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Cluster structure and isoscalar monopole excitation in light nuclei

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Abstract. Isoscalar monopole excitation to cluster states in light nuclei is in general strong as to be comparable with the single particle strength and shares about 20 % of the sum rule value. In the present paper, the isoscalar monopole strength function in ^{16}O is discussed up to $E_x \lesssim 40$ MeV as a typical example. We found that 1) two different types of monopole excitations exist in ^{16}O; one is the monopole excitation of cluster states which is dominant in the lower energy part, and the other is the monopole excitation of the mean-field type such as one-particle one-hole ($1p1h$) which is attributed mainly to the higher energy part, 2) this character of the monopole excitations originates from the fact that the ground state of ^{16}O with the dominant doubly closed shell structure has a duality of the mean-field-type as well as alpha-clustering character, and 3) the monopole strength is much enhanced by the α-type ground state correlation.

1. Introduction

Isoscalar monopole excitation is related to a density fluctuation of nucleus. The typical example is the isoscalar giant monopole resonance (ISGMR) observed as a single bump in medium and heavy nuclei, which exhausts almost 100 % of the energy weighted sum rule (EWSR) value. It is interesting to study what happens for the ISGMR in lighter nuclei. When the nuclear masses decrease from medium nuclei to light nuclei, the isoscalar monopole strengths are in general fragmented. Table 1 shows the experimental monopole strengths of the low-lying states in ^{16}O, ^{12}C, ^{11}B, and ^{4}He [1, 2]. The strengths of the clusters states are rather strong they are comparable with the single particle strength (~ 4 fm^2) and shares about 20 % of the EWSR value [3]. In this paper, we concentrate on discussing the case of ^{16}O.

The histogram in Fig. 1 shows the experimental isoscalar monopole strength function in ^{16}O obtained by the inelastic α scattering [11] (the data below $E_x \sim 10$ MeV is absent due to the experimental condition). The monopole strengths split over wide energy region, and one can see discrete peaks in low energy ($E_x \lesssim 16$ MeV) and gross three-bump structure in higher energy region ($16 \lesssim E_x \lesssim 40$ MeV). The experimental data is compared with the RRPA calculation [12], the result of which is shown by the solid line in Fig. 1. In order to match their calculation to the experimental centroid, the calculated strength function was shifted down in energy by 4.2 MeV and furthermore they normalized it by multiplying the RRPA curve by a factor of 0.25 [11], although their calculation failed to reproduce the 0^+ states found in the low energy region. In the non-relativistic RPA (SRPA) calculations for ^{16}O [13] a significant discrepancy is also revealed as compared with the experimental data, in particular, in the low energy region, although the gross structures at the higher energy region in the RPA calculations are in rather good agreement with the data.

Table 1. Experimental monopole strengths ($M^{\rm exp}$(E0) [fm^2]) in ^{16}O, ^{12}C, ^{11}B, and ^{4}He [1, 2]. In ^{11}B, the isoscalar monopole transition rates (B(E0, IS) [fm^4]) are presented with "a)". $P^{\rm e.w.}$ stands for the percentage of the energy weight strength to the isoscalar monopole EWSR value. The assignment of the structure of each state together with the calculated monopole strength ($M^{\rm cal}$(E0)) is referred from Refs. [4, 5, 6, 7, 8, 9]. h. o. denotes the higher nodal (see text).

