

# Formation of strongly deformed states in entrance channels

A S Zubov<sup>1</sup>, V V Sargsyan<sup>1,2</sup>, G G Adamian<sup>1</sup>, N V Antonenko<sup>1</sup> and W Scheid<sup>3</sup>

<sup>1</sup>Joint Institute for Nuclear Research, 141980 Dubna, Russia

<sup>2</sup>International Center for Advanced Studies, Yerevan State University, 0025 Yerevan, Armenia

<sup>3</sup>Institut für Theoretische Physik der Justus-Liebig-Universität, D-35392 Giessen, Germany

E-mail: adamian@theor.jinr.ru

**Abstract.** The high-spin hyperdeformed (HD) nuclear states treated as dinuclear or quasi-molecular configurations are suggested to be directly populated in heavy ion-collisions at near Coulomb barrier energies.

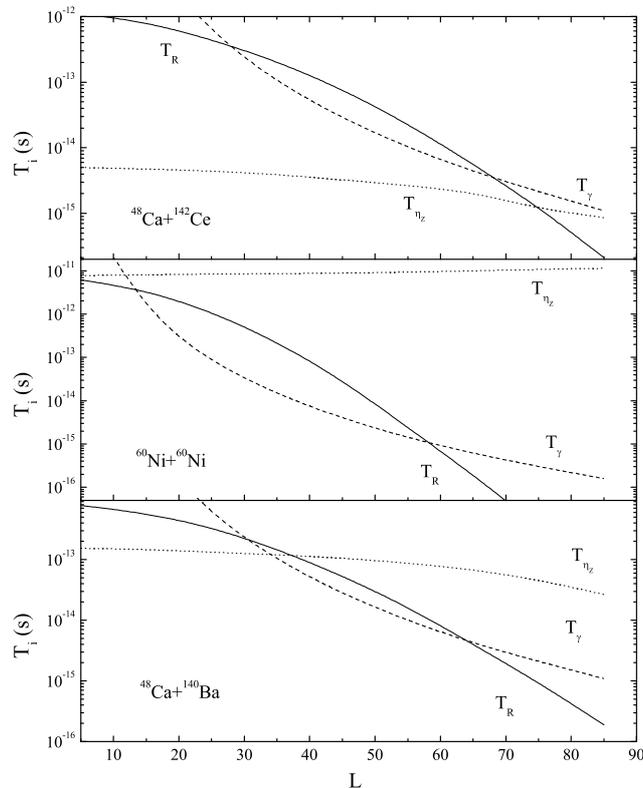
## 1. Introduction

The evidence of low-spin HD states in actinides has been experimentally established in induced fission reactions ( $n,f$ ), ( $t,pf$ ) and ( $d,pf$ ) [1]. The question of experimental indications of high-spin HD states is still open. No convincing evidence for the discrete-line HD rotational bands has been found so far. The question arises how to obtain high-spin HD nuclei. In the high-spin HD studies one can use the experimental and theoretical knowledge about high-spin superdeformed (SD) states and low-spin SD and HD isomers. As shown in shell model calculations [2], the third minimum in the potential energy surface of actinide nuclei belongs to a quasi-molecular configuration of two touching nuclei (clusters). Based on the results of refs. [3, 4, 5] one can be convinced that certain quasi-molecular configurations with dumb-bell shapes have the same quadrupole momenta and momenta of inertia as those measured for the high- and low-spin SD states and low-spin HD isomer states. In light  $\alpha$ -particle nuclei the similarity between the HD and cluster-type states was mentioned in refs. [6, 7, 8]. The HD configurations of  $^{36}\text{Ar}$ ,  $^{40}\text{Ca}$  and  $^{56}\text{Ni}$  seem to be the dinuclear systems (DNS) formed in the reactions  $^{24}\text{Mg} + ^{12}\text{C}$ ,  $^{28}\text{Si} + ^{12}\text{C}$  and  $^{28}\text{Si} + ^{28}\text{Si}$  [7, 9, 10], respectively.

