Numerical model of two-dimensional flow dynamic of explicit splitting method

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Abstract. A solution of partial differential equations has been examined in the 2D Navier-Stokes Hydrodynamic equation by applying a numerical solution. This study aims to discuss the preparation of the circulation model by using a two-dimensional numerical model explicit splitting method. The solution can only be applied if it has fulfilled a stability requirement in determining the time and space steps. In modeling this method, the effects that cause instability such as boundary conditions and initial values of numerical equations was analyzed. Theresults of the rectangular discretization grid arrangement model shows a flow distribution value similar to the SUTRENCH model, which models the linear grid curve. The difference occurred only at the open boundary value in the east, however, the value is not significant to conclude the incompatibility with the verified model. In this study, it is also found that the velocity distribution in front of the jetty is greater than the current distribution in other areas, this is caused by narrowing area passed by the water mass, causing the discharge out of water larger.

1. Introduction

The availability of marine information in the territory of Indonesia is still limited, incomplete and uneven. This has resulted in several areas not been explored, while in other areas excessive exploitation has occurred. For this reason, marine research in the Indonesian still needs further development where the government is currently active implementing research programs and utilization of marine areas. In addition to the utilization for biodiversity transportation and production, the sustainability of the sea itself needs to be maintained.

One of the physical and dynamical phenomena that need to be understood in processes that affect coastal conditions is the currents circulation due to the dynamics of sea water [1, 2]. This condition is important to be known for various activities or events such as inter-island shipping, sea planning, marine engineering which includes energy use, national defense and nasiaonal development [3].

The sea water mass movements that have never stopped moving is necessary to know as the initial stage of starting planning at the sea. Understanding of sea water circulation system can be done in three ways, namely direct measurement, physical models and numerical models [4, 5]. Measurement directly takes a very long time, whereas by carrying out a physical model requires high costs and difficulties in taking the right scale. Therefore, a numerical model is an alternative way that is very helpful in solving

this problem. This study discusses the preparation of circulation models using two-dimensional numerical models explicit methods of spliting.

2. Materials and Method

2.1. Numerical motion formulation

The equation for the 2-D hydrodynamic model developed here is a set of simultaneous partial differential equations. To solve these differential equations a numerical computation method is needed that uses discretization of equations on one grid. In anticipating the preparation of the difference scheme, the equation is formulated in the form of flux.

To determine the equation for finite difference, subtraction and addition operations for finite difference methods are defined as follows [6, 7]:

$$\overline{F(x,y,t)}^x = \frac{F(x+\frac{\Delta x}{2},y,t) + F(x-\frac{\Delta x}{2},y,t)}{2} \tag{1}$$

$$\delta_x F(x,y,t) = \frac{F(x+\frac{\Delta x}{2},y,t) + F(x-\frac{\Delta x}{2},y,t)}{\Delta x} \tag{2}$$

$$\delta_x \overline{F(x,y,t)}^x = \frac{F(x+\Delta x,y,t) + F(x-\Delta x,y,t)}{2\Delta x} \tag{3}$$

and

$$\overline{F(x,y,t)}^{xy} \equiv \overline{\overline{F(x,y,t)}^x}^y \equiv \overline{\overline{F(x,y,t)}^y}^x \tag{4}$$

Bar and delta operators orm commutative and distributive algebra. The variable F (x, y, t) can be written as $F_{i,j}^n$ and Δx as well as Δy are constant horizontal grid intervals. By using the definition of the finite difference operator in equations (1), (2) and (3) the formulation of the vertically integrated regulator equation can be written as follows [6, 7]:

Continuity equation:

$$\delta_t \overline{\eta}^t + \delta_x(\overline{D}^x \overline{U}) + \delta_y(\overline{D}^y \overline{V}) = 0 \tag{5}$$

The equation of motion in the direction of x and y is:

$$\delta_t(\overline{\overline{D}^x \overline{U}})^t + \delta_x(\overline{\overline{D}^x \overline{U}}^x \overline{\overline{U}}^x) + \delta_y(\overline{\overline{D}^y \overline{V}}^x \overline{\overline{U}}^y) - f\overline{\overline{V}}^y D^x$$
$$+ g\overline{D}^x \delta_x \eta - \overline{F}_x^{n-1} = \Phi_x \tag{6}$$

$$\delta_t(\overline{\overline{D}^y \overline{V}})^t + \delta_x(\overline{\overline{D}^x \overline{U}}^y \overline{\overline{V}}^x) + \delta_y(\overline{\overline{D}^y \overline{V}}^y \overline{\overline{V}}^y) + f\overline{\overline{U}}^x D^y$$
$$+ g\overline{D}^y \delta_y \eta - \overline{F}_y^{n-1} = \Phi_y \tag{7}$$

The structure function of φx, φy is composed of the quantity provided by internal mode and calculated by integrating vertically in internal mode.

