Journal of Physics: Conference Series **436** (2013) 012003 doi:10.1088/1742-6596/436/1/012003

Recent developments in nuclear cluster theory

Hisashi Horiuchi

Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
International Institute for Advanced Studies, Kizugawa, Kyoto 619-0225, Japan

E-mail: horiuchi@rcnp.osaka-u.ac.jp

Abstract. The major novelties that have emerged in nuclear cluster theory since the last, 9th, cluster conference is reviewed. Among the marked developments made in many traditional and novel subjects of nuclear cluster physics, four subjects are chosen in this introductory talk. They are 1. neutron-rich nuclei, 2. cluster gas, 3. coexistence of cluster and mean-field dynamics, and 4. *ab initio* approaches. A representative example of the present status of the field is that it is studied in the framework of *ab initio* nuclear theory as well. We discuss a recent *ab initio* calculation of the Hoyle state of ^{12}C, which supports its cluster-gas-like structure.

1. Introduction

Complying with the request of the organizers of the present conference, the purpose of this talk is to review the major novelties in nuclear cluster theory since the last, 9th, cluster conference at Stratford-upon-Avon. Actually, marked progress has been made in many traditional and novel subjects of nuclear cluster theory. I have chosen four subjects among them in this talk. The substantial progress in these subjects shows the importance of cluster dynamics as a basic dynamics of the nuclear many-body system. The topics to be covered are 1. neutron-rich nuclei, 2. cluster gas, 3. coexistence of cluster and mean-field dynamics, and 4. *ab initio* approaches. The discussions are largely based on a recent publication [1]. A representative result of present-day nuclear cluster physics is the application of *ab initio* nuclear theory to the study of the Hoyle state of ^{12}C. We discuss that a recent calculation [2] confirms the cluster-gas-like structure of the Hoyle state.

2. Neutron-rich nuclei: novel types of clustering

2.1. Cluster thresholds and neutron thresholds

In most cases, cluster structure in neutron-rich nuclei consists of two parts: a clustered core and valence neutrons around the core. Therefore the cluster dynamics in neutron-rich nuclei consists of cluster dynamics in the core part and neutron dynamics around the core. The cluster dynamics in the core part is the same as that in stable nuclei. The neutron dynamics around a clustered core is a new ingredient, which does not exist in stable nuclei. Because of this new feature, when the ground state and/or low-lying states have molecular-orbit-type cluster structure, these cluster states do not obey Ikeda's threshold rule proposed for stable nuclei. This is a very distinct character of clustering in neutron-rich nuclei. Since neutron-rich nuclei have weakly-bound valence neutrons, neutron thresholds are located low in excitation energy, but, on the contrary, cluster thresholds are located high in excitation energy. This fact means [1] that the ground state and/or low-lying states having molecular-orbit-type cluster

structure are deeply bound from the cluster thresholds although they are weakly bound from the neutron thresholds. Fig. 1 shows this situation in ^{12}Be, whose ground state has, as a dominant component, molecular-orbit-type clustering with a 2α core.

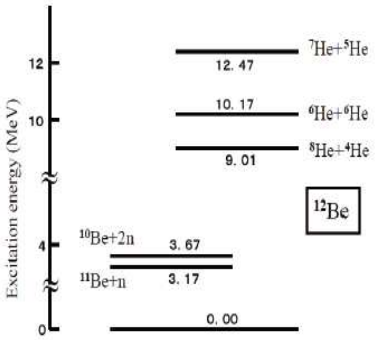

Figure 1. Cluster threshold energies (^{8}He + ^{4}He, ^{6}He + ^{6}He, ^{7}He + ^{5}He) are high, while neutron threshold energies (^{11}Be + n, ^{12}Be + $2n$) are low in ^{12}Be, whose ground state has a dominant cluster structure. The figure is taken from Ref. [1].

