

# Complex-energy shell model description of alpha decay

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**Abstract.** The alpha decay width has been successfully described in the framework of a combined shell and cluster model. It has been found that the shell model does not provide enough alpha clustering. The cluster model cures this deficiency by including the alpha cluster in the model from the very beginning. In the complex-energy shell model attempt is made to produce the clustering needed by a large model space which involves particles in the continuum. An advantage of such a formulation is that the tail of the formation amplitude may be consistent with the decaying boundary condition. The formulation can be applied to systems, like  $^{104}\text{Te}$ , decaying to the continuum.

## 1. Introduction

The theoretical description of the alpha decay in  $^{212}\text{Po}$  was a long standing unsolved problem. From the first successful description of the relative decay given in a semi-microscopic description by Gamow in 1928 [1, 2] up to the satisfactory description of the absolute width given by Varga *et al* [3] 64 years passed. The theoretical enterprise started with an outstanding progress made by Gamow by dividing the decay into two steps and by using the very new idea that Quantum Mechanics can describe the tunnelling of the alpha particle through the Coulomb barrier, providing its probability. Gamow assumed that the alpha particle (formed by four protons and two electrons) is pre-formed inside the nucleus. Thomas [4] made another major step by contributing with a new element: a way to calculate microscopically the formation of the alpha particle by using the then newly invented formalism of nuclear reactions: R-matrix theory. Using time dependent perturbation theory Mang [5] elaborated the connection of the formation amplitude with the Shell Model. Mang's approach happens to be equivalent to Thomas' approach [6]; the absolute width is given by  $\Gamma = 2P(R)\gamma^2(R)$  [4, 5]. The formation stage is incorporated in the reduced width  $\gamma^2(R)$  while the decay stage is represented by the penetrability  $P(R)$ . Alternatively, the absolute width can be calculated from the spectroscopic factor  $S$  and the single-particle width  $\Gamma_{sp}$  as  $\Gamma = S\Gamma_{sp}$  [4, 7].

The first Shell Model calculation of the absolute width involved a single Harmonic Oscillator (HO) configuration [5]. The absolute width obtained was several orders of magnitude far from the experimental one. This failure was caused by the unrealistic rapid falloff of the HO wave function



tail. Harada [8] showed that a mixing of configurations improves the absolute width calculation due to the constructive interference of the HO states in the tail region. But the enhancement was not enough to reproduce the experimental result. Even the more extended HO shell model calculation carried out by Tonozuka and Arima [9] did not produce enough clustering. In order to improve the tail of the formation amplitude, Woods-Saxon wave functions expanded in an HO basis were used [10, 11]. A schematic calculation in the Berggren basis showing the importance of the continuum states was done in ref. [12]. But all the improvements mentioned were not enough to reproduce the experimental  $^{212}\text{Po}$  decay.

Fliessbach [13] noticed for the first time that the normalization of the relative wave function in the decay channel implies a normalization to be interpreted in terms of a resonating-group-like theory. As a consequence, the formation amplitude has to be modified by a renormalization. The first calculation [14] in a single-configuration model space gave a promising result by showing that the absolute width increased by a factor of 100. Unfortunately the systematic calculation done by Tonozuka and Arima in the extended shell model proved that that hope was unjustified.

The final word to explain theoretically the absolute alpha decay width in the nucleus  $^{212}\text{Po}$  came when the Shell Model, Cluster Model and the renormalization were all combined. In ref. [3] Varga *et al* reproduced the experimental absolute width within 10% and predicted 30% of clusterization for the alpha particle in the mother nucleus. It was also suggested that the low-lying yrast spectrum of  $^{212}\text{Po}$  could also be explained in terms of an  $\alpha + ^{208}\text{Pb}$  cluster model [15].