	J_i	J_f	$E_x(J_f)$ [MeV]	$M^{\rm exp}$(E0)	$P^{\rm e.w.}$	Structure of J_f	$M^{\rm cal}$(E0)
^{16}O	0^+_1	0^+_2	6.05	3.55 ± 0.21	3.5%	α+^{12}C(0^+_1)	3.9 [5] (3.88 [10])
		0^+_3	12.1	4.03 ± 0.09	8.9%	α+^{12}C(2^+_1)	2.4 [5] (3.50 [10])
		0^+_4	13.6	no data	–	h. n. α+^{12}C(0^+_1)	2.4 [5] (– – – –)
		0^+_5	14.1	3.3 ± 0.7	6.9%	α+^{12}C(1^-_1)	2.6 [5] (– – – –)
		0^+_6	15.1	no data	–	4α-gas	1.0 [5] (– – – –)
^{12}C	0^+_1	0^+_2	7.65	5.4 ± 0.2	16%	3α-gas	6.7 [6]
^{11}B	$3/2^-_1$	$3/2^-_2$	5.02	$< 9^{\rm a)}$	< 1%	shell-model-like	$0.6^{\rm a)}$ [8]
		$3/2^-_3$	8.56	$96 \pm 12^{\rm a)}$	12%	$\alpha + \alpha + t$	$92^{\rm a)}$ [8]
^{4}He	0^+_1	0^+_2	20.2	1.10 ± 0.16	11%	$3N - N$	1.38 [9]

Recently the structure study of ^{16}O has made a great advance up to $E_x \simeq 16$ MeV around the 4α disintegration threshold. The six lowest 0^+ states of ^{16}O, up to $E_x \simeq 16$ MeV, including the ground state, have for first time been reproduced very well with the 4α orthogonality condition model (OCM) [4]. The six 0^+ states have the following characteristic structures [4]: 1) the ground state (0^+_1) has dominantly a doubly-closed-shell structure, 2) the 0^+_2 state at $E_x = 6.05$ MeV and the 0^+_3 state at $E_x = 12.05$ MeV have mainly α+^{12}C structures where the α-particle orbits around the ^{12}C(0^+_1) core in an S-wave and around the ^{12}C(2^+_1) core in a D-wave, respectively, 3) the 0^+_4 ($E_x = 13.6$ MeV) and 0^+_5 ($E_x = 14.1$ MeV) states mainly have α+^{12}C(0^+_1) structure with higher nodal behavior and α+^{12}C(1^-) structure, respectively, and 4) the 0^+_6 state at 15.1 MeV is a strong candidate of the 4α condensate, $(0S)^4_\alpha$, with the probability of 61 %. The results of 1) $\sim$ 3) are consistent with those obtained by the α+^{12}C(0^+_1,2^+_1,4^+_1) OCM calculation [10]. The 4α OCM reasonably reproduces the experimental monopole strengths within a factor of 1.5 (see Table 1) as well as the decay widths [4, 5]. The purpose of this paper is to study whether the 4α OCM can reproduce the experimental monopole strength function in the low energy region ($E_x \lesssim 16$ MeV) in ^{16}O, a region which is difficult to be treated in the mean-field theory, and to show the excitation mechanism of the cluster states by the monopole transition.

2. Calculated monopole strength function of ^{16}O with 4α OCM

Fig. 2 [5] shows the calculated isoscalar monopole strength function of ^{16}O with the 4α OCM, where we use the calculated monopole matrix elements and the calculated decay widths for the six 0^+ states up to $E_x \simeq 16$ MeV obtained by the 4α OCM calculation, also the experimental excitation energies for the six 0^+ states are employed. We can see a rather good correspondence with the experimental data. The fine structures in the calculated strength function, i.e. one peak at $E_x = 12.1$ MeV (corresponding to the 0^+_3 state), one shoulder-like peak at $E_x = 13.8$ MeV (0^+_4), two peaks at $E_x = 14.1$ MeV (0^+_5) and 15.1 MeV (0^+_6), are well reproduced. On the other hand, it has been recently suggested that some 0^+ states with the dominant α+^{12}C(Hoyle:0^+_2) configuration [14] can contribute to the energy region of the first bump in Fig. 1 ($E_x \sim 18$ MeV) and their contributions are estimated to be less than several % of the EWSR value.