2.2. Clustering in the core and neutron dynamics around the core depend on each other
Cluster dynamics in the core and neutron dynamics around a clustered core do of course interact with each other. A very characteristic example is seen when the so-called molecular σ-orbit is occupied by valence neutron(s). The σ-orbit is the intruder orbit which comes down from the higher shell when the clustering of the core part is increased [1]. In the case of Be isotopes, the σ-orbit around the 2α core corresponds to the $(n_x, n_y, n_z) = (0, 0, 2)$ orbit of the anisotropic harmonic oscillatior model with axial symmetry. In the case of F isotopes, the σ-orbit around the ^{16}O + α core corresponds to the $(n_x, n_y, n_z) = (0, 0, 3)$ orbit of the anisotropic harmonic oscillatior model with axial symmetry. When the σ-orbit is occupied by valence neutron(s), the clustering of the core part is forced to become larger because the enhancement of the core clustering lowers the single-particle enegy of the σ-orbit. For example, many authors have shown that the ground-state configurations of ^{11}Be and ^{12}Be are dominantly of $(\pi_{3/2})^2\sigma$ and $(\pi_{3/2})^2(\sigma)^2$, respectively around the 2α core [1]. Fig. 2 shows predictions [3] of antisymmetrized molecular dynamics (AMD) for three types of excited cluster bands having ^{15}N + α core in F isotopes. All three bands have one proton hole in the $0p$-shell. The bands labelled by σ^2 (first band) and $\sigma\pi$ (second band) have two neutrons and one neutron in the valence σ-orbit, respectively. A very interesting prediction of this figure is that the excited bands of the first and second types come down rapidly to low-excitation-energy region as the neutron number increases, as low as around 2 MeV in ^{27}F and ^{29}F.

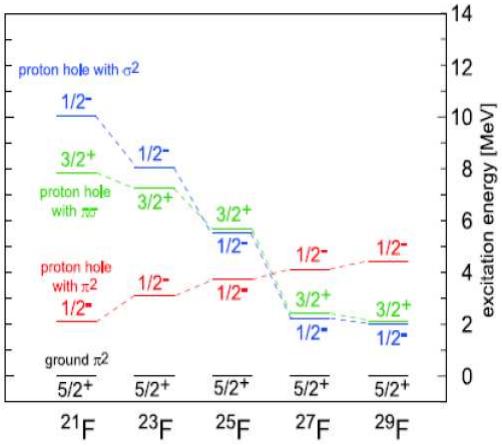

Figure 2. Excitation energies of band-head states of three types of excited cluster bands in F isotopes predicted by AMD [3]. All three bands have a ^{15}N + α core due to one proton hole in the $0p$-shell. The bands labeled by σ^2 and $\sigma\pi$ have two neutrons and one neutron in the valence σ-orbit, respectively.

2.3. Atomic orbits and mixture of molecular and atomic orbits

Recent investigations of the cluster structure in neutron-rich nuclei show the formation of cluster states with atomic-orbital configurations of neutrons at somewhat higher excitation energies. In this configuration, neutrons move not over the whole system but around individual clusters constituting the core. A neutron-rich clustered nucleus having valence neutrons in atomic orbits has a cluster structure similar to that of a stable nucleus with no neutrons moving over the whole system. Namely, the structure of such a necleus is composed of neutron-rich clusters. Representative examples are ^{6}He + ^{6}He and ^{8}He + ^{4}He structures in ^{12}Be. These structures have been studied in Refs. [4, 5, 6, 7, 3, 1]. In Ref. [8] it is reported that the AMD study of ^{22}Ne gives (1) a mean-field-type structure for the ground-state band, (2) $K^\pi = 0_2^+, 0_1^-$, and 1^- bands with molecular-orbit structure around the ^{16}O + α core, and (3) $K^\pi = 0_3^+$ and 0_2^- bands with atomic-orbit structure of ^{18}O + α type. The excitation energy of a cluster state with atomic-orbit structure is rather close to the breakup threshold energy into the constituent neutron-rich clusters.

The existence of molecular-orbital clustering and atomic-orbital clustering exhibits the richness of valence-neutron dynamics, and in Ref. [7] clustering with a mixture of molecular and atomic orbits is also discussed.

2.4. Dineutron

The dineutron is unbound in free space but can be bound inside a nucleus by the Borromean effect. At present ^{11}Li is the only system where dineutron structure has been confirmed experimentally [9]. In ^{11}Li, the dineutron structure (^{11}Li = ^{9}Li + dineutron) is intimately related to the two-neutron halo structure and the breaking of N=8 magicity. A very attractive explanation was given in Ref. [10] where, in addition to the pairing-force effect of valence neutrons, the tensor-force effect in the ^{9}Li core was newly introduced to give a simultaneous explanation of the above three problems, namely the dineutron formation, the two-neutron halo, and the breaking of N=8 magicity. It is discussed that the Pauli blocking in pairing and tensor-force correlations cause that the gap between $0p$ and $1s$ orbits is small (magicity breaking), from which it follows that the $1s$ orbit enhances the halo character, and the parity mixing with approximately equal weights of the opposite parities due to the approximate degeneracy of 0p and 1s orbits induces spatial clustering. The Coulomb breakup cross section was reproduced well, with a large matter radius and a large weight of the $(1s)^2$ component.