## 2. Cluster model

According to the cluster interpretation the HD state can be treated as a cold rotating DNS (such that the internal excitation energy of the DNS is zero). The relative distance  $R$  between the center of two touching clusters corresponds to the minimum of the potential energy surface and, correspondingly, to the minimum of the pocket of nucleus-nucleus interaction potential. The large overlap of the DNS nuclei is hindered by a repulsive nucleus-nucleus interaction potential at smaller relative distances. The pocket of the nucleus-nucleus potential at a given angular momentum contains the quasi-bound states with energies below the potential barrier and with quite long half-lives. The lowest quasi-bound state, which is identical to the HD state can





**Figure 1.** Time  $T_\gamma$  of  $E2$  transition and tunneling times  $T_{R,\eta Z}$  through the barriers in the  $R$  and the  $\eta Z$  coordinates for the HD states formed in the entrance channel of indicated reactions as the function of angular momentum  $L$ .

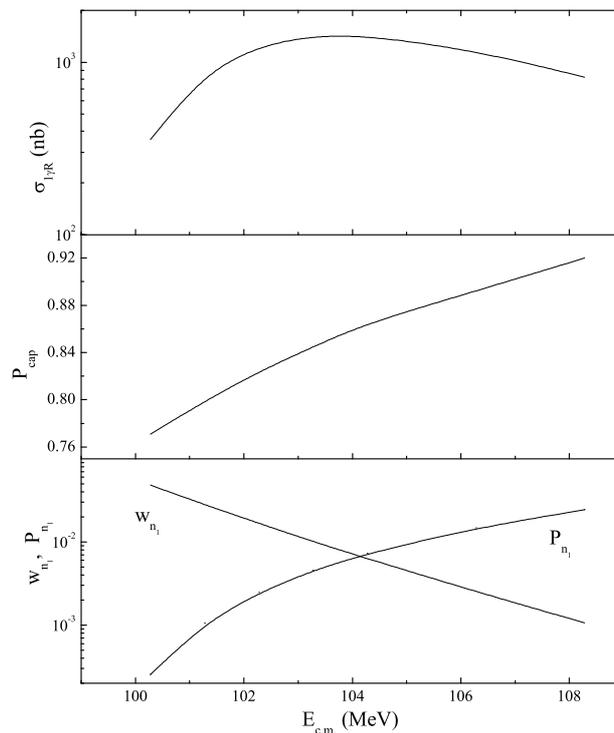
be populated in capture process either by de-excitation (neutron emission) of the initial DNS formed at collision energy above the entrance Coulomb barrier [11], or directly by tunneling through this barrier [5, 12]. Note that the entrance barrier contains the centrifugal term.

### 2.1. HD state formation cross section

The partial cross section  $\sigma_{\text{HD}}(E_{\text{c.m.}}, L)$  for the formation of the HD state at the bombarding energy  $E_{\text{c.m.}}$  in the center-of-mass system depends on the capture probability  $P_{\text{cap}}$ , which is related to the formation of the initial excited DNS and to the probability  $P_{\text{HD}}$  of transformation of this DNS into a HD state by neutron emission [11]:

$$\sigma_{\text{HD}}(E_{\text{c.m.}}, L) = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} (2L + 1) P_{\text{cap}}(E_{\text{c.m.}}, L) P_{\text{HD}}(E_{\text{c.m.}}, L) . \quad (1)$$

For each reaction there is a certain interval of angular momentum  $L$ , where the value  $\sigma_{\text{HD}}$  is maximal and there is a chance to observe the  $\gamma$  quanta from the formed HD states. Outside this interval the identification of HD states seems to be difficult. The probability  $P_{\text{HD}} = \sum_{k=1}^2 P_{n_k} w_{n_k}$  of forming the HD state depends on the probability  $P_{n_k}$  of neutron emission from the  $k$ 'th nucleus of the DNS and on the probability  $w_{n_k}$  of emitting a neutron with large enough kinetic energy to cool the excited DNS to zero internal excitation energy. We assume that the neutron can be evaporated from each fragment of the DNS and the neutron spectrum has Maxwellian form. The probability  $P_{n_k}$  depends on the competition between neutron emission and the DNS decay either in the relative distance coordinate  $R$  or via evolution in the charge



**Figure 2.** Probability  $P_{n_1}$  of neutron emission, probability  $w_{n_1}$  of cold system formation (lower panel), probability of capture  $P_{cap}$  (middle panel) and identification cross section  $\sigma_{1\gamma R}$  of the HD state formed in the  $^{60}\text{Ni}+^{60}\text{Ni}$  reaction at  $L=30-40 \hbar$  (upper panel) as the functions of the bombarding energy  $E_{c.m.}$  in the center-of-mass system.

(mass) asymmetry  $\eta_Z(\eta)$ . The widths of these different processes are calculated within the statistical approach [11].