The goal of this contribution is to show that the experimental absolute width of the elusive alpha decay of  $^{212}\text{Po}$  can also be described within the Shell Model Framework. The way to get at this goal involves two improvements: (1) use a realistic basis of Woods-Saxon eigenfunctions without an HO expansion, and (2) use  $\Gamma = S\Gamma_{sp}$  to calculate the width, where  $\Gamma_{sp}$  is the width obtained in a daughter + alpha single-particle model of the parent nucleus. The effect of (1) is to improve the tail of the shell-model formation amplitude while (2) reduces the sensitivity of the final result to the model space and makes the result independent of a channel radius  $R$ . Other formulations which avoid the use of a channel radius can be found in references [16, 17, 18, 19].

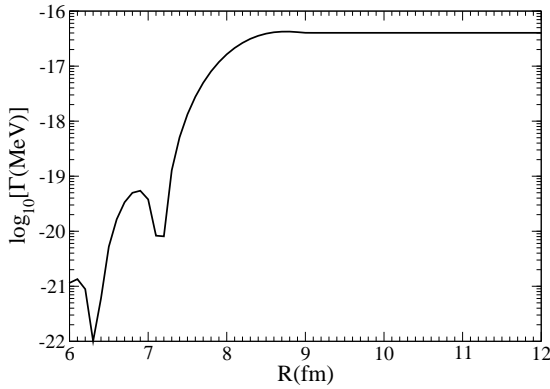
## 2. Formalism

The first ingredient in any many-body calculation is the basis. An extensive shell-model space has to be used in order to gain sufficient amount of clustering. This implies that one has to represent the continuum in the basis somehow. For the Woods-Saxon mean-field used here, this implies that we have to deal with positive-energy states. We will use a complex-energy representation taking a truncated Berggren basis [20]. Since we are not using HO states, we have to deal with nucleon single-particle coordinates and daughter-alpha relative coordinates at the same time. For this reason the six-dimensional integral involved in the formation amplitude must be calculated numerically.

We are going to show results for the alpha decay absolute width calculated by (1) the  $R$ -dependent formula  $\Gamma = 2P(R)\gamma^2(R)$  ( $R$  the channel radius) coming from the R-matrix theory and (2) the  $R$ -independent formula  $\Gamma = S\Gamma_{sp}$ . Both expressions split the decay process into a formation stage, represented by  $\gamma$  and  $S$ , and a decay stage, represented by  $P(R)$  and  $\Gamma_{sp}$ , respectively. The reduced width amplitude  $\gamma(R) = (\hbar^2 R/2\mu)^{1/2}G(R)$  is proportional to the modified formation amplitude  $G(R)$ , which gives the probability amplitude that an alpha particle is formed inside the mother nucleus,

$$G(R) = \int \mathcal{N}^{-1/2}(R, R') g(R') R'^2 dR' \quad (1)$$

where  $\mathcal{N}$  is the norm kernel [21] of the alpha-particle + daughter nucleus problem and  $g$  the



**Figure 1.** Alpha decay absolute width for  $^{212}\text{Po}$  using the R-matrix expression  $\Gamma = 2 P \gamma^2$ . Beyond 9 fm the tail of  $\Gamma$  was replaced by  $H_0^+(kR)$  with  $\frac{\hbar^2 k^2}{2\mu} = 8.986$  MeV.

(conventional) formation amplitude

$$\mathcal{N}(R, R') = \left\langle \mathcal{A} \frac{\delta(R_\alpha - R)}{R^2} \phi_\alpha Y_{00} \Psi_D \left| \mathcal{A} \frac{\delta(R_\alpha - R')}{R'^2} \phi_\alpha Y_{00} \Psi_D \right. \right\rangle,$$

$$g(R) = \int d\Omega_R \int d\xi_\alpha \int d\xi_d \Phi_P^* \phi_\alpha(\xi_\alpha) \Psi_D(\xi_d) Y_{00}(\hat{R}). \quad (2)$$

Here  $\Phi_P$  and  $\Psi_D$  are the parent and daughter wave functions, respectively, both in the ground state;  $\phi_\alpha$  is the alpha intrinsic wave function.