3. Cluster-gas states: novel nuclear structure and non-localized clustering

3.1. 4α condensate-like state in ^{16}O

The Hoyle state in ^{12}C was studied in detail in 1970's by 3α RGM and GCM which succeeded in reproducing almost all experimental data. These RGM and GCM studies predicted a large radius of this state. In 2001 Ref. [11] proposed the interpretation of the Hoyle state as a 3α condensate-like state by using the THSR wave function $\mathcal{A}[\prod_{j=1}^3 \exp(-(2/B^2)\mathbf{X}_j^2)\phi(\alpha_j)]$. In 2003 it was discovered [12] that the 3α RGM/GCM wave functions of the Hoyle state are almost equivalent to a single THSR wave function:

$$|\,\langle\Phi(\text{single THSR})|\Psi(3\alpha\text{RGM/GCM})\rangle\,|^2 \approx 100\% \quad \text{for the Hoyle state.} \tag{1}$$

Studies for the 4α condensate-like state in ^{16}O were reported in Refs. [13, 14]. In ^{16}O, six 0^+ states are observed up to the 6th 0^+ state at 15.1 MeV. The 4α OCM calculation of Ref. [13] could reproduce the energies and decay widths of six 0^+ states up to the 6th 0^+ state. This OCM study assigns the observed 0_6^+ state the 4α condensate-like structure, because the reduced-α-width amplitude $y(r)$ of the calculated 0_6^+ state has a large amplitude only in the ^{12}C(0_2^+) + α channel, whereas the amplitudes in other channels are much suppressed. Since $y(r)$ in the

Hoyle-state (^{12}C(0_2^+)) channel has a very long tail stretching out to about 20 fm and the Hoyle state has a 3α condensate-like structure, one can certainly say that the calculated 0_6^+ state has a character of 4α-condensate-like structure.

The 4α threshold is about 7 MeV higher than the ^{12}C + α threshold. Thus one may think that the width of a 4α-condensate-like state expected to be around the 4α threshold must have a very large decay width. But contrary to this conjecture, the calculated 0_6^+ state has a rather small width of 136 keV. The reason of this small width is simple and twofold. The first reason is the small overlap of the exotic structure of a 4α-condensate-like state with ^{12}C + α channels except the Hoyle-state channel ^{12}C(0_2^+) + α. The overlap is small because the ^{12}C states other than the Hoyle state are compact. The second reason is the very small decay Q-value of the decay to the Hoyle-state channel, which makes the Coulomb-barrier penetration very small although the overlap of this channel with the 4α-condensate-like state is rather large. The width of 136 keV of the calculated 0_6^+ state is in a good accord with the observed value, 166 keV. The above arguments are expected to be general and applicable to heavier $n\alpha$-condensate-like states as well.

Recently Ref. [15] reported that the fine structure of the isoscalar monopole strength function in the low energy region up to about 16 MeV in ^{16}O is rather satisfactorily reproduced within the 4α OCM. It is reported that the observed fine structure can be attributed to the 0_3^+, 0_4^+, 0_5^+, and 0_6^+ states.

3.2. Cluster-gas-like states in various nuclei

Cluster-gas-like states other than ^{12}C and ^{16}O have been studied in various nuclei by many authors. As to ^{11}B (^{11}C), Refs. [16, 17, 18, 19] discussed possible gas-like structure of 2α + t (2α + ^{3}He) for the third $3/2^-$ state at 8.11 MeV (8.56 MeV). Refs. [20, 21, 27, 23, 24, 25] discussed candidates for gas-like states of 3α + p (3α + n) type in ^{13}C (^{13}N). The possibility of $n\alpha$-condensate-like states with $n > 4$ was discussed in Ref. [29]. In Refs. [26, 27, 28], the possiblity of α-gas-like states around core nuclei like ^{16}O and ^{40}Ca was discussed.