## 2.2. HD state identification cross section

One can propose the experimental method of identifying the HD states by measuring rotational  $\gamma$  quanta in the HD band in coincidence with decay fragments of the DNS constituting the HD state [5, 11, 12]. This means that the properties of the formed cold system must fulfill the following conditions:

$$T_\gamma < T_R < T_{\eta_Z}, \quad (2)$$

where  $T_R$  and  $T_{\eta_Z}$  are the tunneling time of the HD DNS decay and that of the transformation in charge (mass) asymmetry, respectively. Condition (2) sufficiently restricts the interval of angular momenta in which it is possible to identify HD states, because the characteristic times of different processes depend on  $L$ . The value of  $T_\gamma$  mainly depends on  $L$  through  $E_\gamma$ . The angular momentum dependence of  $T_R$  and  $T_{\eta_Z}$  is defined by the angular momentum dependence of the corresponding barriers. While the barrier in  $\eta_Z$  is weakly affected by the change of  $L$ , the value of the decay barrier in  $R$  decreases much strongly with increasing contribution of the repulsive centrifugal part of the nucleus-nucleus potential. For example, the values  $T_\gamma$ ,  $T_R$  and  $T_{\eta_Z}$  are presented in fig. 1 as the functions of  $L$  for the HD states formed in the entrance channel of reactions  $^{48}\text{Ca}+^{142}\text{Ce}$ ,  $^{60}\text{Ni}+^{60}\text{Ni}$ , and  $^{48}\text{Ca}+^{140}\text{Ba}$ . The condition  $T_\gamma \leq T_R$  is satisfied only in some interval of  $L$ . At very small angular momentum the time of  $E2$  transition becomes enormous due to the small values of  $E_\gamma(L \rightarrow L-2)$ , while at very large  $L$  the quasi-fission (the DNS decay) barrier vanishes. In addition, the minimum in  $\eta_Z$  in this interval of  $L$  should

**Table 1.** The momenta of inertia  $\mathfrak{S}$ , electric quadruple momenta  $Q_2^{(c)}$ , range of angular momenta  $L$ , optimal bombarding energy  $E_{c.m.}$  in the center-of-mass system and cross sections  $\sigma_{HD}$  and  $\sigma_{x\gamma R}$  ( $x = 1 - 3$ ) calculated for the HD states formed in the entrance channel of the indicated reactions.

Reactions	$\mathfrak{S}$ $\hbar^2/MeV$	$Q_2^{(c)}$ $10^2(e\text{ fm}^2)$	$L_{min} < L < L_{max}$	$E_{c.m.}$ MeV	$\sigma_{HD}$ $\mu\text{b}$	$\sigma_{1\gamma R}$ nb	$\sigma_{2\gamma R}$ nb	$\sigma_{3\gamma R}$ nb
$^{40}\text{Ca}+^{40}\text{Ca}$	38.1	13.9	$20 < L < 30$	69.7	10	123	122	120
			$30 < L < 40$	76.7	7.3	110	100	96
			$40 < L < 50$	86.0	2.8	250	130	75
$^{48}\text{Ca}+^{48}\text{Ca}$	51.5	15.6	$40 < L < 50$	66.5	59	23	4.6	1.1
			$50 < L < 60$	75.1	32	2400	970	390
			$60 < L < 70$	85.2	11	1300	270	83
$^{58}\text{Ni}+^{58}\text{Ni}$	83.0	31.3	$20 < L < 30$	104.4	2.2	1700	1200	760
			$30 < L < 40$	107.6	1.1	450	350	270
			$40 < L < 50$	111.8	0.6	340	210	140
			$50 < L < 60$	116.9	0.3	59	17	6.2
$^{60}\text{Ni}+^{60}\text{Ni}$	89.3	32.9	$20 < L < 30$	99.9	2.2	1400	1200	980
			$30 < L < 40$	103.3	2.0	1400	1400	1200
			$40 < L < 50$	106.7	1.0	770	650	570
			$50 < L < 60$	111.5	0.5	280	180	120
$^{48}\text{Ca}+^{140}\text{Ba}$	138	36.8	$40 < L < 50$	134	0.08	35	19	10
			$50 < L < 60$	137.1	0.07	37	23	14
			$60 < L < 70$	140.8	0.02	11	5.3	2.9

be quite deep to provide the condition  $T_R \leq T_{\eta Z}$ . Thus, from the analysis of fig. 1 one can see that the HD states can be identified by measuring the rotational  $\gamma$  quanta in coincidence with the decay fragments at  $20 < L < 60$  and  $40 < L < 70$  in the reactions  $^{60}\text{Ni}+^{60}\text{Ni}$  and  $^{48}\text{Ca}+^{140}\text{Ba}$ , respectively, and cannot be identified by  $\gamma$  quanta at any angular momenta in the reaction  $^{48}\text{Ca}+^{142}\text{Ce}$ .