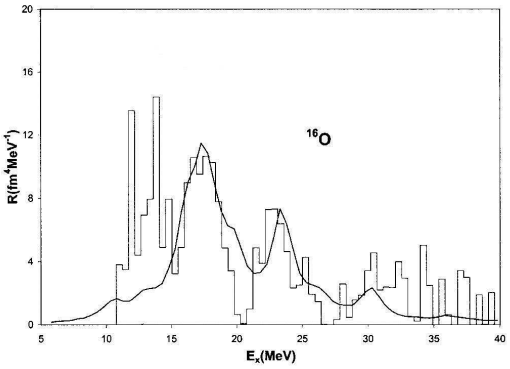

Figure 1. Experimental isoscalar monopole strength function of ^{16}O [11] is shown by the histogram. The real line is the calculated result by the relativistic RPA calculation [12].

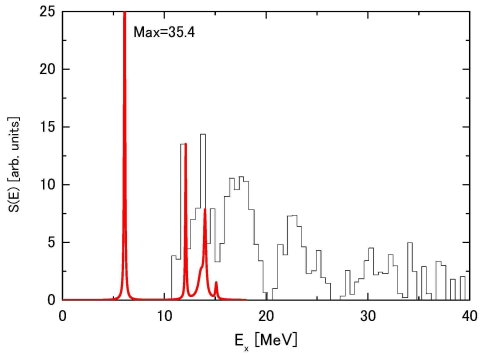

Figure 2. (Color online) Calculated isoscalar monopole strength functions of ^{16}O with the 4α OCM (bold line) [5] and experimental data (thin line: see Fig. 1 [11]).

3. Mechanism of monopole excitation of cluster states from the ground state

It is instructive to discuss the mechanism of why the five α cluster states (0_2^+, 0_3^+, 0_4^+, 0_5^+, and 0_6^+) of ^{16}O are excited relatively strongly from the shell-model-like ground state by the isoscalar monopole transition.

The wave function of the ^{16}O ground state has the dominant $(0s)^4(0p)^{12}$ configuration, corresponding to the SU(3) $(\lambda,\mu) = (0,0)$ wave function. This character is in common in the mean-field calculations and cluster-model calculations, and is also obtained by the no-core shell-model calculation [16]. According to the Bayman-Bohr theorem [15], this doubly closed shell-model wave function is mathematically equivalent to a single cluster-model wave function of α+^{12}C as well as 4α with the total harmonic oscillator (h.o.) quanta $Q = 12$ [3, 5],

$$\begin{aligned}
\det|(0s)^4(0p)^{12}| &= N_0 \times \mathcal{A}\left\{\left[u_{40}(\boldsymbol{\xi}_3, 3\nu)\phi_{L=0}(^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}\phi_{\mathrm{cm}}(\boldsymbol{R}_{\mathrm{cm}}), \quad (1)\\
&= N_2 \times \mathcal{A}\left\{\left[u_{42}(\boldsymbol{\xi}_3, 3\nu)\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}\phi_{\mathrm{cm}}(\boldsymbol{R}_{\mathrm{cm}}), \quad (2)\\
&= \hat{N}_0 \times \mathcal{A}\Big\{\left[u_{40}(\boldsymbol{\xi}_3, 3\nu)\left[u_{40}(\boldsymbol{\xi}_2, 8\nu/3)u_{40}(\boldsymbol{\xi}_1, 2\nu)\right]_{L=0}\right]_{J=0}\\
&\qquad \times\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\Big\}\phi_{\mathrm{cm}}(\boldsymbol{R}_{\mathrm{cm}}), \quad (3)
\end{aligned}$$

where ϕ_{cm} denotes the wave function of the c.o.m. motion of ^{16}O, and $\boldsymbol{\xi}_k$ ($k = 1 \sim 3$) stand for the Jacobi coordinates between the clusters. The functions $\phi(\alpha)$ and $\phi_L(^{12}\mathrm{C})$ represent, respectively, the internal wave function of the α cluster and that of ^{12}C with the angular momentum of L. $N_L(\hat{N}_0)$ is the normalization factor. The relative wave function between the α and ^{12}C clusters in Eqs. (1) and (2) is described by the h.o. wave function $u_{QL}(\boldsymbol{\xi}, \beta)$ with $Q = 4$. Eqs. (1) and (2) mean that the doubly closed shell-model wave function has an α+^{12}C cluster degree of freedom. In addition, Eq. (3) demonstrates that the doubly closed shell-model wave function also possesses a 4α cluster degree of freedom. Thus the ground state of ^{16}O has the mean-field of degree of freedom as well as the cluster degree of freedom. We call this the dual nature.