An important category of the nuclear states strongly related to the cluster-gas-like states is the states that contain cluster(s) having cluster-gas-like structure themselves. Representative examples are the molecular states of ^{12}C + ^{12}C in which one or two ^{12}C cluster(s) is(are) in the Hoyle state (0_2^+). The states of this type were studied both experimentally and theoretically (see Refs. [1, 30]). The 'Hoyle + α' state is discussed by Funaki in this conference.

3.3. Non-localized clustering

The THSR wave function was introduced originally in order to describe cluster-gas-like states. However it was found already in 2003 [12] that the 3α RGM/GCM wave functions for the ground state are equivalent by 93% to a single THSR wave functions:

$$|\langle\Phi(\text{single THSR})|\Psi(3\alpha\text{RGM/GCM})\rangle|^2 \approx 93\% \quad \text{for the ground state.} \tag{2}$$

Recently it was found [31] that the ^{16}O + α RGM/GCM wave functions for both the ground-band states and the $K^\pi = 0_1^-$ band states are almost equivalent to single THSR wave functions:

$$|\langle\Phi(\text{single THSR})|\Psi(^{16}\text{O} + \alpha\ \text{RGM/GCM})\rangle|^2 \approx 100\% \tag{3}$$

$$\text{for ground and } K^\pi = 0_1^- \text{ band states.} \tag{4}$$

The THSR wave function does not appear to represent localized clustering, but the intercluster Pauli repulsion creates non-zero separation between the clusters. We may say that the dynamics prefers non-localized clustering but the Pauli principle makes the system look like consisting of localized clusters. Non-localized clustering in ordinary cluster states looks to be intimately related to cluster-gas-like structure.

4. Coexistence of cluster and mean-field dynamics

It was pointed out in Ref. [17] that the observed $E0$ excitation strength to cluster states is in general large, with its magnitude comparable to the single-particle strength. For example in ^{16}O we have large observed $E0$ values for three excited states, 0_2^+, 0_3^+, and 0_5^+ (3.6, 4.0, and 3.3 fm^2, respectively). In mean-field theories there is no simple way to explain the three observed strong $E0$ transitions of similar magnitudes. On the other hand, cluster model calculations can easily reproduce them. The reason is explained [33] by the duality property of the ground state, which means that the ground state wave function has both mean-field-type and clustering characters. This duality is well-known as the Bayman-Bohr theorem [32].

The dual character of the ground state means that the ground state has both the mean-field-type degree of freedom and the clustering degree of freedom. When the mean-field-type degree of freedom is excited, mean-field-type excited states are created, but when the clustering degree of freedom is excited, clustered excited states are formed. This argument is explained in detail in Refs. [34, 35, 1, 36].

5. Ab initio approaches

5.1. Ground and the first-excited 0^+ states of ^{4}He

In an *ab initio* study of ^{4}He [37] the calculated ground and first-excited 0^+ states are compared. A very important result is that the probabilities of S, P and D components of the 0_2^+ state are very different from those of the 0_1^+ state but are almost the same as those in ^{3}H (and ^{3}He). For example $P_D = 14.08\%$ in 0_1^+ while it is 8.74% in 0_2^+, which is close to 8.70% in ^{3}H. This result and the fact that the 0_2^+ state is located near the $3N + N$ threshold suggest strongly that the 0_2^+ state has $3N + N$ cluster structure. In fact the reduced-width amplitude of the 0_2^+ state in the $3N + N$ channel is stretched in the outer spatial region while that of the 0_1^+ state is mostly in the inner spatial region. What is remarkable here is that a clustering effect is inferred not only from the density distribution but also from the existence of a tensor-force effect.

5.2. Ab initio cluster model calculation with microscopic R-matrix method

A very good example which shows that the effective-force approach cannot be a substitute of the *ab initio* approach is given by the $^2\mathrm{H}(d,\gamma)^4\mathrm{He}$ reaction in the astrophysical low-energy region. Fig. 3 shows an *ab initio* cluster model calculation with the microscopic R-matrix method for $^2\mathrm{H}(d,\gamma)^4\mathrm{He}$ [38]. In the effective-force approach, only relative D wave is existent and therefore in the astrophysical low-energy region the reaction rate becomes very small due to the centrifugal barrier. However, in the *ab initio* approach, relative S wave can also exist due to the tensor force, and that allows the reproduction of the data.