Using condition (2), one can estimate the probability of emitting  $x$   $\gamma$  quanta from the HD state just before its decay in  $R$  as

$$P_{x\gamma R} = \frac{\Lambda_R(L-2x)}{\Lambda_{tot}(L-2x)} \prod_{k=0}^{x-1} \frac{\Lambda_\gamma(L-2k)}{\Lambda_{tot}(L-2k)}. \quad (3)$$

Here  $\Lambda_{\gamma,R,\eta} = \hbar/T_{\gamma,R,\eta Z}$  are the rates of different competing processes (rotational  $\gamma$  quantum emission and tunneling in  $R$  and  $\eta Z$ ), while  $\Lambda_{tot} = \Lambda_\gamma + \Lambda_R + \Lambda_{\eta Z}$ . Then one can estimate the cross section of emitting at least  $x$   $\gamma$  quantum from the HD state just before its decay in  $R$

$$\sigma_{x\gamma R} = \sum_{L=L_{min}}^{L_{max}} \sigma_{HD}(E_{c.m.}, L) \sum_{x'=x}^{L/2} P_{x'\gamma R}. \quad (4)$$

The values of  $L_{min}$  and  $L_{max}$  are defined by eq. (2). Since the detection of  $\gamma$  quanta in coincidence with the decay products of HD state provides the identification of HD state, the cross section

$\sigma_{x\gamma R}$  is also the identification cross section. This should be taken into account in planning the experiments. Measuring more than one  $\gamma$  quantum will provide the experimentalists an opportunity to determine the momenta of inertia of the analyzed HD states. One can compare the values of  $\sigma_{x\gamma R}$  with the cross section of the formation of the HD state calculated for the same interval of angular momenta  $\sigma_{HD} = \sum_{L=L_{min}}^{L_{max}} \sigma_{HD}(E_{c.m.}, L)$ .

### 3. Results of calculations

Table 1 displays the calculated values of various characteristics of HD states formed in the entrance channel of several reactions. These characteristics include the moment of inertia, electric quadrupole moment, optimal bombarding energy, range of angular momenta and the cross section of formation with emitting up to 3  $\gamma$  quanta before decaying in  $R$ . With increasing angular momentum the value of  $\sigma_{HD}$  decreases due to the increase of the DNS decay in  $R$ . The choice of the reaction is influenced by several reasons. In the entrance channel of the reaction the DNS should have a local potential minimum which is populated, and after neutron emission leads to the formation of the cold quasi-bound state, treated here as a HD state. There should exist a range of angular momentum satisfying conditions (2). As one can see from table 1, symmetric reactions lead to larger  $\sigma_{x\gamma R}$  if more neutron-rich nuclei are used. One can compare, for example, the reactions  $^{40}\text{Ca}+^{40}\text{Ca}$  ( $^{58}\text{Ni}+^{58}\text{Ni}$ ) and  $^{48}\text{Ca}+^{48}\text{Ca}$  ( $^{60}\text{Ni}+^{60}\text{Ni}$ ). Comparing the calculated values of  $\sigma_{1\gamma R}$ ,  $\sigma_{2\gamma R}$  and  $\sigma_{3\gamma R}$  one can see that for some reactions these cross sections decrease rather slowly with the number of emitted  $\gamma$  quanta. This means that an articulate rotational band of the formed HD state can be observed experimentally. For example, in the  $^{60}\text{Ni}+^{60}\text{Ni}$  reaction at initial  $L = 40 - 50$  the calculated value of  $\sigma_{10\gamma R} = 260$  nb. Note that the  $1n$ -emission channel is the optimal evaporation channel for the formation of the HD state.