On the other hand, the isoscalar monopole operator of ^{16}O can be decomposed into the internal part (acting on the constituent clusters) and relative part (with respect to the

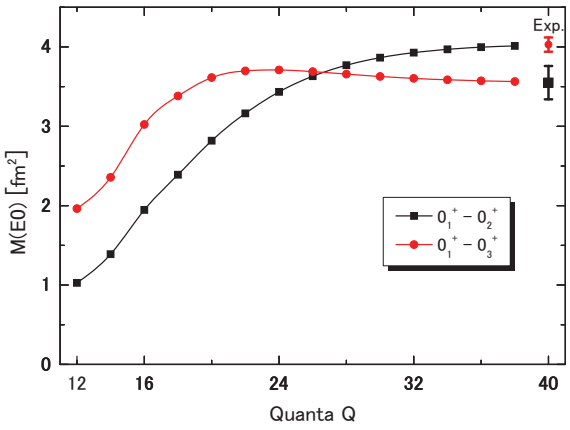

Figure 3. (Color online) Dependence of the monopole strength M_Q(E0) on the model space of the ^{16}O ground state wave function characterized by the harmonic oscillator quanta Q with the α+^{12}C(0_1^+,2_1^+,4_1^+) OCM [3]. The square and circle points correspond to $M_Q(\mathrm{E0}; 0_1^+ \to 0_2^+)$ and $M_Q(\mathrm{E0}; 0_1^+ \to 0_3^+)$, respectively, and the experimental data are also shown there.

clusters) [3, 5]:

$$\mathcal{O}^{\mathrm{IS}}_{\mathrm{E0}}(^{16}\mathrm{O}) = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}})^2 = \mathcal{O}^{\mathrm{IS}}_{\mathrm{E0}}(\alpha) + \mathcal{O}^{\mathrm{IS}}_{\mathrm{E0}}(^{12}\mathrm{C}) + 3\boldsymbol{\xi}_3^2 \quad (4)$$

$$= \sum_{k=1}^{4} \mathcal{O}^{\mathrm{IS}}_{\mathrm{E0}}(\alpha_k) + \sum_{k=1}^{4} 4(\boldsymbol{R}_{\alpha_k} - \boldsymbol{R}_{\mathrm{cm}})^2, \quad (5)$$

where $\boldsymbol{R}_k$ denotes the c.o.m. coordinate of the k-th α cluster in the 4α system. ¿From Eqs. (1), (2), and (4), we can understand the reasons why the 0_2^+ and 0_3^+ states of ^{16}O with the $\alpha + ^{12}$C cluster structure are excited by the monopole transition as follows: 1) the ground state of ^{16}O is of the SU(3) $(\lambda, \mu) = (0, 0)$ nature with the α-clustering degree of freedom, 2) the relative part of the monopole operator referring to the α+^{12}C relative motion, $3\boldsymbol{\xi}_3{}^2$, in Eq. (4) can activate the α-cluster degree of freedom, and 3) the $\alpha + ^{12}$C states are excited by the monopole operator.