2.2. Boundary condition

Horizontal boundary conditions are applied to open boundaries in the form of waters and closed boundaries in the form of land. On the horizontal open boundary on the surface is given the following elevation boundary conditions:

$$\eta = \varsigma \tag{8}$$

where ς is the observation of the water level elevation. For horizontal boundary conditions of land, the semi-slip boundary conditions are written as follows:

$$\frac{\partial \bar{v}}{\partial n} = 0 \tag{9}$$

This boundary condition illustrates that the speed in the normal direction to the boundary condition of land is equal to zero.

2.3. The arrangement of grid

The grid arrangement (staggered grid) for external methods can be seen in Figure 1. Through the figure, it can be seen how the external model horizontal calculation system ([7]).
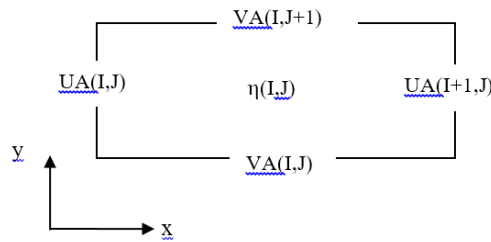

Figure 1. Grid for 2D external models

2.4. Time step determination

The determination of the time step for calculating momentum equations in an external (vertically integrated) mode is carried out based on Courant-Friedrichs-Levy (CFL) computing stability requirements with the following formulation [6]:

$$\Delta t_E \le \frac{1}{C_t} \left| \frac{1}{\partial x^2} + \frac{1}{\partial y^2} \right|^{-1/2} \tag{10}$$

where Ct = 2 (gH) 1/2 + Umax; Umax is the maximum speed that might occur. There are other options for time steps but for practicality, the CFL stability conditions are used which are quite strict. The time step limitation used by the model is generally 90% in accordance with this stability requirement. Internal mode has a longer time step as long as the external mode effect has been removed. The criteria for determining the time step used for this method are analogous to external mode, namely:

$$\Delta t_I \le \frac{1}{C_t} \left| \frac{1}{\partial x^2} + \frac{1}{\partial y^2} \right|^{-1/2} \tag{11}$$

where Ct = 2 C + Umax; Ct is the maximum speed of internal gravity wave which is generally in the order of 2 m/s, and Umax is the maximum advective speed. For certain ocean conditions, the comparison between internal time steps and external time steps $\Delta tI / \Delta tE = DTI / DTE$ is 10–80.

There is an additional limitation in determining the internal mode time step because it involves horizontal diffusion of momentum or is symbolized in scalar A = AM or A = AH. The criteria used is:

$$\Delta t_I \leq \frac{1}{4A} \left| \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right|^{-1}$$

(12)

The criteria for determining the internal mode time step due to rotation are:

$$\Delta t < \frac{1}{f} = \frac{1}{2\Omega \sin \Phi}$$

(13)

where AH is horizontal diffusivity, Ω is the angle of rotation of the earth and Φ is the latitude of the place. The two time step determination criteria above are not as strict as the time step determination criteria based on CFL criteria.

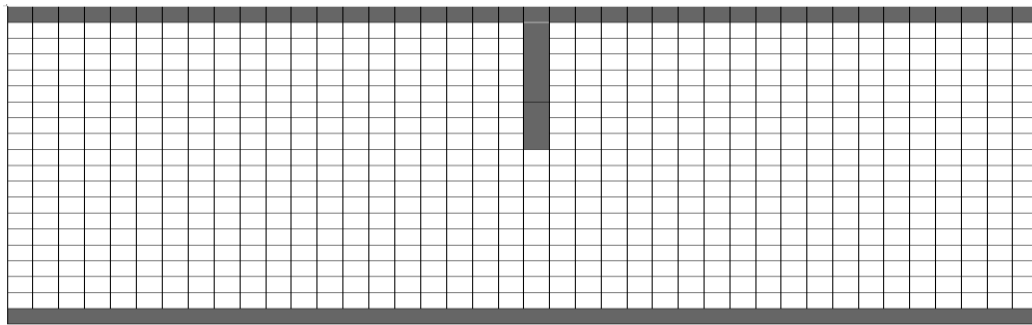## 3. Results and Discussion

3.1. Staggered model

Figure 2. Rectangular grid model used in this case.

