99\times10^{+4}$	$9.5\times10^{+3}$
^{252}No	0.758	$1.01\times10^{+0}$	$2.4\times10^{+0}$
^{254}No	0.757	$1.13\times10^{+1}$	$5.1\times10^{+1}$
^{256}No	0.751	6.95×10^{-1}	$2.9\times10^{+0}$
^{256}Rf	0.755	3.30×10^{-1}	3.6×10^{-1}
^{260}Sg	0.747	2.27×10^{-3}	3.6×10^{-3}
^{266}Sg	0.744	$1.94\times10^{+0}$	$2.6\times10^{+1}$
^{264}Hs	0.742	1.88×10^{-4}	1.0×10^{-4}
^{266}Hs	0.740	7.59×10^{-4}	2.3×10^{-3}
^{270}Hs	0.747	5.60×10^{-1}	
^{270}Ds	0.734	2.55×10^{-5}	1.0×10^{-5}
^{284}Cn	0.730	$9.39\times10^{+0}$	$9.8\times10^{+0}$
^{288}114	0.725	4.37×10^{-1}	$1.9\times10^{+0}$
^{292}116	0.720	1.82×10^{-2}	3.3×10^{-2}

In summary, we have proposed a folded Woods-Saxon α-cluster potential. The advantage of this cluster potential is that no new free parameter is to be introduced, which makes the predictions reliable. Moreover, it leads to a consistent description of single-particle and cluster motions. Within the framework of the quantum-tunnelling picture, the potential has been successfully applied to the α-decay of some excited states of ^{24}Mg with excitation energies of $E_x \approx$ 10–15 MeV and to the ground states of superheavy nuclei. The calculated decay widths or half-lives agree reasonably with experimental values.

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