An example for the dependence of  $\sigma_{1\gamma R}$  and that of the factors  $P_{cap}$ ,  $P_{n_1}$  and  $w_{n_1}$  on  $E_{c.m.}$  is shown in fig. 2. The capture probability slowly increases with  $E_{c.m.}$  and does not affect much the location of the maximum of the function  $\sigma_{1\gamma R}(E_{c.m.})$ . This location is mainly determined by  $P_{n_1}$  and  $w_{n_1}$ . While  $P_{n_1}$  increases,  $w_{n_1}$  decreases with increasing  $E_{c.m.}$ . The final curve representing the function  $\sigma_{1\gamma R}(E_{c.m.})$  has a maximum, which is usually about 8–10 MeV higher than the value of the entrance Coulomb barrier. The location of this maximum gives us the optimal bombarding energy for each considered reaction.

### 4. Summary

We propose to consider the reactions  $^{48}\text{Ca}+^{48}\text{Ca}$  and  $^{58,60}\text{Ni}+^{58,60}\text{Ni}$  as good candidates for the production and experimental identification of the HD states. The estimated identification cross sections  $\sigma_{\gamma R}$  for the HD states formed in these reactions are of the order of 1  $\mu\text{b}$ . We suggested a new experimental method for the identification of the high-spin HD state by measuring the consecutive rotational  $E2$   $\gamma$ -transitions in the HD band in coincidence with decay fragments constituting the HD state. Observing these signatures would be a unique proof of the idea that HD states are cluster-type states, and further that quasi-molecular configurations also exist in heavier nuclei. Then the concept of nuclear molecular states would not be restricted to light and medium nuclei, but would spread over whole mass region.

### References

- [1] Buck B B, Britt H C, Garrett J D and Hansen O 1972 *Phys. Rev. Lett.* **28** 1707  
Baumann F F and Brinkmann K Th 1989 *Nucl. Phys. A* **502** 271  
Krasznahorkay A et al 1998 *Phys. Rev. Lett.* **80** 2073  
Krasznahorkay A et al 1999 *Phys. Lett. B* **461** 15
- [2] Cwiok S, Nazarewicz W, Saladin J X, Plociennik W and Johnson A 1994 *Phys. Lett. B* **322** 304
- [3] Royer G and Haddad F 1995 *J. Phys. G* **21** 339
- [4] Shneidman T M, Adamian G G, Antonenko N V, Ivanova S P and Scheid W 2000 *Nucl. Phys. A* **671** 119
- [5] Adamian G G, Antonenko N V, Nenoff N and Scheid W 2001 *Phys. Rev. C* **64** 014306

- Adamian G G, Andreev A V, Antonenko N V, Nenoff N, Scheid W and Shneidman T M 2003 *Heavy Ion Phys.* **19** 87
- Adamian G G, Andreev A V, Antonenko N V, Nenoff N, Scheid W and Shneidman T M 2003 *Acta Phys. Pol. B* **34** 2147
- Adamian G G, Antonenko N V and Scheid W 2003 *Fizika B* **12** 21
- Kuklin S N, Adamian G G, Antonenko N V and Scheid W 2008 *Int. J. Mod. Phys. E* **17** 2020
- [6] Cindro N (1978) *J. Phys. G* **4** L23 (1978)
- Greiner W, Park J Y and Scheid W *Nuclear Molecules* (World Scientific, Singapore, 1995).
- [7] Sanders S J, Szanto de Toledo A and Beck C 1999 *Phys. Rep.* **311** 487
- [8] Cseh J, Lévai G, Ventura A and Zuffi L, 1998 *Phys. Rev. C* **58** 2144;  
Darai J, Cseh J and Jenkins D G 2012 *Phys. Rev. C* **86** 064309
- [9] Beck C *et al* 2000 *Phys. Rev. C* **63** 014607  
Rousseau M *et al* 2002 *Phys. Rev. C* **66** 034612
- [10] Sciani W, Otani Y, Lépine-Szily A, Benjamim E A, Chamon L C, Lichtenthäler Filho R, Darai J and Cseh J (2009) *Phys. Rev.* **80** 034319  
Cseh J, Darai J, Sciani W, Otani Y, Lépine-Szily A, Benjamim E A, Chamon L C and Lichtenthäler Filho R 2009 *Phys. Rev. C* **80** 034320
- [11] Zubov A S, Sargsyan V V, Adamian G G, Antonenko N V and Scheid W 2010 *Phys. Rev. C* **81** 024607
- [12] Zubov A S, Sargsyan V V, Adamian G G, Antonenko N V and Scheid W 2010 *Phys. Rev. C* **82** 034610