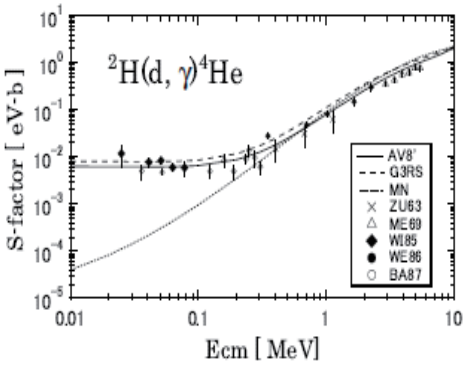

Figure 3. Astrophysical S-factor of the $^2\mathrm{H}(d,\gamma)^4\mathrm{He}$ reaction. Calculated results with the AV8' force (solid), with the realistic force G3RS (dashed) and the effective force MN (dotted) are compared with experiments.

5.3. Ab initio calculation of the Hoyle state

As we mentioned in the introduction, one of the representative features of present-day nuclear cluster physics is that the study of the Hoyle state of ^{12}C is an important and challenging frontier topic of the *ab initio* approach to nuclear physics.

The calculations in the no-core shell model (e.g., Ref. [39]) have difficulties in obtaining sufficiently low excitation energies of the Hoyle state and other cluster states although mean-field-type states are well reproduced. The Hoyle state was also studied recently by lattice simulation with effective field theory in Ref. [40]. Here we show the recent result by the quantum Monte Carlo approach [2]. In Fig. 4 we compare the density distributions $r^2\rho(r)$ of the Hoyle state (0_2^+) and the ground state (0_1^+) of ^{12}C as result from the quantum Monte Carlo calculation [2] (left figure) and the 3α THSR wave function [41] (right figure). Although the quantum Monte Carlo calculation has not fully converged for the Hoyle state, we clearly see a good correspondence of the density distributions between the quantum Monte Carlo approach and the 3α THSR wave function. Thus we see that the quantum Monte Carlo results are consistent with the gas-like structure of the Hoyle state.

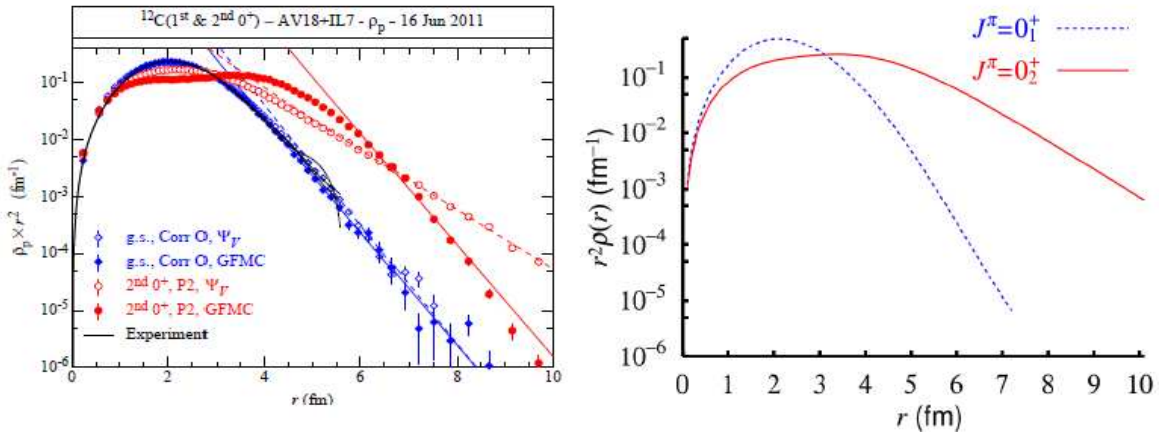

Figure 4. Comparison of the density distributions $r^2\rho(r)$ of the Hoyle state (0_2^+) and the ground state (0_1^+) of ^{12}C. Quantum Monte Carlo calculation (left figure) and 3α THSR wave function (right figure).

6. Summary

Marked progress has been made in many traditional and novel subjects of nuclear cluster physics. Four subjects were discussed in this talk. The developments in these subjects show the importance of clustering among the basic modes of motion of nuclear many-body systems.