The spectroscopic factor  $S = \int dR R^2 G^2(R)$  gives the probability that an alpha particle and the daughter ground state is formed. The penetration factor  $P(R) = kR/|H^+(\chi, kR)|^2$  is the probability that a pre-formed alpha particle can penetrate through the section of the Coulomb barrier between  $R$  and  $\infty$  ( $H^+$  being the outgoing spherical Coulomb function), while the single-particle absolute width  $\Gamma_{sp}$  gives the width of the alpha decay of a system formed by a structureless daughter and a structureless alpha particle. To determine  $\Gamma_{sp}$ , one needs the daughter-alpha relative wave function  $u(R)$ , which is obtained as a solution of the daughter-alpha Schrödinger equation with outgoing boundary condition, and the single-particle absolute width is obtained from [22]

$$\Gamma_{sp} = \frac{\hbar^2 \text{Re}(k)}{\mu} \frac{|u(R)|^2}{|H_0^+(kR)|^2}. \quad (3)$$

### 3. Results

The neutron and proton single-particle model spaces needed for the calculation of the formation amplitude, were defined as the two major valence shells with widths smaller than 1 MeV for both  $^{212}\text{Po}$  and  $^{104}\text{Te}$ . The  $^{212}\text{Po}$  wave function was taken as the seniority zero part of the Tonozuka and Arima [9] wave function from their  $N7$  model space. (But the Tonozuka–Arima basis was spanned by HO states, while here we use Woods–Saxon states with outgoing boundary condition.) The wave function for the nucleus  $^{104}\text{Te}$  was constructed as the product of the two-proton and two-neutron wave functions calculated using a separable force. The relative wave function for the calculation of the penetrability and the single-particle absolute width (3) was calculated with the program [23] using  $Q_\alpha(^{212}\text{Po}) = 8.986$  MeV and  $Q_\alpha(^{104}\text{Te}) = 5.151$  MeV. Figure 1 shows the absolute width  $\Gamma = 2 P \gamma^2$  for the  $^{212}\text{Po}$  alpha decay with the tail substituted by  $H_0^+(kR)$ . The asymptotic value of  $\Gamma$  is 36 times smaller than the experimental one.

Table 1 shows the absolute width  $\Gamma = S \Gamma_{sp}$  for  $^{212}\text{Po}$  and  $^{104}\text{Te}$  alpha decays. The respective single-particle absolute width used are  $\Gamma_{sp}(^{212}\text{Po}) = 0.1247 \times 10^{-12}$  MeV and

$\Gamma_{sp}({}^{104}\text{Te}) = 0.162 \times 10^{-12}$  MeV. The norm kernel was represented by its eigenvalues and eigenfunctions. The eigenvalue problem was solved in an equidistant shifted Gaussian basis, with Gaussian centres at  $R_k = k \Delta R$  and  $\Delta R = 0.56$  fm (for details see Ref. [24].) The terms belonging to eigenvalues larger than 0.001 were taken into account. The agreement with experiment for  ${}^{212}\text{Po}$  is much better with this method.

**Table 1.** Alpha decay absolute width calculated using the expression  $\Gamma = S\Gamma_{sp}$ .

	$\Gamma$ (MeV $\times 10^{14}$ )	$\Gamma_{exp}$ (MeV $\times 10^{14}$ )
${}^{212}\text{Po}$	0.14	0.153
${}^{104}\text{Te}$	0.083	—

#### 4. Discussion and conclusions

The absolute width obtained from  $\Gamma = 2P(R)\gamma^2(R)$  should not depend on  $R$  around and beyond the nuclear surface where the interaction is dominated by the Coulomb interaction. But the approximations involved in the R-matrix formalism make this ideal condition unrealizable [25]. Another alarming fact is that the penetrability  $P(R)$  and the reduced width amplitude  $\gamma$  are calculated using different procedures, which makes the matching inconsistent. On the other hand the absolute width from  $\Gamma = S\Gamma_{sp}$  has no  $R$  dependence because none of  $S$  and  $\Gamma_{sp}$  depends on  $R$ . Another important advantage of calculating the absolute width using  $\Gamma = S\Gamma_{sp}$  is that this formula proves to be less sensitive to the model space [24]. The R-matrix expression uses the formation amplitude, which is very sensitive to the collective behaviour of the wave function at the nuclear surface, which depends on the model space, while the spectroscopic factor involves an integral which strongly diminishes the sensitivity to the model space. This explains the difference between the results obtained for the  ${}^{212}\text{Po}$  decay in the two approaches.