The grid arrangement model (staggered grid) developed in this study is an unchanged grid model for each step space (rectangular grid model) (Figure 2), while the grid model carried out by van Rinj [8] is a linear grid curve model (Figure 3). However, making the grid model is done to facilitate calculations. Therefore, it is expected that the modeled results will not go too far.
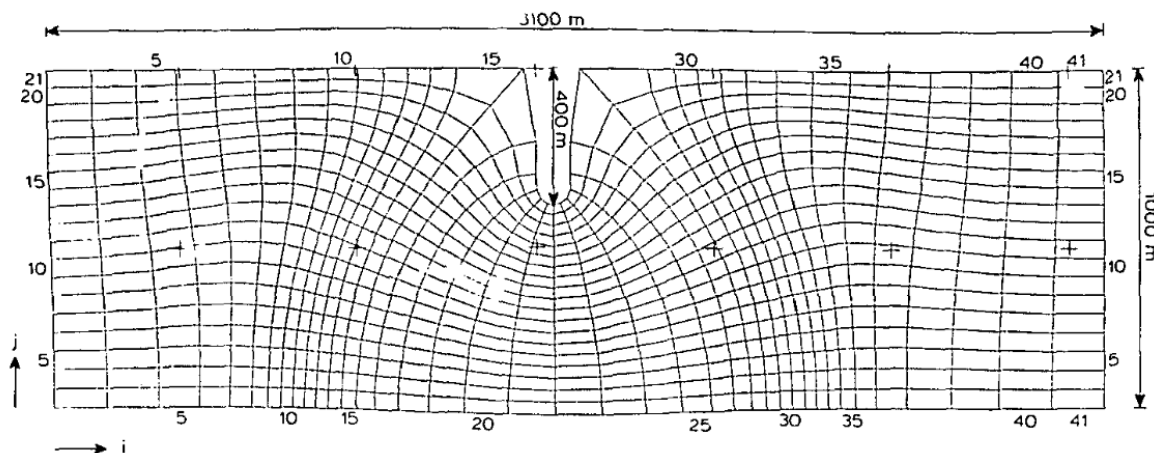

Figure 3. Model kurva linier grid yang dilakukanoleh Van Rijn (1987)[8]

From the developed grid arrangement model, it can be seen that the modeled grid is simple compared to the arrangement model developed by Van Rijn [8]. Each grid space (dots) made in this case has the same size, namely x = 50 meters and y = 77.5 meters.

3.2. Flow speed

This test is carried out to compare the current patterns produced between developed explicit splitting models with theVan Rijn [8] model. This test is made for a flow in a channel with a constant width (1000 m) and a constant depth of 6.18 meters in the west (Figure 3.). The canal is closed in North-South and open in West-East, which is partially covered by a dam in the middle of the North with a length of 400 m and a width of 100 m with a canal length of 3100 m. The water depth in the east is 6 m with a water density ρ = 1000 kg/m^3.

With the input given above, it is simulated for a while until a steady state is reached. The time taken to achieve a steady state with an explicit splitting model is 1000 times iteration with a time step (ΔtE) of 0.1 seconds. Figures 4 and 5 show the depth averaged velocity generated by using the explicit splitting model in steady state. The type of speed on the western boundary (inflow) is 0.65 m / s. The maximum flow velocity right in front of the jetty (dam) is 1.72 m/s, this value has the same result as obtained by the Van Rijn [8] model.

Table 1.shows the results obtained through the model made and the comparison with the results obtained by Van Rijn [8].
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Table 1. Comparison of the results of the study model and the SUTRENCH model [8]

No	Test case	Model		Verificat-ion
		Van Rijn	Study	
1	Flow speed	Umax=1.72 m/s	Umax=1.702 m/s	similar
2	Velocity in A in	U = 1.72 m/s	U = 1.70 m/s	
3	Velocity in B in	U = 1.26 m/s	U = 1.26 m/s	similar
4	Velocity in C in	U = 1.36 m/s	U = 1.35 m/s	
5	Velocity in D in	U = 1.39 m/s	U = 1.41 m/s	