References

[1] Horiuchi H, Ikeda K and Kato K 2012 *Prog. Theor. Phys. Suppl.* **192** 1
[2] Wiringa R B 2012 *Wilhelm und Else Heraeus-Seminar on Nuclear Ground-State Properties of the Lightest Nuclei: Status and Perspectives* p 501
[3] Kanada-En'yo Y and Kimura M 2010 *Lecture Notes in Physics* **818** 129
[4] Freer M *et al* 2001 *Phys. Rev.* C **63** 034301
[5] Descouvemont P and Baye D 2001 *Phys. Lett.* B **505** 71
[6] Kanada-En'yo Y and Horiuchi H 2003 *Phys.Rev.* C **68** 014319
[7] Ito M, Itagaki N, Sakurai H and Ikeda K 2008 *Phys. Rev. Lett.* **100** 182502

[8] Kimura M 2007 *Phys. Rev.* C **75** 034312
[9] Nakamura T *et al* 2006 *Phys. Rev. Lett.* **96** 252502
[10] Ikeda K, Myo T, Kato K and Toki H 2010 *Lecture Notes in Physics* **818** 165
[11] Tohsaki A, Horiuchi H, Schuck P and Röpke G 2001 *Phys. Rev. Lett.* **87** 192501
[12] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke G 2003 *Phys. Rev.* C **67** 051306(R)
[13] Funaki Y, Yamada T, Horiuchi H, Röpke G, Schuck P and Tohsaki A 2008 *Phys. Rev. Lett.* **101** 082502
[14] Funaki Y, Yamada T, Tohsaki A, Horiuchi H, Röpke G and Schuck P 2010 *Phys. Rev.* C **82** 024312
[15] Yamada T, Funaki Y, Myo T, Horiuchi H, Ikeda K, Röpke G, Schuck P and Tohsaki A 2012 *Phys. Rev.* C **85** 034315
[16] Fujita Y *et al* 2004 *Phys. Rev.* C **70** 011306(R); Kawabata T *et al* 2004 *Phys. Rev.* C **70** 034318
[17] Kawabata T *et al* 2007 *Phys. Lett.* B **646** 7
[18] Kanada-En'yo Y 2007 *Phys. Rev.* C **75** 024302
[19] Yamada T and Funaki Y 2010 *Phys. Rev.* C **82** 064315
[20] Sasamoto Y *et al* 2006 *Mod. Phys. Lett.* A **82** 2393
[21] Fujimura H *et al* 2004 *Phys. Rev.* C **69** 064327
[22] von Oertzen W, Freer M and Kanada-En'yo Y 2006 *Phys. Rep.* **432** 43
[23] Milin M and von Oertzen W 2002 *Eur. Phys. J.* A **14** 295
[24] Yamada T and Funaki Y 2008 *Int. J. Mod. Phys.* E **17** 2101
[25] Tohsaki A 2008 *Int. J. Mod. Phys.* E **17** 2106
[26] Itagaki N *et al* 2007 *Phys. Rev.* C **75** 037303
[27] von Oertzen W 2006 *Eur. Phys. J.* A **29** 133
[28] Kokalova Tz *et al* 2005 *Eur. Phys. J.* A **23** 19
[29] Yamada T and Schuck P 2004 *Phys. Rev.* C **69** 024309
[30] Ito M, Hirabayashi Y and Sakuragi Y 2002 *Phys. Rev.* C **66** 034307
[31] Zhou B *et al* 2012 *Phys. Rev.* C **86** 014301
[32] Bayman B F and Bohr A 1958/59 *Nucl. Phys.* **9** 596
[33] Yamada T, Funaki Y, Horiuchi H, Ikeda K and Tohsaki A 2008 *Prog. Theor. Phys.* **120** 1139
[34] Horiuchi H 2009 *Int. Journ. Mod. Phys.* A **24** 2205
[35] Horiuchi H 2010 *Lecture Notes in Physics* **818** 57
[36] Horiuchi H 2012 *Rom. Journ. Phys.* **57** 227
[37] Hiyama E, Gibson B F, Kamimura M 2004 *Phys. Rev.* C **70** 031001(R)
[38] Arai K, Aoyama S, Suzuki Y, Descouvemont P and Baye D 2011 *Phys. Rev. Lett.* **107** 132502
[39] Roth R, Langhammer J, Calci A, Binder S and Navrátil P 2011 *Phys. Rev. Lett.* **107** 072501
[40] Epelbaum E, Krebs H, Lee D and Meisner U 2011 *Phys. Rev. Lett.* **106** 192501
[41] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke G 2005 *Eur. Phys. J.* A **24** 321