Here, we discuss the important role of the α-cluster-type ground-state correlation significantly enhancing the monopole strength. This is studied with the α+^{12}C(0_1^+,2_1^+,4_1^+) OCM [3], which reproduces the monopole strengths of the 0_2^+ and 0_3^+ states in ^{16}O [10] (see Table 1). The component of the SU(3) $(\lambda, \mu) = (0, 0)$ wave function with the total h.o. oscillator $Q = 12$, i.e. $(0s)^4(0p)^{12}$, in the ground state of ^{16}O, is about 90 % in the α+^{12}C OCM calculation as well as the 4α OCM [3, 4, 5, 10]. The model space of the α+^{12}C OCM is characterized by the total harmonic oscillator quanta Q of ^{16}O. The remaining component of about 10 % in the ground-state wave function corresponds to the α-type ground-state correlation with $Q > 12$. When the pure SU(3) $(\lambda, \mu) = (0, 0)$ wave function is adopted as the ^{16}O ground-state wave function, the monopole strengths to the 0_2^+ and 0_3^+ states are about half or one third smaller than the experimental data, although the calculated values reproduce the order of magnitude of the experimental values (see Fig. 3). When the model space of the ground state increases from the lowest value of $Q = 12$, the calculated monopole strengths to the 0_2^+ and 0_3^+ states are gradually increasing and reproduce the experimental values at $Q \simeq 30$ within a factor of 1.13, as shown in Fig. 3. This means that the component with $Q > 12$ in the ground-state wave

function (corresponding to the α-type ground-state correlation) gives a coherent contribution to enhancement of the monopole strengths. From these results, we can learn that the α-cluster-type ground-state correlation plays the important role in reproducing the monopole strengths.

The reason why the 0_6^+ state of ^{16}O with the 4α-gas-like character is excited by the monopole transition can be also understood from the property of the ground state of ^{16}O. The doubly closed shell-model wave function is mathematically equivalent to the single 4α cluster wave function with $Q = 12$ in Eq. (3). This equation means that the ground state of ^{16}O inherently has a 4α-cluster degree of freedom. The relative part (or second term) of the monopole operator in Eq. (5) can excite the relative motion among the 4α particles. In other words, the monopole operator has an ability to populate democratically 4α particles by $2\hbar\omega$ with respect to the c.o.m. coordinate of ^{16}O. The resultant state, thus, has some amount of the overlap with the 4α-gas-like state or $\alpha + {}^{12}\mathrm{C}(0_2^+)$, i.e. 0_6^+, with the 4α-condensate-like structure [4]. It is noted that the mechanism of the 4α-gas-like state being populated by the monopole transition is similar to that of the Hoyle state (0_2^+) with the 3α-gas-like structure, excited by the monopole transition, although the ground state of ^{12}C has a shell-model-like compact structure with the main configuration of SU(3) $(\lambda, \mu) = (0, 4)$ [3].

As for the 0_5^+ state, its main configuration is $\alpha + {}^{12}\mathrm{C}(1_1^-)$. According to the Bayman-Bohr theorem [15], the SU(3) $(0, 0)$ state of ^{16}O has no component of the $\alpha + {}^{12}\mathrm{C}(1_1^-)$ channel. However, the monopole strength to the 0_5^+ state is as large as 3 fm^2 [5]. This is the reason that the 0_5^+ state has small but important components of the $\alpha + {}^{12}\mathrm{C}(0_1^+, 2_1^+, 0_2^+)$ configurations. Since these three configurations can be excited from the ground state of ^{16}O by the monopole operator as discussed above, their respective contributions are coherently added to provide the relatively large monopole strength to the 0_5^+ state. Concerning the 0_4^+ state, the situation is similar to the case of the 0_5^+ state. The 0_4^+ state has also small but non negligible components of the $\alpha + {}^{12}\mathrm{C}(0_1^+, 2_1^+, 0_2^+)$ channels, which contribute to the monopole strength for the 0_4^+ state.

4. Summary

The isoscalar monopole excitation is useful to search for cluster states in light nuclei. Two different types of the monopole excitations and the duality of the ground state discussed in this paper seem to exist in general in light nuclei [3, 5]. It is desirable that systematically study the isoscalar monopole excitations in $4n$ and neutron-rich nuclei as well as light hypernuclei may be studied.

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