The results obtained in Table 1 are the results of a cannal flow at the marked area point (Figure 4) which aims to verify the results obtained by Van Rijn [8] resulted in almost similar value.
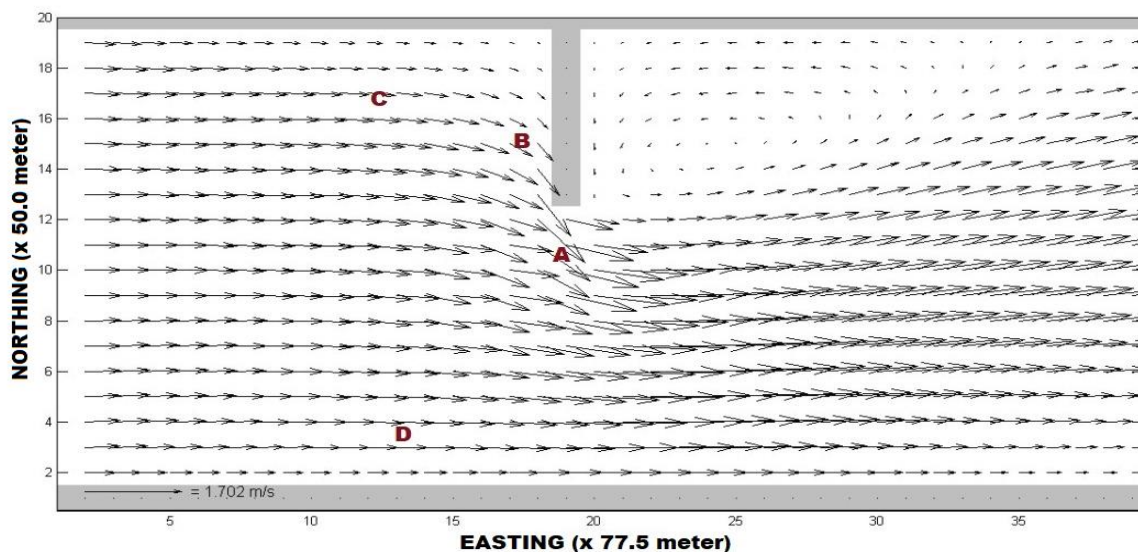

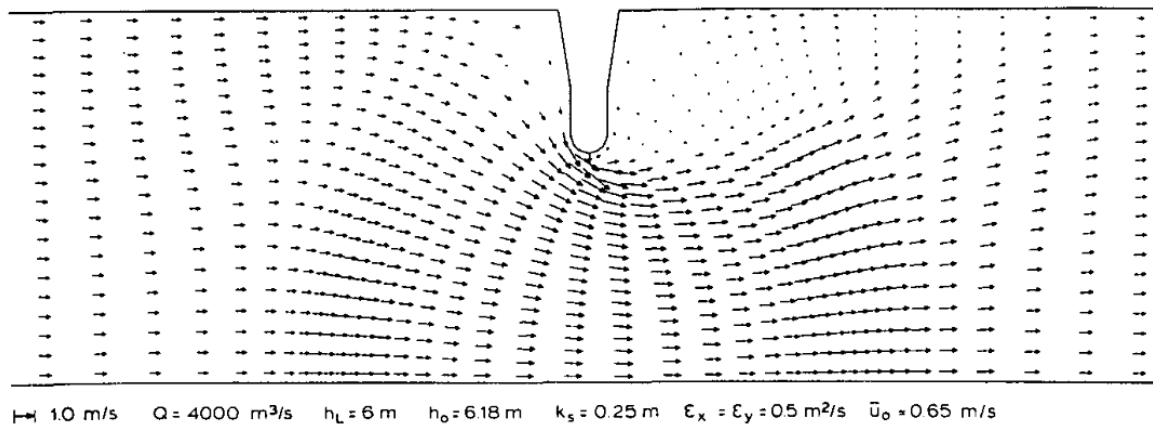**Figure 4**. Pattern of the West-East open channel current in a steady state state

Figure 5. Current vector produced by Van Rijn's model [8] where the maximum accelerating flow near the head of the dam is 1.72 m/s.

4. Conclusions

Based on the results of the model, it can be concluded that the development of explicit splitting model has been very good to be used as a reference model. This is marked by the similarity of verification results from the SUTRENCH model developed by van Rijn. For further purposes this model is feasible to be developed or tried in a real area where several factors still need improvement to see the validation of the model being developed.

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