A microscopic formulation of the formation amplitude for alpha particle decay in an extended Woods-Saxon basis including the continuum spectrum (Berggren bases) was presented. To preserve the correct tail of the wave function, no harmonic oscillator expansion was used. The absolute width calculated using the spectroscopic factor for  ${}^{212}\text{Po}$  is consistent with the experimental result. This shows the feasibility of the Shell Model for calculating the alpha width.

As a byproduct of the single-particle complex energy shell model presented here, we calculated the alpha decay width of the nucleus  ${}^{104}\text{Te}$ , whose all valence orbits lie completely in the continuum. An upper bound for the life-time of this nucleus was calculated and found to be  $0.55 \times 10^{-6}$  s. The  $\Gamma$  value obtained in this work shows that the decay model involving the mother wave function has to be treated as in the case of  ${}^{212}\text{Po}$ .

Finally, let us mention the intriguing 1% found in this Shell Model calculation for the amount of clusterization of the  $\alpha$  particle compared with 30% obtained using the Cluster plus Shell Model formalism [3].

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## References

- [1] Gamow G 1928 *Z. Physik* **51** 204
- [2] Condon E U and Gurney R W 1928 *Nature* **122** 439
- [3] Varga K, Lovas R G and Liotta R J 1992 *Nucl. Phys. A* **550** 421
- [4] Thomas R G 1954 *Prog. Theor. Phys.* **12** 253
- [5] Mang H J 1957 *Z. Physik* **148** 582
- [6] Zeh H D 1962 *dissertation* (Univ. Heidelberg)
- [7] Arima A and Yoshida S 1972 *Phys. Lett. B* **40** 15
- [8] Harada K 1962 *Prog. Theor. Phys.* **27** 430
- [9] Tonozuka I and Arima A 1979 *Nucl. Phys. A* **323** 45
- [10] Insolia A, Liotta R J and Maglione E 1988 *Europhys. Lett.* **7** 209
- [11] Delion D S and Suhonen J 2000 *Phys. Rev. C* **61** 024304
- [12] Lenzi S M, Dragún O, Maqueda E E, Liotta R J and Vertse T 1993 *Phys. Rev. C* **48** 1463
- [13] Fliessbach T and Mang H J 1976 *Nucl. Phys. A* **263** 75
- [14] Fliessbach T, Mang H J and Rasmussen J O 1976 *Phys. Rev. C* **13** 1318
- [15] Astier A, Petkov P, Porquet M-G, Delion D S and Schuck P 2010 *Phys. Rev. Lett.* **104** 042701
- [16] Harada K and Rauscher E A 1968 *Phys. Rev.* **169** 169
- [17] Kadmsky S G and Kalechits V E 1970 *Yad. Fiz.* **12** 70
- [18] Jackson D F and Rhoades-Brown M 1977 *Nature* **267** 593
- [19] Jackson D F and Rhoades-Brown M 1977 *Ann. Phys. (N. Y.)* **105** 151
- [20] Berggren T 1968 *Nucl. Phys. A* **109** 265
- [21] Fliessbach T 1976 *Z. Physik A* **277** 151
- [22] Kruppa A T and Nazarewicz W 2004 *Phys. Rev. C* **69** 054311
- [23] Ixaru L Gr, Rizea M and Vertse T 1995 *Comp. Phys. Commun.* **85** 217
- [24] Id Betan R and Nazarewicz W 2012 *Phys. Rev. C* **86** 034338
- [25] Descouvemont P and Baye D 2010 *Rep. Prog. Phys.* **73** 036301